What's in Your Numeracy Toolkit? NCTM 2016

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Important Number Sense Vocabulary:

Counting (verbal): Knows and says the number sequence

1-1 Correspondence: Says one number name for each object counted

Cardinality:

Conservation:

Inclusion:

=: Understands "the same as" in one way or balance of

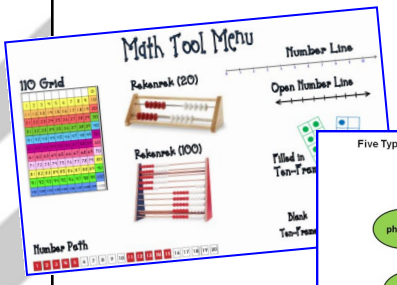
Unitizing:

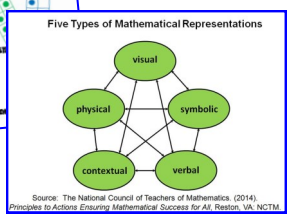
Magnitude: Understanding about relationships of numbers (distance from zero, which is bigger, use mental number line)

Subitizing:

Part-Part-Whole: Once students begin to understand inclusion, they begin to consider parts of a number. (6 is made up of 5 and 1, 4 and 2, 3 and 3).

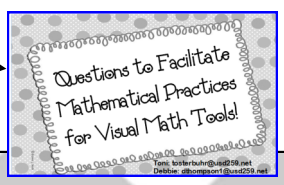
Compensation: Children begin to see parts of the whole and then are able to compensate. (5 + 1 equals 6 then I also know that 4 + 2 equals 6 because 4 is one less than 5 and 2 is one more than 1; 1 was removed from 5 and given to 2).





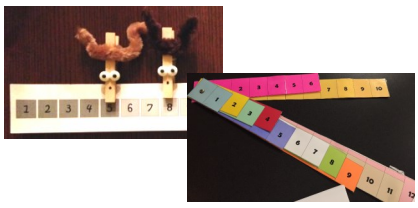
FREE: Check out the PDF posted under our online session containing:

- templates of all the tools
- Addition & Subtraction Situations Table
- Math Tool Menu
- Question Cards
- This handout & more…



Number Sense Tools	* Provides teacher insight into students current number concept developments based on interaction * Develops students ability to visualize ("see") and think about number * Allows students a way to see multiple ways to approach a problem * Decontextualizes and solves story situations * Develops Fluency (Flexibility, Accuracy & Efficiency) *Essential Understanding* Series: http://www.nctm.org/catalog/productsview.aspx?id=129	**Free Number Talks PPTs:** http://schoolwires.henry.k12.ga.us/Page/37070 **Situation Task Cards:** http://www.k-5mathteachingresources.com/ http://gregtangmath.com/materials **Pairs wonderfully with *Number Talks* and *Classroom Discussions*:** http://mathsolutions.com/common-core-support/
Rekenrek 2 rows of 10 beads, 5 red, 5 white	* Group counting (move away from always counting by ones) * Decomposition-combinations of numbers (1-9 & teen numbers) * Subitizing, Inclusion, Unitizing * Compensation strategies * Associative and communitive properties	**Interactive:** http://www.didax.com/rekenrek/app **Rekenrek Activities:** http://catalog.mathlearningcenter.org/store/product-6720.htm
Five Frames 1x5 array **Ten Frames** 2x5 array	* Subitizing * 1-1 counting * Benchmark numbers 5 and 10 * Teen Numbers * Decomposition and composition (part-part-whole, missing parts) * Comparison of quantities, pre-place value, double and near doubles	**Interactive:** http://illuminations.nctm.org/ **Ten Frames resources:** Didax.com Google: Doug Clements, subitizing

110 Grid or Hundreds Board
10x10 array, numbers in squares
	* Repeated structure of numbers (algorithm of +1)
	* Benchmark numbers (1, 5, 10, 20, 25, 50, 100, etc)
	* Magnitude of number
	* Pre– and place value (base-ten system)
	* Connect number/name (language of count can be so confusing)
	  * Teen means ten; first two "teen" words do not have –teen, thirteen instead of threeteen, fifteen instead of fiveteen, seven-teen does not look like 17.
	  * Pattern switches after 20: Words for 20-100 says tens first, then ones
		Interactive:
		http://www.abcya.com/interactive_100_number_chart.htm
		http://nlvm.usu.edu/en/nav/vlibrary.html
		 

Number Path Discrete, count model 1 2 3 **Layered Number Path**	* Counting forward and backward (and from other numbers besides one) * Comparing (more, less and equal) * Magnitude (distance from a number)	
Number Line Continuous, length	* Counting forward and backward * Comparing (more, less and equal) * Magnitude (distance from a number) * Parts of a whole (fractions)	http://nlvm.usu.edu/en/nav/vlibrary.html 

Common Addition and Subtraction Situations (pg 88 in CCSS)

Shading taken from OA progression

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Taken from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3

	Total Unknown	Addend Unknown	Both Addends Unknown[1]
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

Blue shading indicates the four Kindergarten problem subtypes. Students in grades 1 and 2 work with all subtypes and variants (blue and green). Yellow indicates problems that are the difficult four problem subtypes or variants that students in Grade 1 work with but do not need to master until Grade 2.

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Questions to Facilitate Mathematical Practices for Visual Math Tools!

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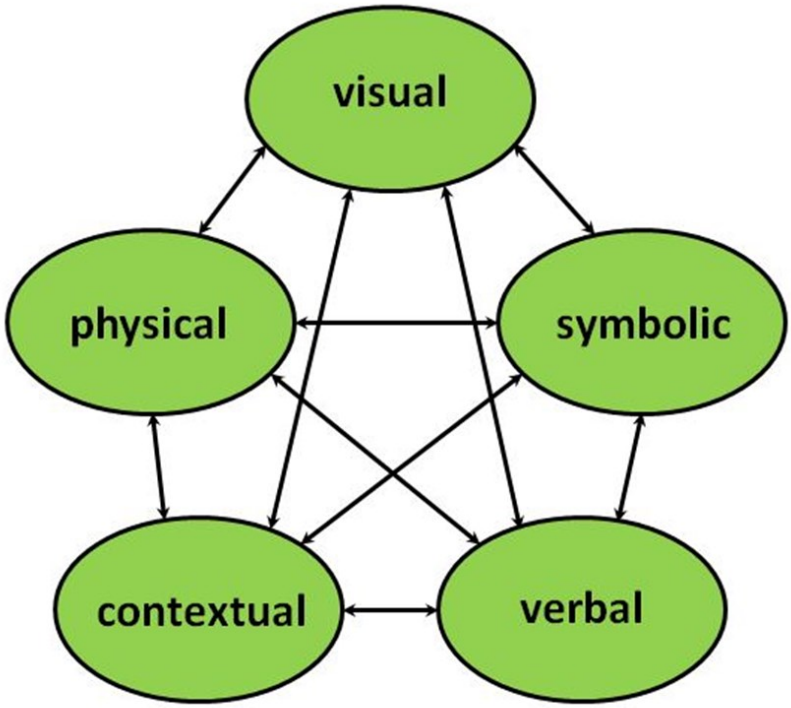
1

