Session #164
Understanding Quadratic Functions through Transformations

Carrie Hair, chair@washoeschools.net
Washoe County School District, Reno NV

Dr. Jenny Salls, jsalls@washoeschools.net
Washoe County School District, Reno NV

Opening Task:

From Illustrative Mathematics:

https://www.illustrativemathematics.org/content-standards/HSF/IF/B/4/tasks/1279

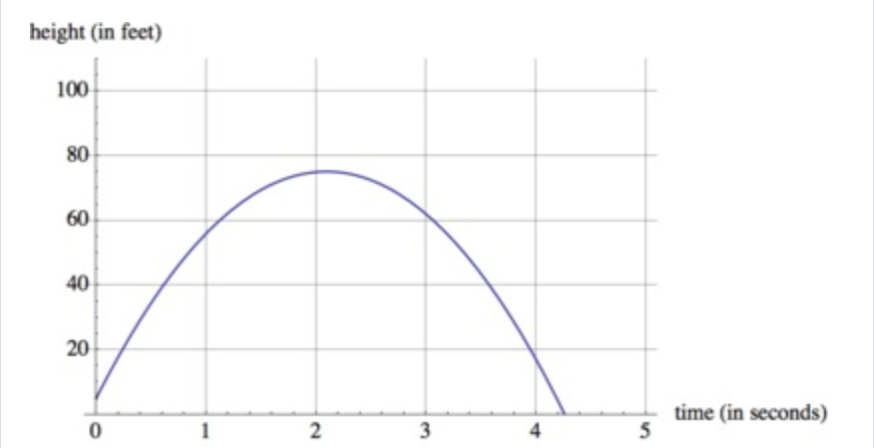
Suppose Brett and Andre each throw a baseball into the air.

The height of Brett's baseball is given by

$$h(t) = -16t^2 + 79t + 6$$

Where h is the height in feet and t is the time in seconds.

The height of Andre's baseball is given by the graph below:



Brett claims that his baseball went higher than Andre's, and Andre says that his baseball went higher.

Who is right?

How long is each baseball airborne?

Construct a graph of the height of Brett's throw as a function of time on the same set of axes as the graph of Andre's throw (if not done already), and explain how this can confirm your claims to parts (a) and (b).

Principles to Actions (pg. 11)

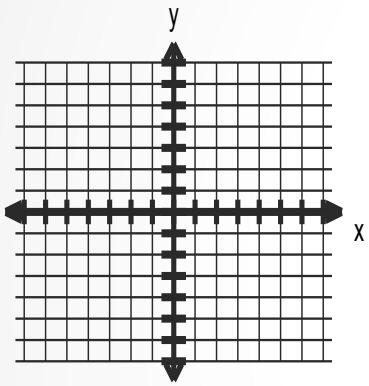
Beliefs about teaching and learning mathematics	
Unproductive beliefs	**Productive Beliefs**
…	…
The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve problems.	The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.
The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.	The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies, and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.
…	…

Quadratic Functions Unit Overview

Lesson 1	• Compare quadratic transformations with absolute value transformations.
Lesson 2	• Identify key aspects of quadratic functions.
Lesson 3	• Solve Quadratic Equations using graphs.
Lesson 4	• Apply key aspects of graphs to real world situations.
Lesson 5	• Solve quadratic equations using square roots.
Lesson 6	• Identify the vertex and zeros of quadratic functions from vertex form.
Lesson 7	• Identify the vertex and zeros of quadratic functions and graph from standard form (factorable).
Lesson 8	• Convert vertex form to standard form. • Convert standard form to vertex form by completing the square. • Solve quadratic equations by completing the square.
Lesson 9	• Derive and use the quadratic formula from completing the square. • Understand the structure of the quadratic formula
Lesson 10	• Use the quadratic formula to solve quadratic equations. • Use the quadratic formula to determine zeros of quadratic functions • Identify the number of real zeros. • Choose the best method to solve quadratic equations
Lesson 11	• Modeling with quadratic functions.

