Name ______________________________ Date __________ Period _________

Drop and Catch

The Scenario

Your team will be dropping a ball from different heights and measuring the rebound each time. You will be studying the relationship between drop height and rebound height.

Prediction

Will the data (drop height, rebound height) represent a function?
Why or why not? If it will be a function, what type of function and why?

Linear	Quickly
Quadratic	Slowly
Non-Linear	Equivalent
Exponential	Initial Value
Constant	Rate of change

____

____

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The Experiment

Pick a drop height that is less than or equal to 100 cm. Hold the ball next to the meter stick so that the bottom of the ball is at the drop height. Release the ball and then **catch** it when it returns to its highest point. (This may take you a few times to practice.) Record the height of the bottom of the ball at its highest point.

Repeat this for four other starting heights, recording your drop and rebound height each time in the table to the right.

Drop Height	Rebound Height

Graph the Data

Determine which is the independent and dependent variable, label the axes and determine how to scale the axes and then graph your data from the table above.

Data Analysis

1) Decide if you think the situation should be represented by a discrete or continuous graph. If continuous, connect a single line or curve through your points. If discrete, leave the graph as a series of points.

2) Decide what, if any, pattern you see in the graph. Record your ideas below.

3) Discuss the following question and record your thoughts below: Is the graph linear, quadratic, exponential or other? Why? What about the data would make it this way? Note: Consider the scenario, table and graph in your answer.

4) Write an equation that comes close to matching your graph. Explain the meaning of any numbers in the equation.

5) Predict the rebound height of the ball were the ball dropped from 120 cm? Find and describe two methods you could use to make this prediction. Record your prediction on the chart paper in front of the room.

6) If the ball rebounded 80 cm, from what height was it dropped?

7) Explain how the graph and the equation of this activity would likely change if we used a tennis ball instead.

Comparing results

Using Inside-Outside circle, each group member will share their answers to questions 4, 5 & 7 with students from other groups.

Teacher Directions

Materials:
Bouncy Ball (Racquet ball works well)- 1 per group
Meter Stick (1 per group)

Objective:
Students will predict and the collect and analyze data to determine if the relationship between rebound height of a ball and its drop height represents a linear, exponential or other type of function.

Directions:
Pass out the activity sheet and have a student read the opening scenario. Show the students the ball and meter stick and, if needed, do a quick demonstration.

Give the students 3 minutes to silently complete the prediction section, encouraging them to use a few words from the vocabulary box. Give them a minute to share their predictions with an elbow partner.

Give the students a few minutes to read the directions for the experiment and then question to class to make sure they understand. Ask, "How many trials will you complete?" "What are you measuring?" Once you are sure the class understands the experiment, have the materials manager from each group come get a ball and meter stick. Encourage groups to drop the ball onto a hard surface such as a desk or cement floor. Note: If a group so desires, allow them to use a second meter stick to try a distance greater than 100 cm.

Once the data have been collected, have each student create the graph of their data. Using Think-Pair-Share, ask the class which is the dependent variable and which is the independent variable and why. After 30 seconds of silent think time, have them share their thoughts with an elbow partner and then select students at random to share. Guide the class to see that rebound height is dependent upon drop height; therefore drop height is the independent variable. Have the students label their axes and then give them a minute to decide how to scale their graph. Let them share ideas with their group, but allow them to make poor choices in scaling to be able to struggle and learn.

Have each group choose a leader to facilitate a discussion of the analysis questions. Allow 10 minutes for the groups to discuss, while you circulate and ask questions to help guide them. Next, use inside-outside line to have students share their thinking with other students not in their group. To do this, number off the groups so that half of the groups are number 1's and half are number 2's. Explain that they will be facing a partner and will reach across to shake hands, introduce themselves, and then share their thoughts to questions 3, 4 and 7. Have all the 1's line up in a straight line, taking their activity sheet with them. Then have the number 2's stand facing a 1, also bringing their activity sheet. Remind them to shake hands, introduce themselves and then have the number 1's share their ideas for questions 3, 4 & 7 for 60-90 seconds followed by the number 2's sharing their thoughts for 60-90 seconds. Ask all number 2's to put their hand up in the air and have each of them step 1 person to the right, with the person at the end walking down to the other end. Repeat the same process two more times, so that each student has at least 3 partners they explain and listen to thinking about for questions 3, 4 and 7. Have students return to their seats. Provide a brief summary as to why the data represent a linear function (see below) and then ask groups to share the equation they wrote for the data. The equations should all be similar! Ask what the meaning of the slope is in context, leading the class to note that it is the percent of the original height the ball rebounds. Note that this will be very important when doing the upcoming activity drop and watch.