Five Types of Mathematical Representations



Source: The National Council of Teachers of Mathematics. (2014).
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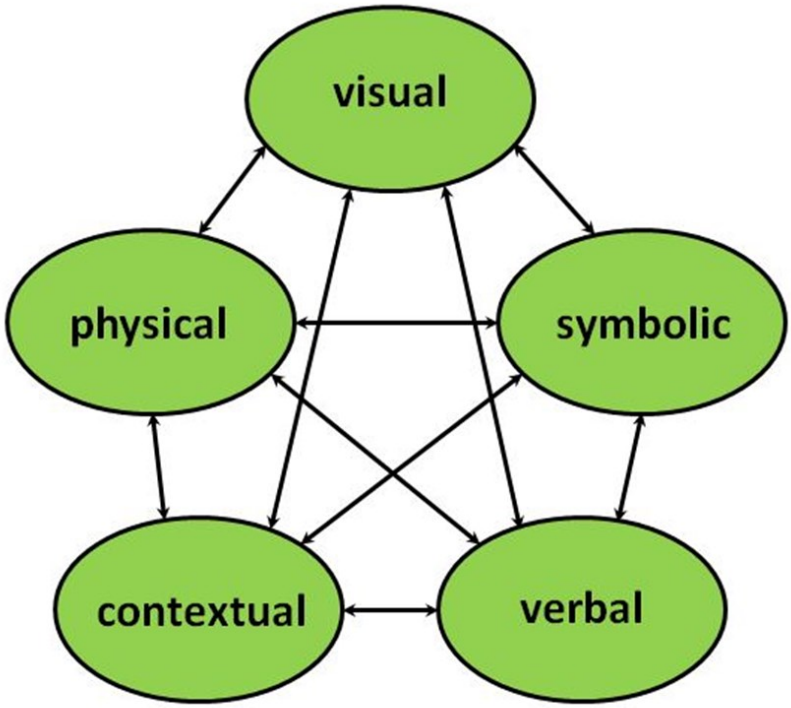

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Math Tool Menu

110 Grid

									0
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11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110

Rekenrek (20)

Rekenrek (100)

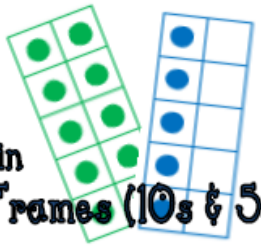
Number Path

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

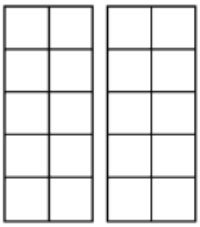
Number Line

Open Number Line

Filled in Ten-Frames (10s & 5s)



Blank Ten-Frames



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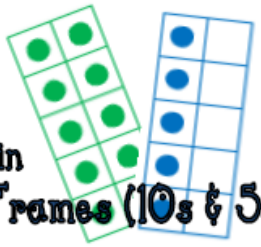

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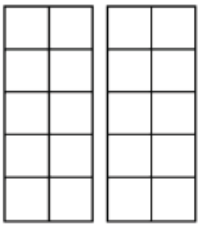
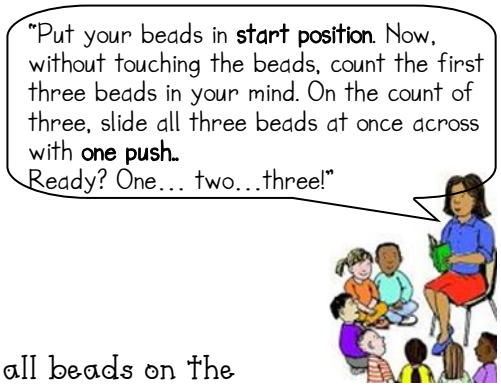


Rekenreks



Rekenreks are simple, but powerful manipulatives that use beads to help students make mental images of numbers and move away from always counting by ones. (not an abacus)

Important to understand: This tool has a starting position with all beads on the right.! Any beads on the left show the solution. This helps develop decomposition of number. Guide students with number of "pushes" you want them to use. (ie, use one push on top row, or two pushes; one on top & one on bottom row, etc.)

Chunking beads in sets help students with using 5 and 10 as anchors during adding and subtracting which is more efficient than one-by-one counting and helps students understand inclusion and cardinality because all (7) beads must be pushed to see or "have" (7) in all.

This tool provides learners with the **visual models** they need to discover number relationships and develop a variety of addition and subtraction strategies, such as doubles plus or minus one, making tens, and compensation, thereby leading to automaticity of basic facts.

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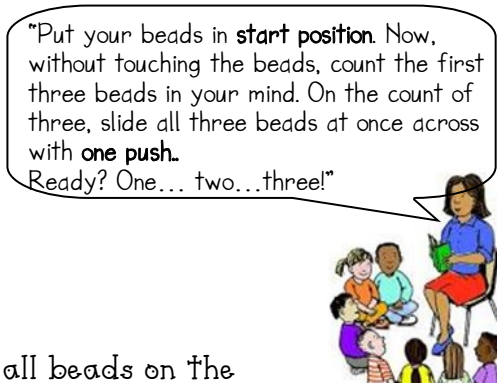

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REKENREK

Adapted from *Number Sense Routines*

- How many beads in all do you see? How many beads are on top row? Bottom row?
- Can you show a way to make _____? Can you show a different way to make the same number?
- How many do you need to add /take-away to make _____?
- What can we do to make _____?
- How many are hiding behind the white panel (or sticky note/paper)?

Subitizing:
- *Without counting each bead, how do know there are ____?*

Combinations of ten:
- Show an amount (8) of beads, ask, "how many more is needed to make 10?"

Teen Quantities:
Show me (12) with two pushes: (one ten, two ones) (students say, I see 12 ones or 1 ten and 2 ones)

Compensation strategy:
- Play "Sum it Up." Place 4 beads in the 'middle" of top row(2 red, 2 white). Make 4 another way (must take 1 red away to add one white: 1 red, 3 white)

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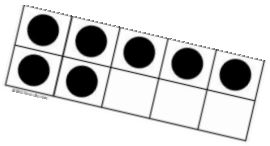
Five & Ten-Frames

"Ten-ness" is one of the best gifts a child can receive in primary education. It is the "UNIT" understanding that allows students to think flexibly and be able to look at a quantity such as 14 and know it is one-ten, 4 ones or 14 ones or 10-ones and 4 more ones. This is a prerequisite for place-value understanding.



Using frames helps...

- provide teacher insight into students current number concept developments based on how they interact with ten-frames.
- students use "anchor" numbers 5 and 10.
- develop students ability to visualize ("see") and think about number in terms of parts/relationships such as part-part-whole, missing parts, more and less, pre-place value, double and near double, combinations of 5/10, compose/decompose, subset, DISTANCE, etc.
- students share and learn strategies from each other.

TIPs:

- Consider modifying/scaffolding skills with use of five-frames
- Always fill the top row first, starting on the left, the same way you read. (builds mental image of number)
- Ask questions that focus on number of dots/spaces, saying one/two or more/less than the number of dots, saying the "ten fact" (4+6=10),
- Connect to daily math routines (attendance, number of the day, survey, etc.)

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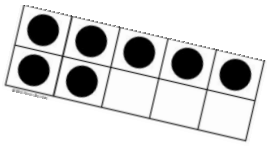

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FIVE AND TEN-FRAMES

Adapted from *Teaching Student-Centered Mathematics*

1. Getting to Know Five-Frames

- How many dots?
- How many spaces?

