

# Moving Beyond Balanced Scales: Contextualizing the Concepts of Equality



**Lincoln Campbell**

Associate Director of Mathematics

asc<sup>e</sup>nd  
CHARTER SCHOOLS

asc<sup>e</sup>nd

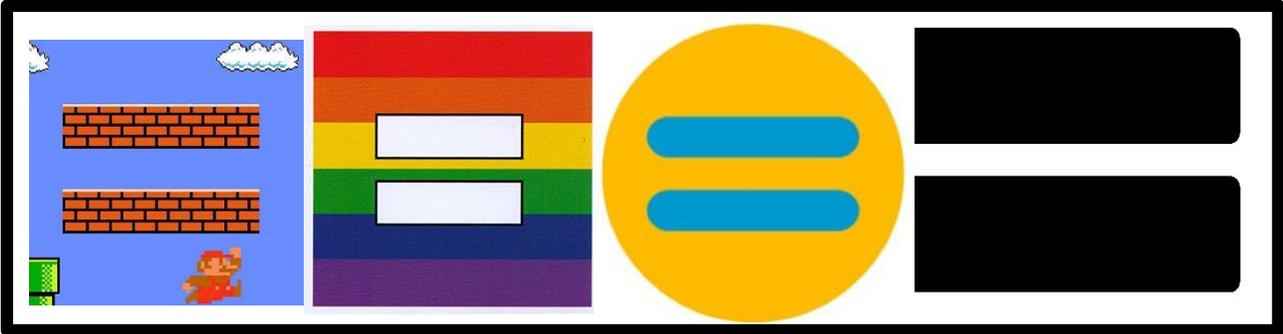
# Overview

- I. Why is Equality Important?
- II. How well do Students Understand Equality?
- III. How do Misconceptions about Equality Form?
- IV. How can we Address Equality Misconceptions?



# I. Why is Equality Important?

# The Importance of Equality



## **II. How well do Student Understand Equality?**

# Do Now

**Think, Pair, Share: How would students in your class respond to the following question:**

*What number would you put in the box to make this a true number sentence?*

$$8 + 4 = \square + 5$$

# Actual Student Responses

$$8 + 4 = \square + 5$$

Student Responses / Percent Responding				
	$\square = 7$	$\square = 12$	$\square = 17$	$\square = 12$ and $17$
1 <sup>st</sup> and 2 <sup>nd</sup> Grade	5%	58%	13%	8%
3 <sup>rd</sup> and 4 <sup>th</sup> Grade	9%	49%	25%	10%
5 <sup>th</sup> and 6 <sup>th</sup> Grade	2%	76%	21%	2%

Carpenter, Frank, Levi et al, Thinking Mathematically 2003

# What Students Think about the Equal Sign

## Turn and Talk:

Anticipate how your students would answer each question.

The following questions are about this statement:

$$3 + 4 = 7$$

↑

- (a) The arrow above points to a symbol. What is the name of the symbol?
- (b) What does the symbol mean?
- (c) Can the symbol mean anything else? If yes, please explain.

# Actual Student Responses

**Think, Pair, Share:** How would you categorize the student responses in groups A, B and C?

<p><b>A - RELATIONAL</b></p> <ul style="list-style-type: none"><li>• “It means that what is to the left and right of the sign mean the same thing” (Grade 6)</li><li>• “The same as, same value” (Grade 7)</li><li>• “The left side of the equals sign and the right side of the equals sign are the same value” (Grade 8)</li></ul>	<p><b>B - OPERATIONAL</b></p> <ul style="list-style-type: none"><li>• “What the sum of the two numbers are” (Grade 6)</li><li>• “A sign connecting the answer to the problem” (Grade 7)</li><li>• “The total” (Grade 8)</li><li>• “How much the numbers added together equal”</li></ul>
<p><b>C - OTHER</b></p> <ul style="list-style-type: none"><li>• “It means equal to”</li><li>• “3 plus 4 equals 7”</li></ul>	

Knuth, Alibali, Hattikudur et al, Mathematics Teaching in the Middle School 2008

# **III. How do Misconceptions about Equality Form?**

# Introducing the Equal Sign

- **When is the equal sign first introduced in Common Core State Standards?**

- **Work with addition and subtraction equations.**

CCSS.MATH.CONTENT.1.OA.D.7

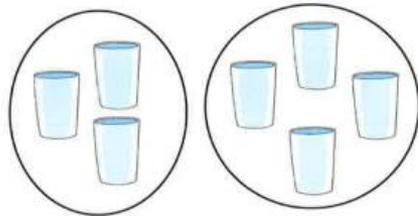
Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false?  $6 = 6$ ,  $7 = 8 - 1$ ,  $5 + 2 = 2 + 5$ ,  $4 + 1 = 5 + 2$ .

# Analyzing Concepts of Equality in Textbooks

add +

equals =

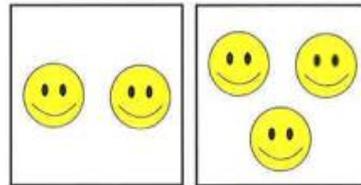
Count the objects in each set. How many are there in all?