Why the data represent a linear function:
Most recreational balls have a coefficient of restitution, meaning that they are built to rebound a given percent. This number is seen on a baseball or softball and is what makes the ball "regulation", as it would not be fair to have some balls that were "bouncier". Since the COR is a set percent, the change in rebound height is proportional to the change in drop height and thus the data are linear, with the COR being the constant of proportionality.

Name ______________________________ Date __________ Period _________

Counting Candies

The Scenario

You will be given a cup of candies which you will pour out onto a plate. You will count the total number of canides and then eat (or discard) all those who do not have a letter representing the candy brand showing. The candies that do have a letter showing will be placed back into the cup and poured out again. You will repreat the process of eating the ones without the letter showing, counting the ones with the letter showing and then putting those back into the cup for the next trial.

Prediction

Will the data (# of pours, # of candies with letter showing) represent a function?
Why or why not? If it will be a function, what type of function and why?

Linear	Quickly
Quadratic	Slowly
Non-Linear	Equivalent
Exponential	Initial Value
Constant	Rate of change

____

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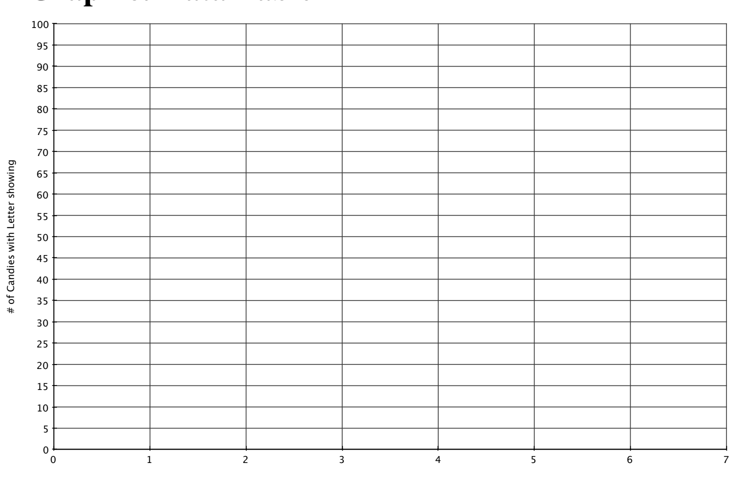
____

____

The Experiment

1. Count the number of candies you have to begin with and record this number under 0 pours.
2. Pour out the candies onto a plate and eat or discard all those that do not have a letter showing. Count those who do have a letter showing face up and record that number on the table under pour #1.
3. Put the candies with the letter showing back into the cup and pour them out onto the plate again. Eat or discard the ones without the letter showing face up and count the number with the letter showing; record this number in the table.
4. Repeat this process until you have less than 5 but MORE than 0 candies left.
5. Plot the data on the graph provided.

Graph & Data Table



# of Pours	# of candies with letter showing face up
0	
1	
2	
3	
4	
5	
6	
7	

Using a Graphing Calculator

1. Based upon the table and the graph, what type of function would you say is represented by the relationship between number of pours and number of candies with the letter showing? Why?

____

____

2. Use your graphing calculator to enter your data from the table. This will most likely happen by looking for STAT on the calculator and then Enter Data.

3. Once the data are entered, have your calculator determine the equation for the exponential function that most closely matches the data. To do this, choose STAT and then CALC and from the list, choose Exp Reg (for exponential regression). When you hit enter, you should have an equation. Record this equation below.

Equation for my data: ____

Enrichment: Use the graphing calculator to graph the equation you just entered to see how close the curve is to matching the data points.

Analysis Questions

After the equations from all groups are listed for you to see, study the equations and answer the following questions.