2. Getting to Know Ten-Frames
(using blank frames with counters. *Tip: watch for students that can adjust count w/out removing all)

- Show me "_____"
- Now, show me "_____"

3. Ten Frame Flash-Card

- How many dots?
- How many spaces?
- What is one/two more?
- What is one/two less?

Conceptual Subitizing: Which has more/less?

4. Part-Part-Whole Relationships

- What are the dots plus the spaces?
- What is a number model that represents your ten-frame?

5. Develop Pre-place Value

A set of ten is important in understanding. 13 is ten ones and 3 more and also 13 ones.

- How many ones are there?
- How many tens?

6. Make Ten on a Ten-Frame

- How many more do you need to make ten?
- How do you know?

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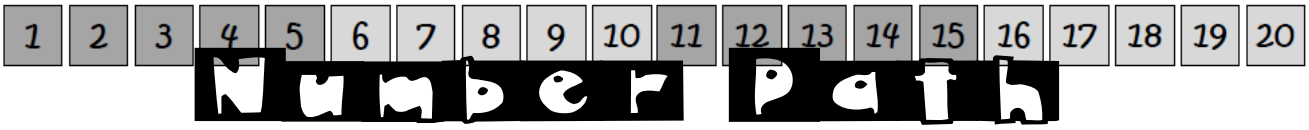
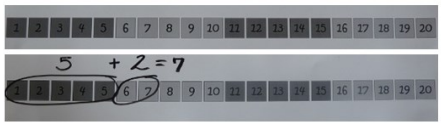


- A **number path** is a **count** model; the numbers are represented by a rectangle and each rectangle can be counted. (A picture graph is a great example in which identical pictures are used to show how many in all. A cloud or sun stands for ONE thing.)
- A **number line** is a **length** model; each number is represented by its length or distance from zero. With a number line children have to count the length units and not the numbers. (number line caution: some kids start the count with zero or starting in middle (between 0 and 1). When young children are still making sense of numbers, a number path can help solidify and build their number understanding, build confidence and accuracy when solving problems. A bar graph used a number line to show the length of the bars in the graph.

Consider making a vertical number path, connecting it to days in a month of school or collecting data such as weather, temperature, surveys. Students visualize how numbers change and grow as they begin writing two-digit numbers which can be more intuitive than a horizontal number path since the number becomes larger as it goes up.

Connect the Number Path and Rekenrek!

The change in color for every group of five numbers (red/white) helps students to see the symbolic representation to the concrete.



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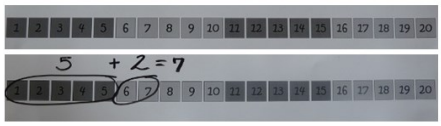

NUMBER PATH

Counting:
When learning the number sequence forward and backward, count and touch each number on the number path, pausing and ask,

- What number comes next? How do you know?
- What number is before? After?
- Starting at (28), is it closer to zero or 50? Is ____ closer to 50 or 100?
- Using a 0-20 path, stating 10 is the "middle" pull a number card and explain where the number should be placed and how you know.
- What does 7 and 8 have in common with 17 and 18?

Comparing (more, less, equal or same)

- Which is bigger/greater ___ or ____.? Show me on the number path.
- Which is smaller/less than ___ or ____? How do you know? can you prove it on the number path?
- How much less is ____ than ____?
- How much of a difference is there between _____ and _____?
- How many (counters, years, etc.) between (5) and (7)?

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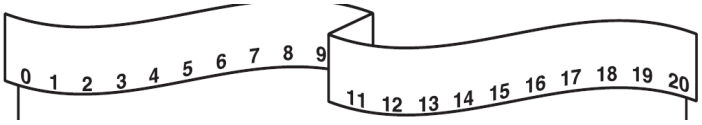


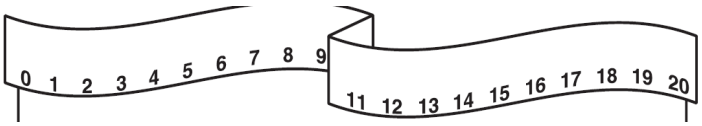
Number Line

The **number line** can do much more than simply help children count or recall the sequence of numbers. It is a tool for helping children reinforce cardinality, develop greater flexibility in mental arithmetic as they actively construct mathematical meaning, develop number sense, come to understand number relationships, and develop powerful strategies for addition and subtraction.

TIPs:

- Connect to 110 Grid, thermometer, clock, number of the day growing number line routine,
- Notice "zero" holds a spot
- Model the language of mathematics! (forward, backward, halfway to 10, closer to five, even, odd, greater than, less than, one more/less, two more/less, growing pattern, increase, decrease, distance (spaces) between, tick marks, "This MUCH is five".
- Effective tool to help students decontextualize story problems

Relative Magnitude: refers to the size relationship one number has with another—is it much larger, much smaller, close, or about the same? Distance of a number from zero. (VDW. p. 142)



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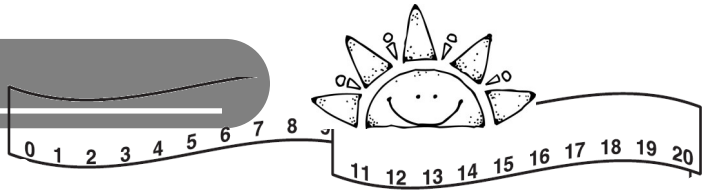
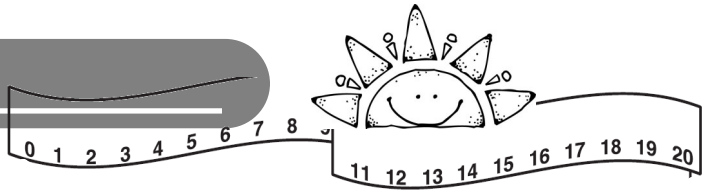


- What can you tell me about the number line? (growth pattern)
- What number comes before ____? What number comes after _____?
- What number is between ____ and ____?
- _____ is 2 more than (3 more than, etc.) _____?
- What multiples of ten is _____ between? What is _____ groups of _____?
- _____ plus how many more make _____. (do many examples to make 10)
- _____ minus how many make _____.
- Let's count by _____ starting at _____.
- Which are farther apart ____ and ____ or ____ and _____?

Pick a number. Ask students what they know about that number? (Encourage use of "math talk" with vocabulary such as less than, greater than, skip counting, counting in groups, even/odd, between, before, after, etc.

**Number line questions provided by Kim Sutton*

NUMBER LINE



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100s or 110 Grid

What is a Hundreds Chart/Grid?

Usually: A 10x10 grid with numbers 1 to 100 printed in the squares. Number Grids are "return sweep" number lines to showcase patterns & save space.