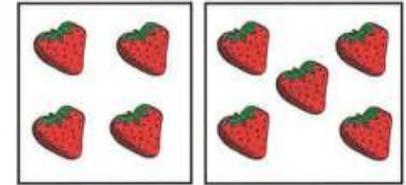
$$3 + 4 = \boxed{7}$$



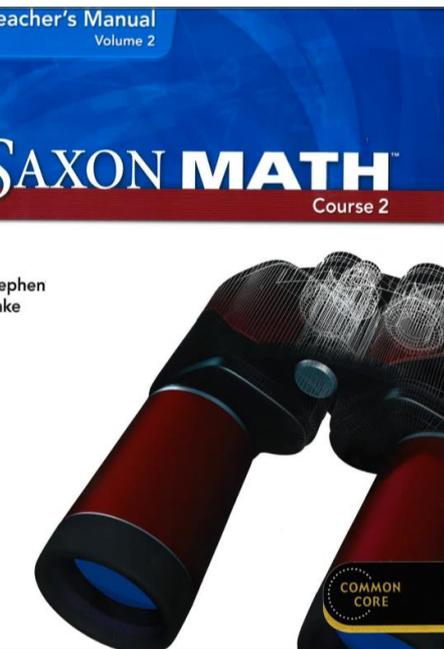
Add. Count how many there are in all.



$$2 + 3 = \boxed{\phantom{00}}$$



$$4 + 5 = \boxed{\phantom{00}}$$



## 2 New Concepts

### Instruction

Write this sentence on the board:

*Lucy said Ben went swimming.*

**"Who went swimming, Lucy or Ben?"**

Help students see that without punctuation there is no way of knowing.

"Lucy," said Ben, "went swimming."

Lucy said, "Ben went swimming."

Help students make the connection between the elements of grammar and the elements of mathematics.

#### Grammar

phrases  
sentences  
punctuation

#### Mathematics

expressions  
equations  
order of operations

Tell students that in this lesson, they will be translating phrases into expressions and sentences into equations.

(continued)



## 3.1 Making Addition Stories

### Objectives

- Understand the meaning of addition.
- Make number stories for addition.
- Write addition equations using + and =.

### Notes

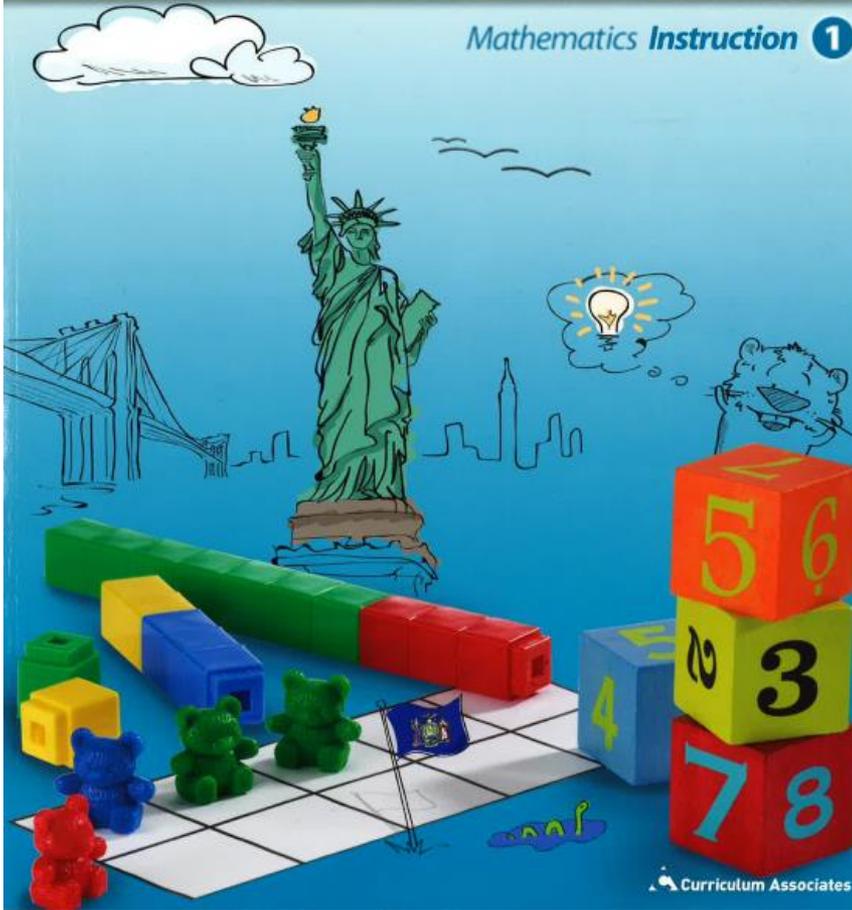
In this chapter, students will review the meaning of addition and write addition equations (number sentences) using the symbols or signs "+" and "=" to represent the mathematical process of finding the total number of objects in two sets.

The "+" sign means "put together" and the "=" sign means "is the same as." Although you generally read an addition equation such as, " $5 + 2 = 7$ " as "5 plus 2 equals 7," this equation represents a variety of situations, such as "5 and 2 more make 7" or "When you put 5 and 2 together, you get 7."