Lesson 1: Transformation example

Graph: $f(x) = -1/2|x - 1| + 3$



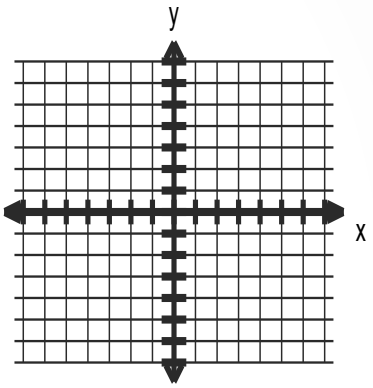
Domain:

Range:

Describe the transformations

Graph: $f(x) = -1/2(x - 1)^2 + 3$

x	y
-1	
0	
1	
2	
3	



Domain:

Range:

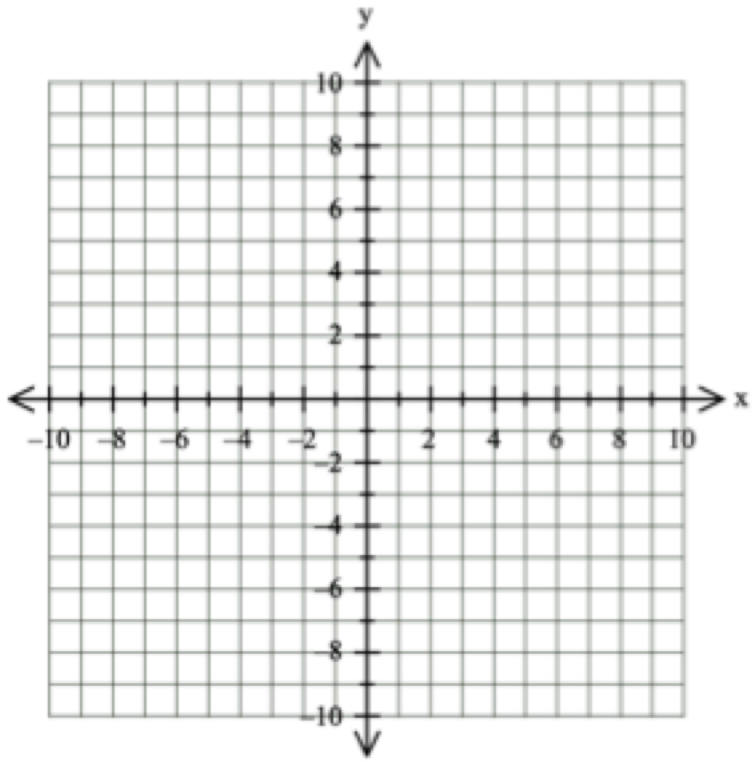
Describe the transformations

How are the graphs of absolute value functions and quadratic functions the same? Different?

Part 2: Understanding Quadratic Functions Graphically

Part 1: Graph the following quadratic function: $f(x) = (x-3)^2 - 4$.

The name for the graph of quadratic function is a _____.



Label the parts of a graph of a quadratic function:

Vertex

Line/Axis of Symmetry

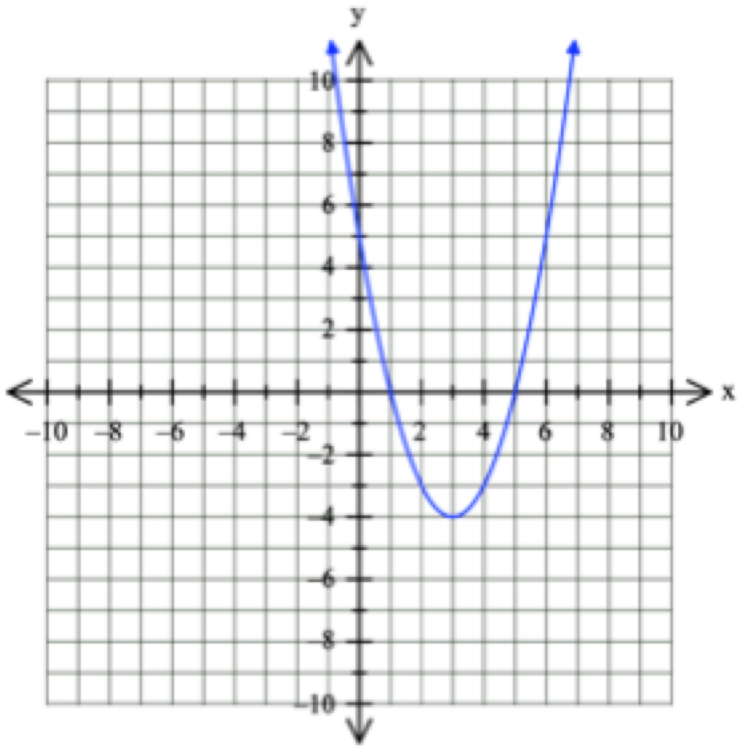
Real Zeros

y-intercept

Part 2: Understanding Quadratic Functions Graphically

Part 1: Graph the following quadratic function: $f(x) = (x-3)^2 - 4$.

The name for the graph of quadratic function is a _____Parabola_____.



Label the parts of a graph of a quadratic function:

Vertex: (h, k), minimum/maximum point

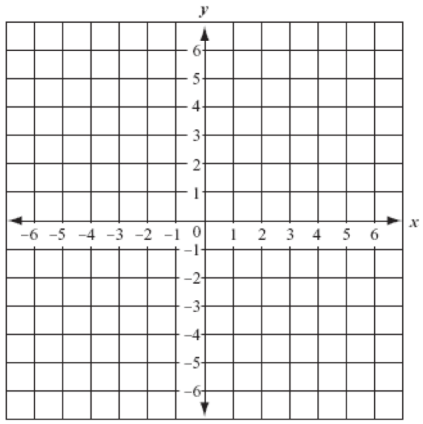
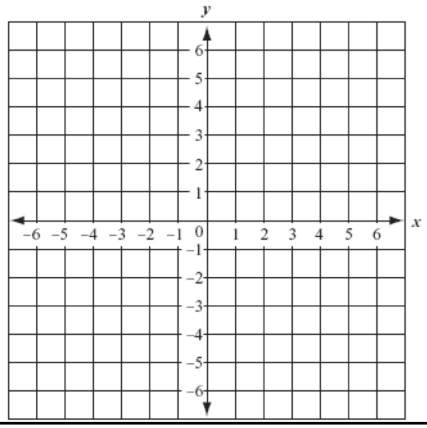
Line/Axis of Symmetry vertical line of reflection, x = h, comes from the x-coordinate of the vertex

Real Zeros x-intercepts, where the graph crosses the x-axis, the y-coordinate is always.

y-intercept where the graph crosses the y-axis, the x-coordinate is always.

Part 2: (continued)

Try these!

$f(x) = (x+3)^2 - 1$	$f(x) = -2(x-3)^2 + 8$
Predict Vertex: Axis of Symmetry: Real Zeros:	Predict Vertex: Axis of Symmetry: Real Zeros:
Verify Graphically 	Verify Graphically 

Part 2: (continued)

Think about it!