Can also:

⇒ Be arranged in vertical columns of ten to see the repeating tens number more easily. The tens at the bottom "summarize" the count

⇒ Start with 0 to keep all "30s" on one row

Hundreds Chart/Grid helps students:

- Allows students to easily see the repeated structure of numbers (benchmarks of 1, 5, 10, 20, 25, 50, 100, etc, magnitude of number, etc.)
- Develops initial understanding of place value and base-ten system
- Connect number/name (language of count can be so confusing)
  - Teen means ten
  - First two "teen" words do not have –teen
  - Thirteen instead of threeteen, fifteen instead of fiveteen
  - Seven-teen does not look like 17.
  - Pattern switches after 20: English words for 20-100 says ones second
  - Decade/teen word confusion: fourteen and forty, sixteen and sixty

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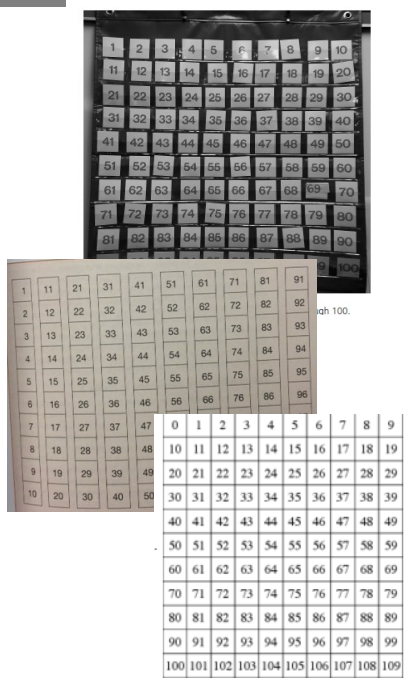


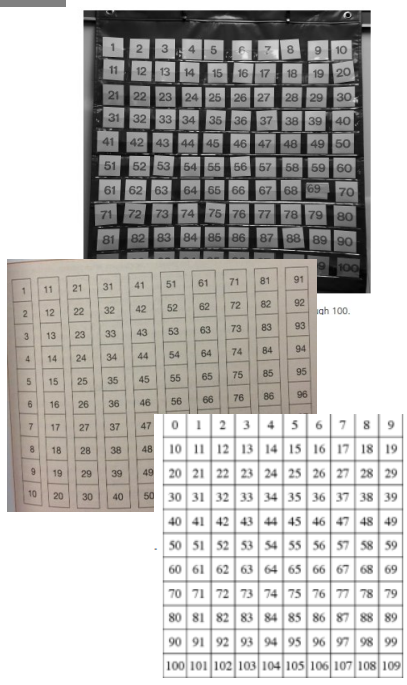
- What do you notice about the number ____?
- How many digits are in the number ____?
- What number comes before/after?
- Start counting (forward/backward) from the number ____.
- Can you count by 10s going down the right-hand column?
- How many groups of ten are in the number _____

Building relationships of numbers:

- Is your number greater than_____
- Is your number between _____ and _____
- Does your number have a(___) tens? Or (___) ones?
- What is 10 more?
- What is 10 less?
- How many more to make a ten?

What does a 110-grid have in common with a number line?

110 GRID



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IMPORTANT NUMBER SENSE VOCABULARY

Subitizing: *Quickly knows how many without counting. (two types: conceptual, perceptual)*

Cardinality: *Understands the final number in a sequence is the total number of objects counted.*

Conservation: *Understands "three" is always "three" regardless of the formation or size.*

Hierarchical Inclusion: *all numbers require the previous number (3 is nested inside 4)*

Unit:

- Discrete Unit: *counting unit of exact things*
- Continuous Unit: *measurement unit, things can be divided into smaller units*

Magnitude: *relationship of numbers (such as distance from zero, 10 closer to 10 or 100?)*

Anchor/Benchmark : *numbers such as 5, 10, 20, 100 that help students compute by de-composing quantities into friendlier numbers*

Compensation: *decreasing from one part and increasing another leaves the quantity unchanged.*

Fluency: *flexibility, accuracy, efficiency*

"=" sign: *same as", or "balance"*

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ADDITION & SUBTRACTION FACT STRATEGIES

Counting On: Use when adding 1 or 2 to a given number.

Fact with 0: Use when one of the addends is 0. (especially helpful with story problems)

Doubles: add two of the same number together (2+2 or 8+8)

Doubles +1: Find a double hidden in the fact where one addend is one more than the other.

Combinations of Ten: Group the numbers to find expressions that would equal 10.

Make a Ten: Use with addend of 8 or 9 building up to 10 + adding on the rest.

Doubles +2: Finding a double hidden in the fact where one addend is two more than the other.

+9: When an addend is 9, then just add 10 to the other addend and take 1 away.

Subtraction Fact Strategies:

Think Addition: Using known addition fact to sol e the subtraction problem (13-5, think what goes with 5 to make 13?) *Number line is a helpful mental tool!

Related Equations: think of the related + and—facts to recall the missing number.

Build up Through the Ten: Used when the subtrahend is 8 or 9. (14-9, 9 and 1 is 10 and 4 more makes 5.)

Back Down Through Ten: Working backward with 10 as a "bridge" (15-6: take 5 away from 15 to get to ten, take 1 more away, leaving 9).

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ADDITION & SUBTRACTION FACT SITUATIONS

http://www.corestandards.org/Math/Content/mathematics-glossary/Table-1/

Common addition and subtraction. [1]

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? **2 + 3 = ?**	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? **2 + ? = 5**	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? **? + 3 =5**
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now?**5-2 = ?**	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?**5 – ? = 3**	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?**? -2 = 3**

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ADDITION & SUBTRACTION SITUATIONS

	TOTAL UNKNOWN	ADDEND UNKNOWN	BOTH ADDENDS UNKNOWN [2]
PUT TOGETHER / TAKE APART [3]	Three red apples and two green apples are on the table. How many apples are on the table? **3 + 2 = ?**	Five apples are on the table. Three are red and the rest are green. How many apples are green? **3 + ? = 5, 5-3 = ?**	Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? **5 = 0 + 5, 5 + 0 5 = 1 +4, 5 = 4 +1 5 = 2 + 3, 5 = 3 + 2**
COMPARE	DIFFERENCE UKNOWN	BIGGER UNKNOWN	SMALLER UNKNOWN
	("How many more?" version):Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have then Julie? **2 + ? = 5, 5 – 2 = ?**	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?(Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? **5 – 3 = ?, ? + 3 = 5**	(Version with "more"):Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?(Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? **5 – 3 = ?, ? + 3 = 5**

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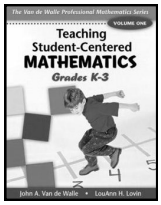
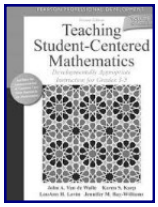
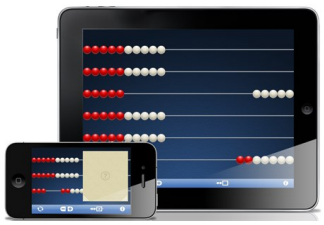
AWESOME NUMBER SENSE RESOURCES

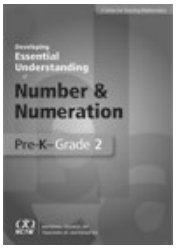


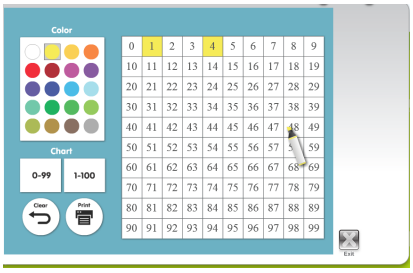


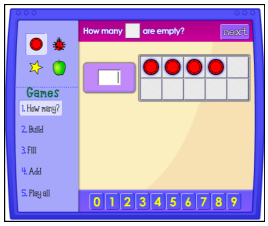
http://catalog.mathlearningcenter.org/apps/number-rack