The equal sign means that both sides evaluate to the same number and does not dictate that what comes after it has to be the answer to the addition problem. When representing the types of addition stories your students will encounter in this chapter, the equal sign is written after the parts, as generally you write the calculations you must do first (i.e.,  $2 + 5$ ), and then find the answer and write it. The equal sign can also be written before the parts:  $7 = 2 + 5$ . You can also write an expression on either side of the equal sign:  $8 + 2 = 7 + 3$ .

# Ready<sup>®</sup> New York CCLS

Mathematics Instruction 1



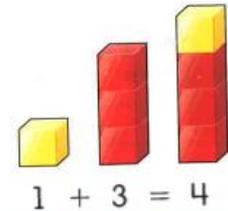
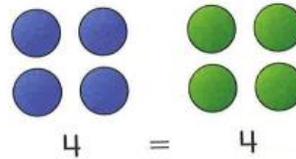
## Understand The Equal Sign

What does = mean?

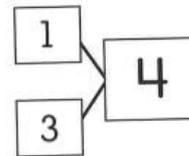


= is the **equal sign**.

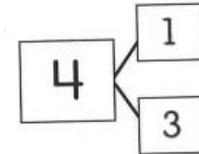
= means **is the same as**.



**Think** The total can go to the left or right of = .



$$1 + 3 = \underline{\quad}$$



$$\underline{\quad} = 1 + 3$$

**Talk About It**

$$4 + 2 = 5$$

$$5 = 2 + 3$$

Are both number sentences true?  
How do you know?

# **IV. How can we Address Equality Misconceptions?**

# Concepts of Equality Routine

- Students are engaged in a discussion about the meaning of the equal sign.
- The context of this discussion is true/false and open number sentences.
- The number sentences provide a focus for students to articulate their ideas and to challenge their conceptions.
- The discussions assist in developing ways of thinking and communicating that embody the principles of algebraic reasoning.
- Students articulate mathematical principles that often are not explored or stated.
- Students must justify the principles that they propose in ways that convince others, and they must recognize and resolve conflicting assumptions and conclusions.

# Video of David

**While you watch, listen to David's explanation to see determine how he understands the equal sign.**

**What teacher moves did the teacher use to push relational and algebraic thinking?**

# Concepts of Equality Routine

Alex has \$20.50 in his bank account and makes a \$7.25 withdrawal. Greg has \$19.50 in his bank account and makes a \$6.25 withdrawal. Do Alex and Greg both have the same amount of money in their accounts now?

$$20.50 + (-7.25) = 19.50 + (-6.25)$$

**Apply and extend previous understandings of operations with fractions.**

CCSS.MATH.CONTENT.7.NS.A.1

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

# Equality Strings

**Running Equality Strings (wrong):**

$$3 + 5 = 8 + 2 = 10 + 5$$

vs.

**Valid Equality Strings (right):**

$$3 + 5 = 6 + 2 = 8 + 0$$

# Adding Context and Movement

If the left side of a bench press has two 10 pound weights and two 5 pound weights and the right side of the bar has one 25 pound weight, how much more weight should be added so that the weight on both sides of the bar are equal?



This could be expressed as:

$$10 + 10 + 5 + 5 = 25 + x$$

# Key Takeaways

- Instead of “equals” say “is the same as”
- Instead of saying  $x$  or box, say “what number?” or “some number”
- Use Concepts of Equality Routine Strings
- Use Equality Strings
- Make meaning through Context

# Resources

## Concepts of Equality Routine Strings

San Diego Public Schools, Mathematics Routines Bank  
September 2004 ([online link](#))

Carpenter, Thomas P., Megan Loef. Franke, and Linda Levi. *Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School*. Portsmouth, NH: Heinemann, 2003. Print.

Knuth, Mcneil, Stephens, et al. "The Importance of Equal Sign Understanding in the Middle Grades." *Mathematics Teaching in the Middle School* 13.9 (2008): n. pag. Web.

# **Closing Thoughts: Connecting Liberal Arts to Math**

# Mathematical, Moral and Legal Equality

It is fitting that these questions of equality should differ from one another and yet be the same, for equality is itself a relationship that uniquely straddles the gap between “different” and “same”: things that are equal, being distinct things, are necessarily different; and yet, being equal, they are also the same. Moreover, the general concept of equality that underlies particular relationships of equality has both a fixed element, which remains the same in all its usages (at least regarding persons and things we perceive through the senses), and a variable term, which can differ greatly from one statement of equality to another. The key to understanding the meaning and rhetorical force of ‘equality’ in law and morals, I believe, lies in identifying the kinds of variable terms that enter into moral and legal statements of equality.

Westen, Peter Speaking of Equality: An Analysis of the Rhetorical Force of ‘Equality’ in Moral and Legal Discourse, Princeton University Press, Jul 14, 2014

# Questions?

*Find out more about how we do math at Ascend!*



The Ascend approach

## Lincoln Campbell

Associate Director of Mathematic

[lcampbell@ascendlearning.org](mailto:lcampbell@ascendlearning.org)