1. What relationships do you see between the following:

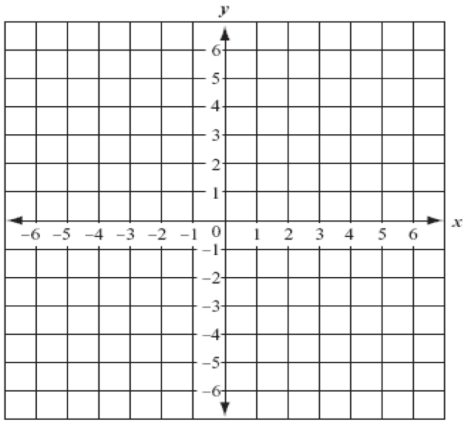
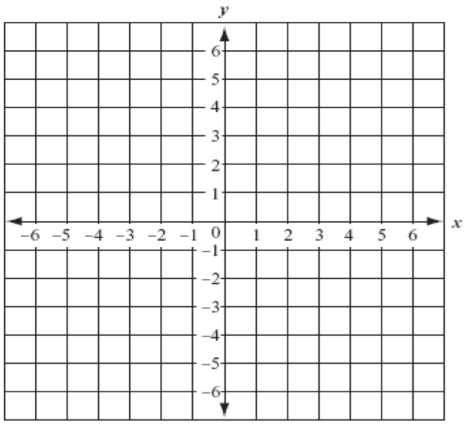
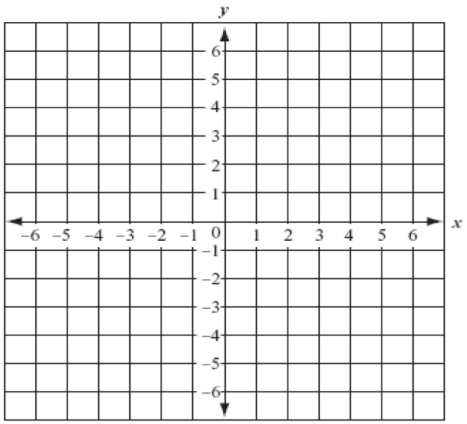
Vertex and the Line of Symmetry

Vertex and the Real Zeros

Line of Symmetry and the Real Zeros

Part 2: (continued)

Part 2: Graph the following quadratic functions

$f(x) = -\frac{1}{2}(x+1)^2 - 2$	$f(x) = (x+1)^2$	$f(x) = (x-2)^2 - 4$
		
# of real zeros:	# of real zeros:	# of real zeros:

Think about it!

1. How can you predict the number of real zeros from the equation?

Background: Solving Absolute Value Equations by graphing

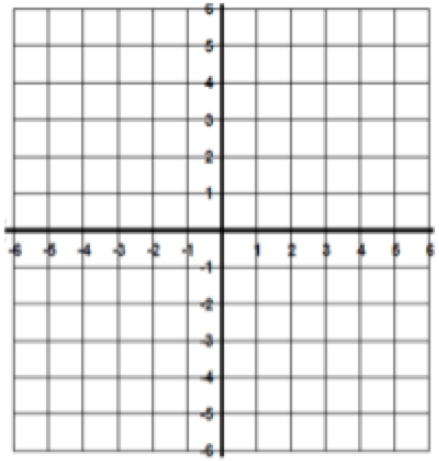
Part 2: Solving Absolute Value Equations graphically and algebraically

Solve the following absolute value equations graphically

7. $|x-2|=4$

$f(x)=$

$g(x)=$

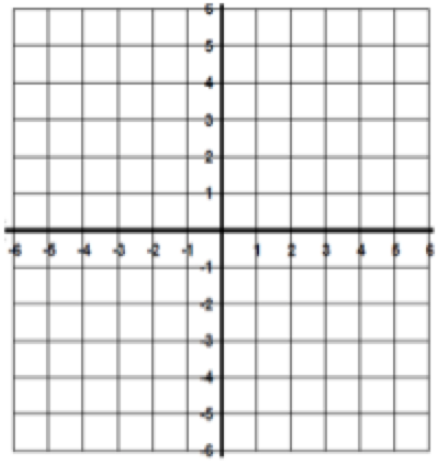


The solution(s):

8. $-|x-1|+2=5$

$f(x)=$

$g(x)=$

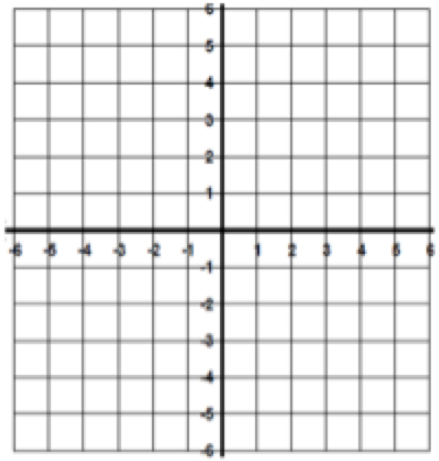


The solution(s):