General Form of an Exponential Function: $y = a(b)^x$

1. For which value(s) of x does $f(x) = 0$? What does this mean in the context of this experiment?

____

2. What was your value for a in your equation? How similar is it to other groups? What do you think it represents in the candy scenario?

____

____

3. What was your value for b in your equation? How similar is it to other groups? What does b represent in the context of the candy scenario? Does your value for b make sense? Why or why not?

____

____

General Form of an Exponential Function: $y = a(b)^x$

a represents _______________________________

b represents _______________________________

Teacher Directions

Materials:
Cup (1 per pair)
Candies with a letter on one side (consider skittles or M&Ms)- about 90-110 per pair (count them out ahead of time and have them in the cups).
Plate- 1 per group (to pour candies on)
Graphing Calculators or App- 1 per pair

Objective:
By conducting and analyzing an experiment in which candies are poured out and only those with a letter showing are kept, counted and poured out again, students will come to understand what it means for data to represent an exponential function. Students will use graphing calculators to generate the equation that best matches their data and compare equations to come to an understanding of the value of *a* and *b* in the exponential function $y = a(b)^x$.

Directions:
Pass out the activity sheet and have a student read the opening scenario. Show the students the candies, and, if needed, do a quick demonstration.

Give the students 3 minutes to silently complete the prediction section, encouraging them to use a few words from the vocabulary box. Give them a minute to share their predictions with an elbow partner.

Give the students a few minutes to read the directions for the experiment and then question to class to make sure they understand. Ask, “”What is the first thing you need to do?” “Which candies do you count?” “What do you do with the candies that have a letter showing face up?” “How do you know when to stop?” Once you are sure the class understands the experiment, divide the class into pairs and have one student from each pair come get a cup with candies and a plate. Give the class about 10-15 minutes to complete the experiment.

Once the data have been collected, have each student create the graph of their data. Use think-pair-share to have students answer question #1. Guide the discussion to help the students see why the data is not linear and why it might be exponential.

Pass out the graphing calculators or have students take out the device they will use for this part of the activity. It is strongly recommended to go through Using a Graphing Calculator steps 2 and 3 as a class (while you model on the document camera). Once all pairs have entered their data, walk the class through how to calculate the exponential regression equation and then have each group share aloud or come record their equation on the board so that all students can see all equations.

Direct the class’ attention to the data analysis section. Explain the general form of an exponential function as shown in the textbox. Use think-pair-share (allow up to 2 minutes for think time followed by 2 minutes to share with a partner) for the three questions. Have multiple students share their observations and then summarize for the class the meaning of *a* (the starting or initial value) and *b* (the percent or ratio by which the data will change on each time). Close out the lesson by asking students to complete the final textbox with their own words.

A big idea for teachers and perhaps students: *b* should be about ½ because the probability that the candy will land "letter up" is EXACTLY ½, and when you perform (essentially) "coin flips" a large number of times, you expect that you will observe "about" ½ of them will be "letter up." In general, when you have an experiment that you perform a large number of times, the proportion of "successes" (however "success" is defined) will be approximately equal to the probability of "success" of one trial of the experiment. Here in this case, the "coin flip experiment", the probability of "success" is ½.

Name ________________________________ Date ___________ Period __________

Drop and Watch

The Scenario

Your team will be dropping a ball from 100 cm and watching measuring consecutive rebound heights. You will be studying the relationship between bounce number and rebound height. (The ball will continue to rebound until finally coming to rest).

Prediction

Will the data (Bounce #, rebound height) represent a function?
Why or why not? If it is a function, what type of function and why?

Linear	Quickly
Quadratic	Slowly
Non-Linear	Equivalent
Exponential	Initial Value
Constant	Rate of change

____

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____

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The Experiment

Hold a ball so that the bottom of the ball is at 100 cm from the ground. **Drop** the ball. **Watch** how high the bottom of the ball rebounds on the first, second, third and fourth bounces. (This may take you a few times and some division of labor among your teammates.)

Record the height of the bottom of the ball after each of the successive bounces.

Bounce Number	Rebound Height
0	100 cm
1	
2	
3	
4	

Graph the Data

Determine which is the independent and dependent variable, label the axes and determine how to scale the axes and then graph your data from the table above.