Number Rack for iPhone, iPad, and the Web



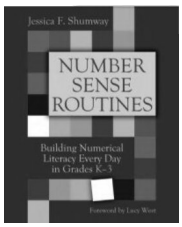






Electronic ten-five and ten frame

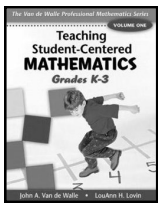
http://illuminations.nctm.org/

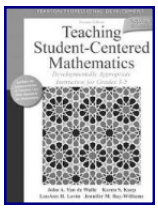


http://illuminations.nctm.org/
(awesome k-5 CCSS math activities)

Electronic 100s chart:
http://www.abcya.com/interactive_100_number_chart.htm

AWESOME NUMBER SENSE RESOURCES





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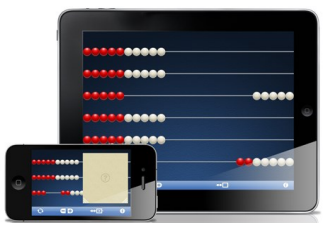
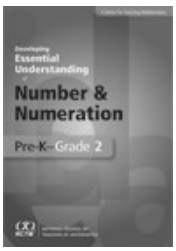
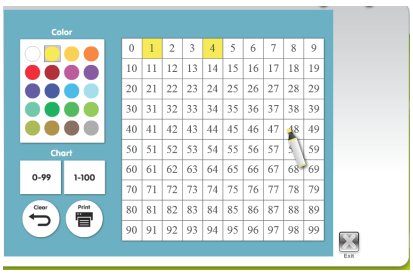
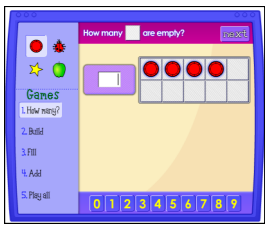


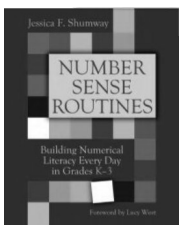






Electronic ten-five and ten frame

http://illuminations.nctm.org/



http://illuminations.nctm.org/
(awesome k-5 CCSS math activities)

Electronic 100s chart:
http://www.abcya.com/interactive_100_number_chart.htm

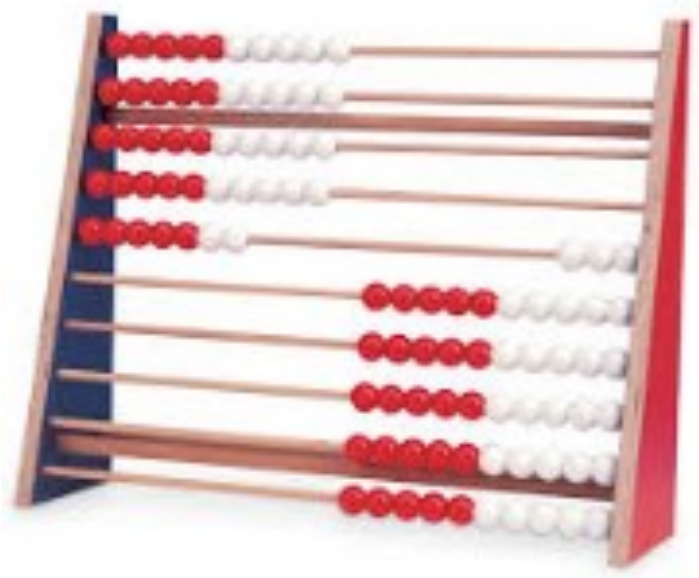
Math Tool Menu

110 Grid

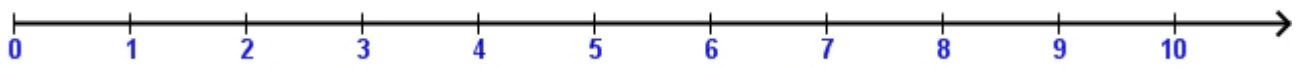
									0
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110

Rekenrek (20)

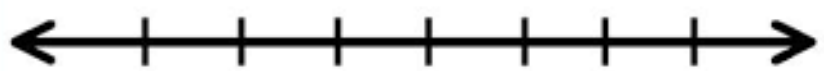
Rekenrek (100)



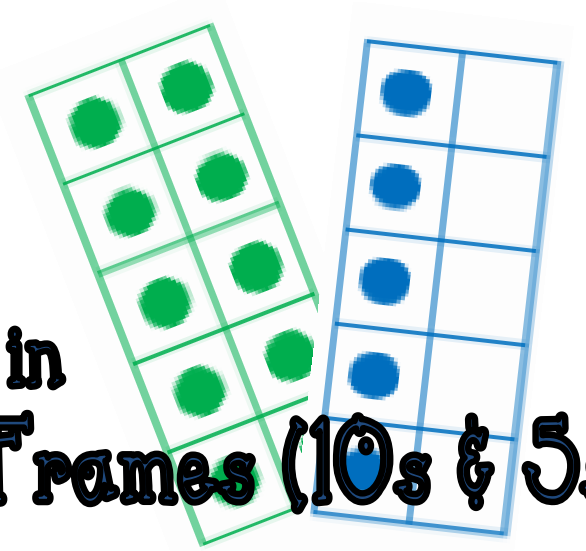
Number Line



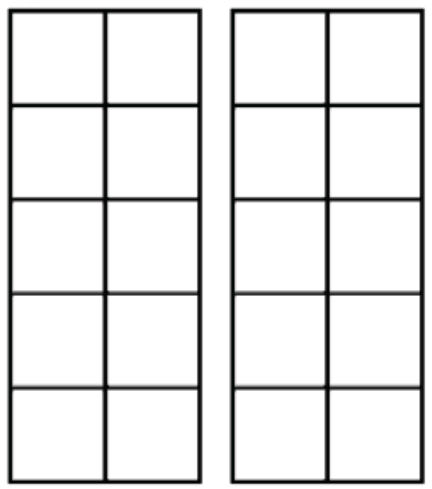
Open Number Line



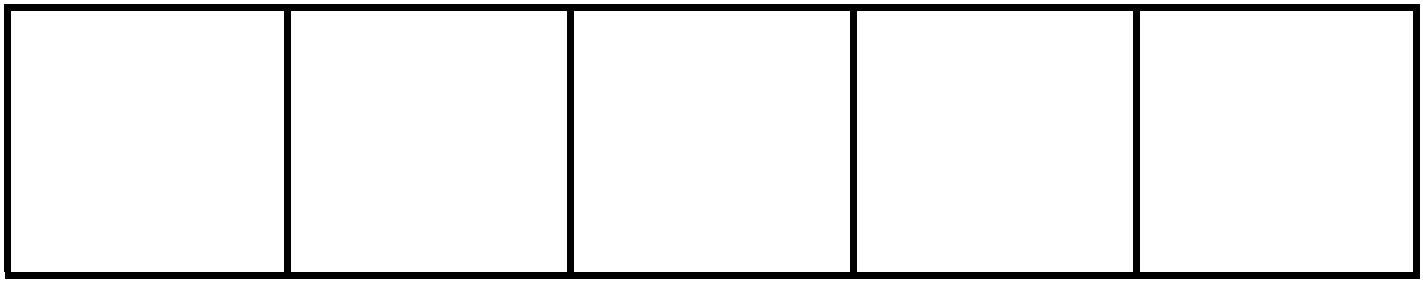
Filled in Ten-Frames (10s & 5s)