9. $\frac{1}{2}|x-1|+2=-x+4$

$f(x)=$

$g(x)=$



The solution(s):

Part 3: Connecting Quadratic Functions to Solving Quadratic Equations

(HSA.REI.B.4, HSA.REI.D.11)

Achieve the Core (Mini-Assessment):

http://achievethecore.org/page/976/quadratic-equations-mini-assessment

The table below contains single equations in a single variable. Check the appropriate box to show whether there is no real solutions, exactly 1 real solution, or exactly 2 real solutions.

		No Real Solutions	Exactly 1 Real Solution	Exactly 2 Real Solutions
a.	$(a+5)^2 = 25$			
b.	$(n-5)^2 = 25$			
c.	$(z+5)^2 = -25$			
d.	$(x-5)^2 = 0$			
e.	$16-(l+5)^2 = 25$			
f.	$(f+1)^2 = (f+2)^2$			
g.	$5b^2 = 5b^2 + 1$			

Part 4: The "In Between"

This is a 3 week unit. From this point we focus on the algebra…

- Find the vertex and zeros algebraically from vertex form and solve quadratic equations within this structure.
- Find the vertex and zeros algebraically from factorable standard form and solve quadratic equations within this structure.
- Learn about completing the square as a strategy for converting between forms, solving quadratic equations, and its connection to the quadratic formula.

BUT … Always connecting it back to a graphical interpretation!

Lesson 9: Understand the structure of the quadratic formula

Vertex Form: $f(x) = a\left(x + \frac{b}{2a}\right)^2 - \frac{(b^2 - 4ac)}{4a}$

vertex: $\left(\frac{-b}{2a}, -\frac{b^2-4ac}{4a}\right)$

Quadratic Formula: $x = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Part 5: Bringing it All Together

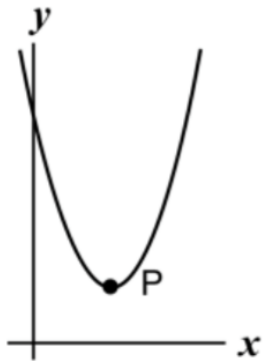
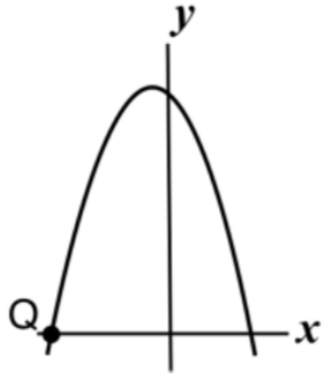
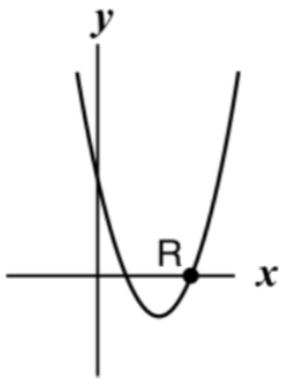
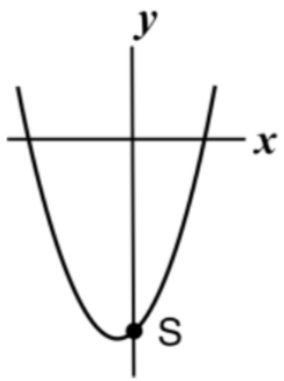
The Mathematics Assessment Project

http://map.mathshell.org/lessons.php?unit=9245&collection=8

Quadratic Functions

1. Here are 4 equations of quadratic functions and 4 sketches of the graphs of quadratic functions.

A. $y = x^2 - 6x + 8$	B. $y = (x-6)(x+8)$	C. $y = (x-6)^2 + 8$	D. $y = -(x+8)(x-6)$

1.	2.	3.	4.
			

a. Match the equation to its graph and explain your decision.

b. Write the co ordinates of the points: P (....,....) Q(....,....) R (....,....) S (....,....)

Questions????