Data Analysis

1) Decide if you think the situation should be represented by a discrete or continuous graph. If continuous, connect a single line or curve through your points. If discrete, leave the graph as a series of points.

2) Decide what, if any, pattern you see in the graph. Record your ideas below.

3) Discuss the following question and record your thoughts below: Is the graph linear, quadratic, exponential or other? Why? What about the data would make it this way? Note: Consider the scenario, table and graph in your answer.

4) Write an equation that comes close to matching your graph (think back to the Counting Candies to write an equation). Explain the meaning of any numbers in the equation.

5) Predict the rebound height of the ball on the 8^{th} bounce? Find and describe two methods you could use to make this prediction.

6) Predict how the data and graph would change if the ball were dropped from 200 cm instead of 100 cm.

7) Explain how the graph and the equation of this activity would likely change if we used a tennis ball instead.

Comparing results

Using Inside-Outside circle, each group member will share their answers to questions 4, 5 & 7 with students from other groups.

Teacher Directions

Materials:
Bouncy Ball (Racquet ball works well)- 1 per group
Meter Stick (1 per group)

Objective:
Students will predict and the collect and analyze data to determine if the relationship between bounce or rebound number and rebound height represents a linear, exponential or other type of function. They will apply their learning about exponential functions to write an equation to match the data and explain the meaning of *a* and *b* in the equation.

Directions:
Pass out the activity sheet and have a student read the opening scenario. Show the students the ball and meter stick and, if needed, do a quick demonstration.

Give the students 3 minutes to silently complete the prediction section, encouraging them to use a few words from the vocabulary box. Give them a minute to share their predictions with an elbow partner.

Give the students a few minutes to read the directions for the experiment and then question to class to make sure they understand. Ask, "How is this experiment different than Drop and Catch?" "What are you measuring?" Once you are sure the class understands the experiment, have the materials manager from each group come get a ball and meter stick. Encourage groups to drop the ball onto a hard surface such as a desk or cement floor. Note: If a group struggles measuring, have them do a few trials focused only on a certain bounce number or consider letting them use a cell phone to video or take a picture of the ball.

Once the data have been collected, have each student create the graph of their data. Using Think-Pair-Share, ask the class which is the dependent variable and which is the independent variable and why. After 30 seconds of silent think time, have them share their thoughts with an elbow partner and then select students at random to share. Guide the class to see that rebound height is dependent upon bounce number; therefore the number of bounces is the independent variable. Have the students label their axes and then give them a minute to decide how to scale their graph. Let them share ideas with their group, but allow them to make poor choices in scaling to be able to struggle and learn.

Have each group choose a leader to facilitate a discussion of the analysis questions. Allow 10 minutes for the groups to discuss, while you circulate and ask questions to help guide them. Next, use inside-outside line to have students share their thinking with other students not in their group. To do this, number off the groups so that half of the groups are number 1's and half are number 2's. Explain that they will be facing a partner and will reach across to shake hands, introduce themselves, and then share their thoughts to questions 3, 4 and 7. Have all the 1's line up in a straight line, taking their activity sheet with them. Then have the number 2's stand facing a 1, also bringing their activity sheet.

Remind them to shake hands, introduce themselves and then have the number 1's share their ideas for questions 3, 4 & 7 for 60-90 seconds followed by the number 2's sharing their thoughts for 60-90 seconds.   Ask all number 2's to put their hand up in the air and have each of them step 1 person to the right, with the person at the end walking down to the other end. Repeat the same process two more times, so that each student has at least 3 partners they explain and listen to thinking about for questions 3, 4 and 7. Have students return to their seats. Provide a brief summary as to why the data represent an exponential function; i.e., the ball is rebounding a certain percent of it's previous height for each consecutive bounce. Ask groups to share the equation they wrote for the data and what the meaning of *a* and *b* are in the equation, relating this back to Counting Candies. The equations should all be similar! Ask how the value of *b* relates to the equations they found while doing Drop and Catch, and guide them to see that it is the slope or percent rebound from the linear function.

Close out this lesson by asking students to do a quick write to describe when data represent an exponential function.