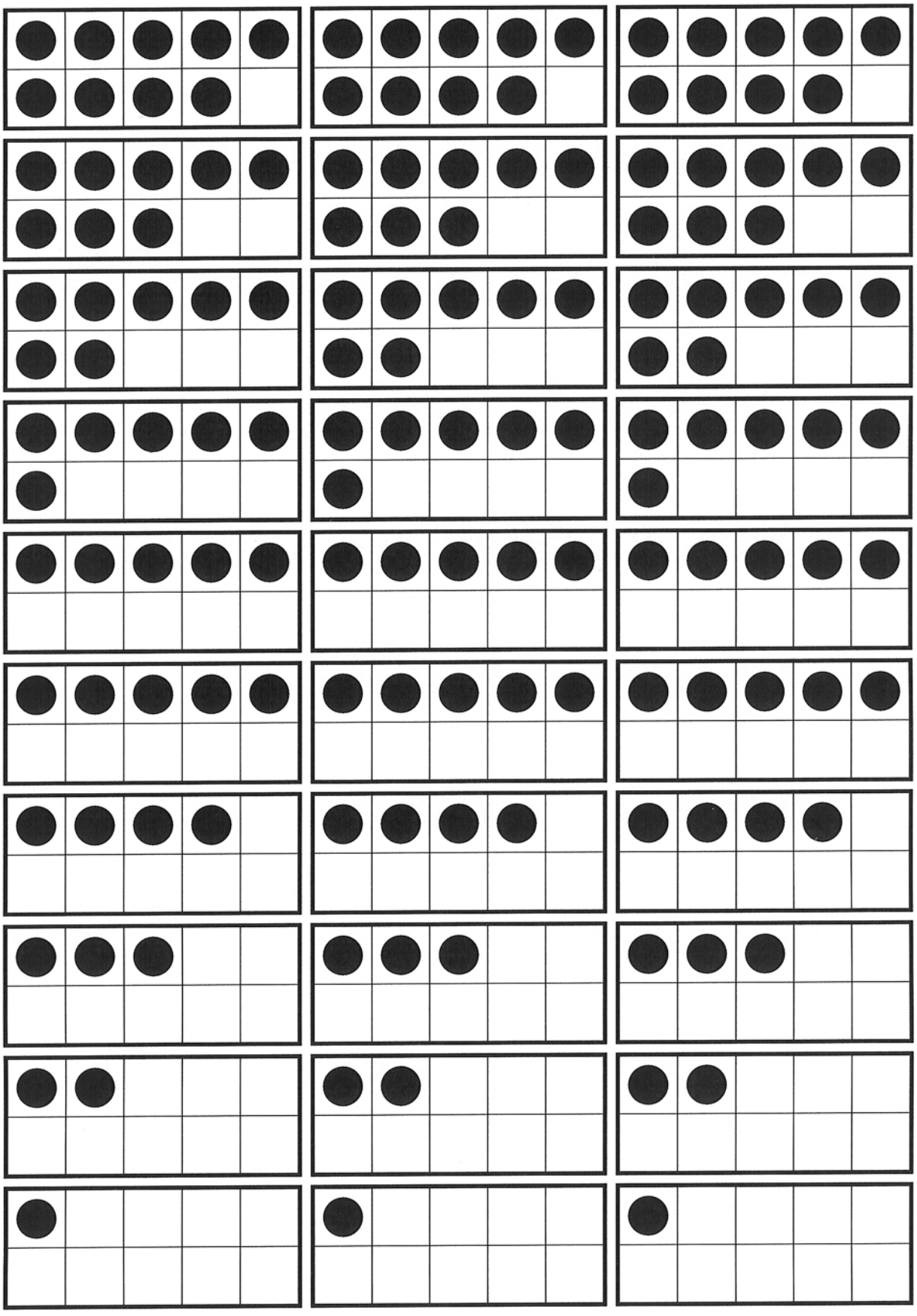
Blank Ten-Frames

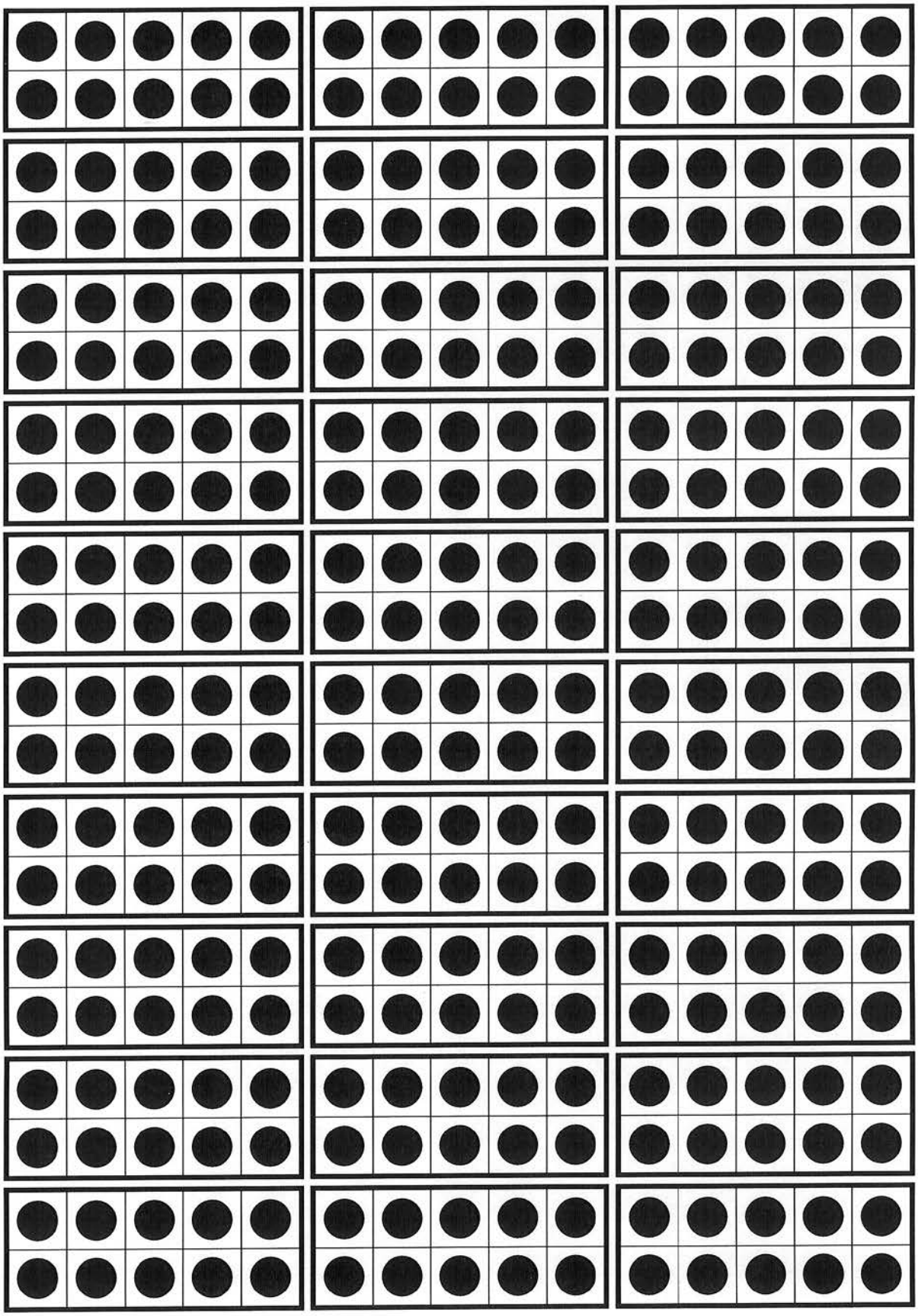


Number Path



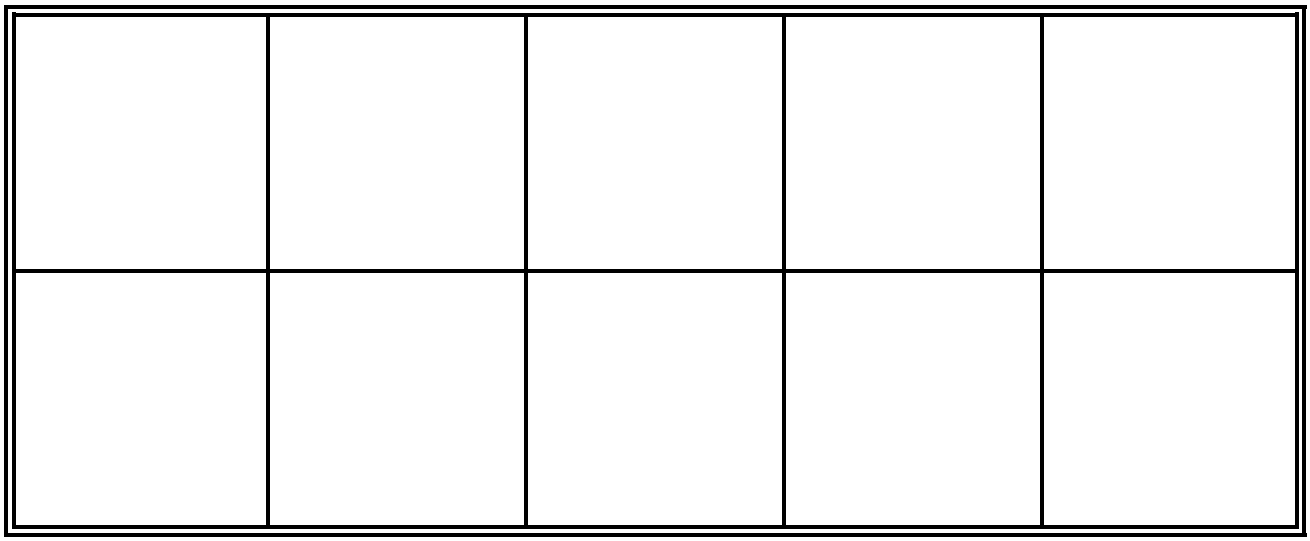
