The Development of Modeling Integers in a Translation/Relativity Context

The Tree Problem

Angela and her dad decided to plant a tree in their backyard. Angela found a spot in the yard that she wanted to plant the tree. Her dad told her to move the tree from that spot to the right ___ feet. Angela moved the tree to the position that her dad wanted, but then she decided to move it ___ feet to the left. Where did Angela eventually end up planting the tree?		
Session 1	Session 2	Session 3
(10, 12)	(6, 18)	(12, 20)

Directions of Activity

1. Solve the problem.
2. Write a number sentence that matches.
3. Which of these number sentences match the story? And, why?

Example of Cards Placed and Jace's Response

Card Placed	Jace's Response
-6 + -18 = □	"No, because that would equal negative twenty-four."
-6 + 18 = □	"Yeah, I agree with that."
6 – 18 = □	"Yeah, that one works…Because eighteen is greater than six and that's in the second place and six is in the front…They moved it to the right six feet. But then they moved it to the ... Oh no. I don't agree with it. Because that doesn't equal twelve. It equals negative twenty-four…I mean, not, negative twelve, I mean. Sorry."
18 – 6 = □	"Yeah, because I think that would equal twelve."
-18 – 6 = □	"No, because I think that would equal negative twelve."
6 + -18 = □	"No, because I think it would equal negative twelve too…Because that mean that it would go into the negatives. Instead of like (points at number line), I'd just stick with this. Because if you go to the right, I think that would be negative. And, if you go all the way to the left that would equal regular numbers."

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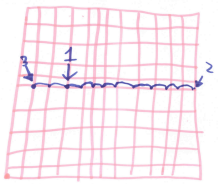
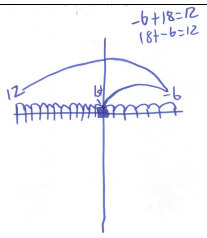
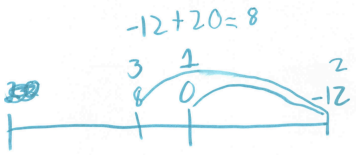
Components to Jace's Modeling of The Tree Problem

	Session 1	Session 2	Session 3
Drawing			
	Draws full coordinate plane. Numbers by positions rather than relative numbers on a scale. Uses whole numbers.	Draws coordinate plane, which is really used as a number line with each unit equally partitioned. Numbers with relative numbers on a scale. Uses negative integers.	Draws empty number line. Numbers with relative numbers on a scale. Uses negative integers.
Number Sentence	Not able to write number sentence.	Writes two number sentences and uses negative integers. -6 + 18 = 12 18 + -6 = 12	Writes one number sentence with negatives. -12 + 20 = 8
Cards that Matched	-10 + 12 = □ 12 – 10 = □	-6 + 18 = □ 6 – 18 = □ 18 – 6 = □	-12 + 20 = □ -20 – 12 = □ 12 + -20 = □

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