Copy on yellow Bristol paper

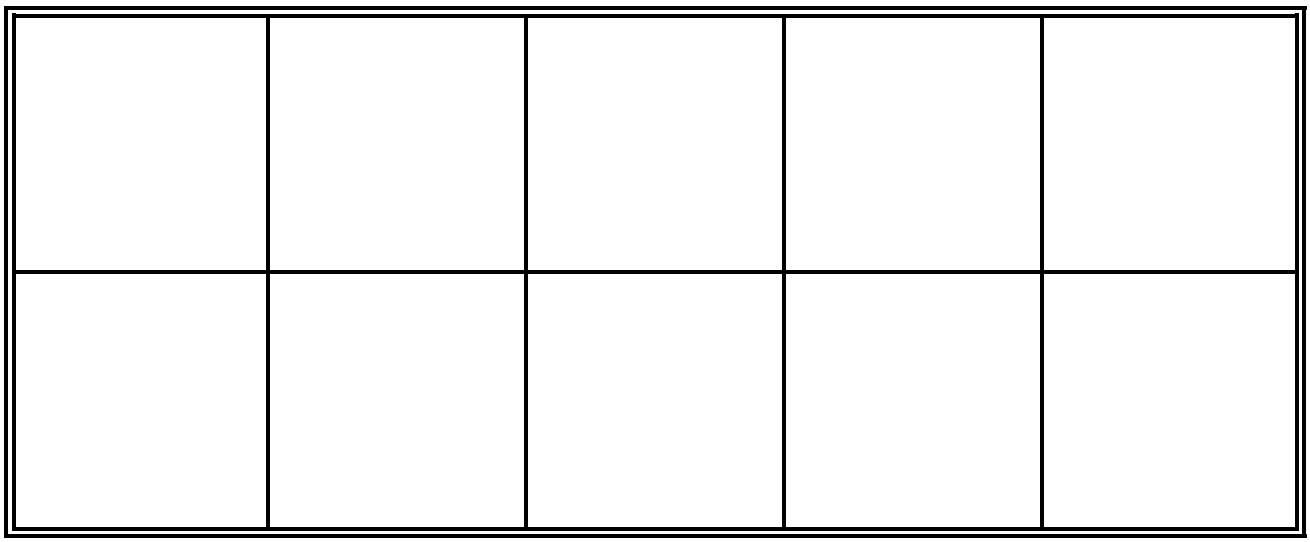


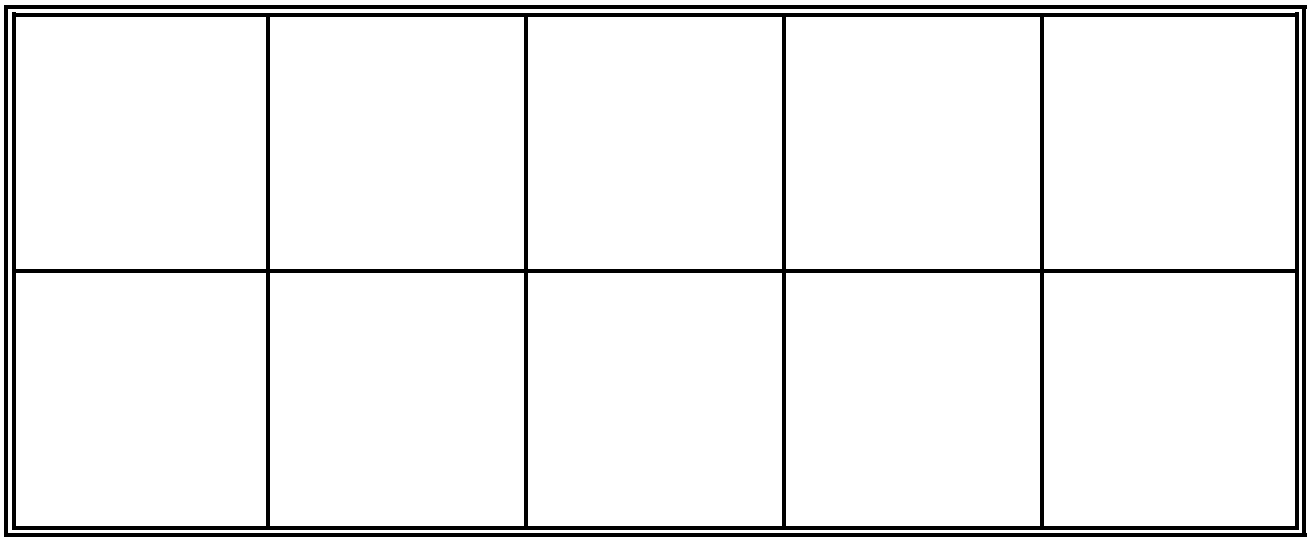


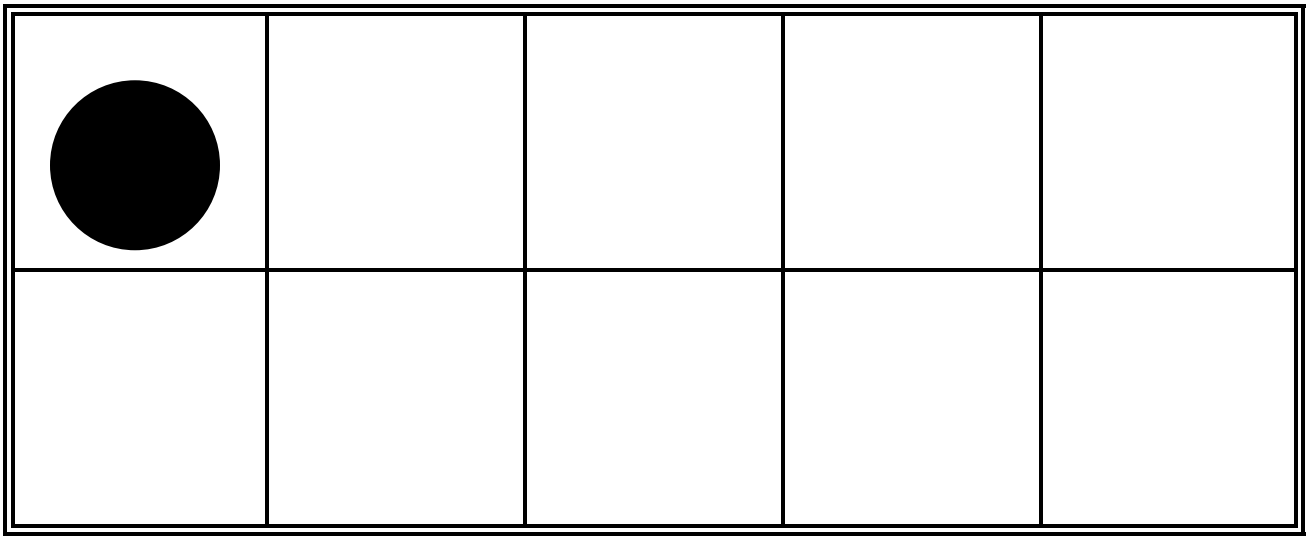
Copy on blue Bristol paper

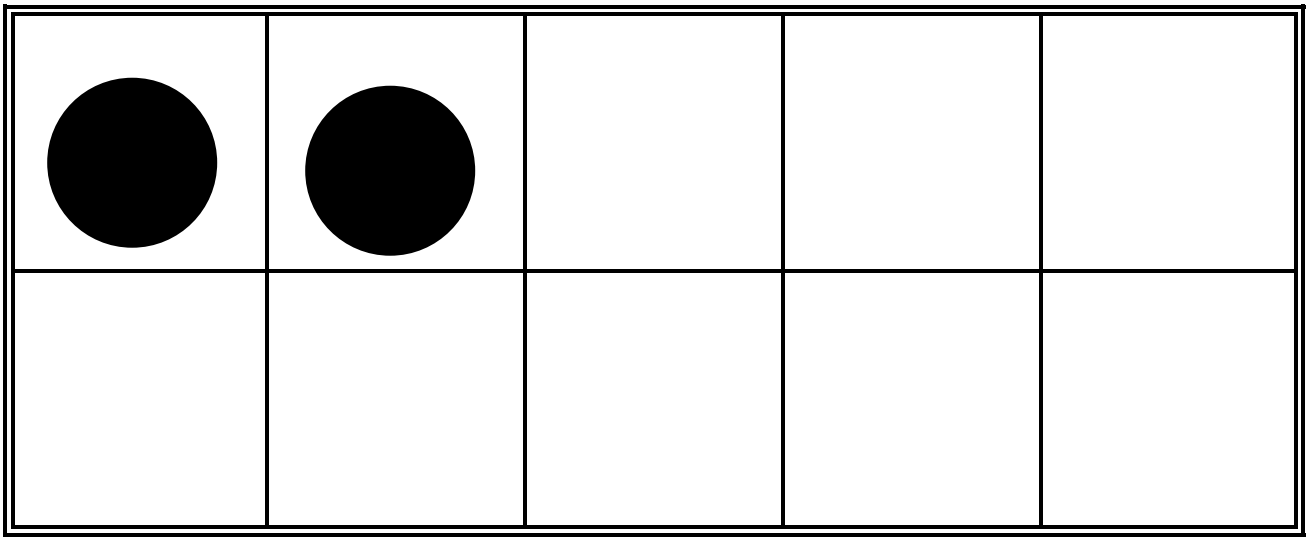
BLM 17—Little ten-frames

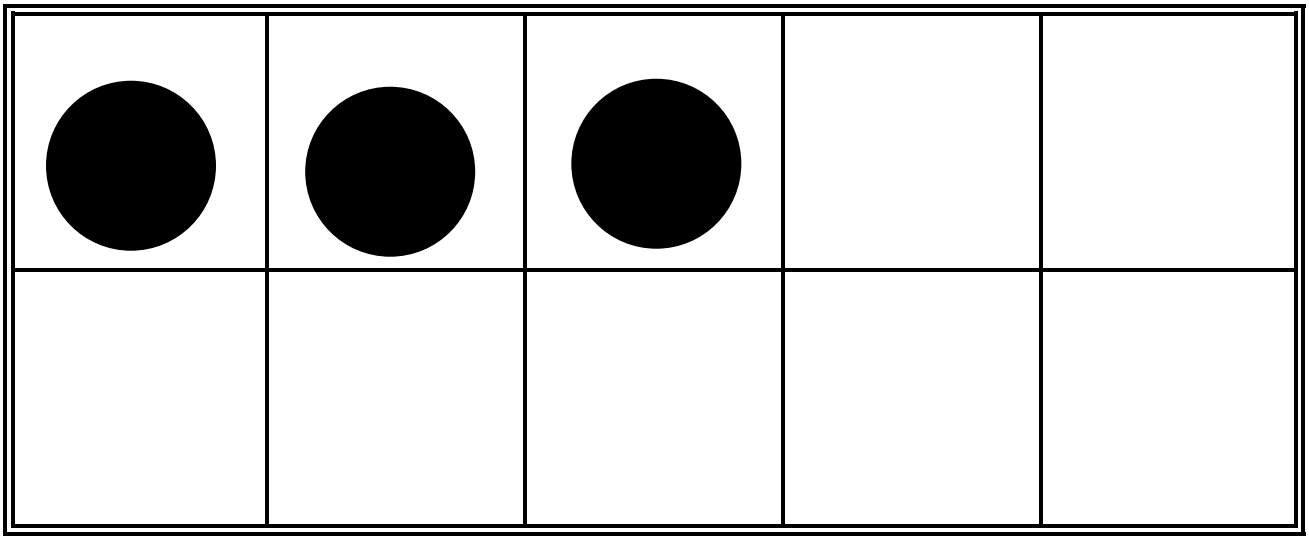


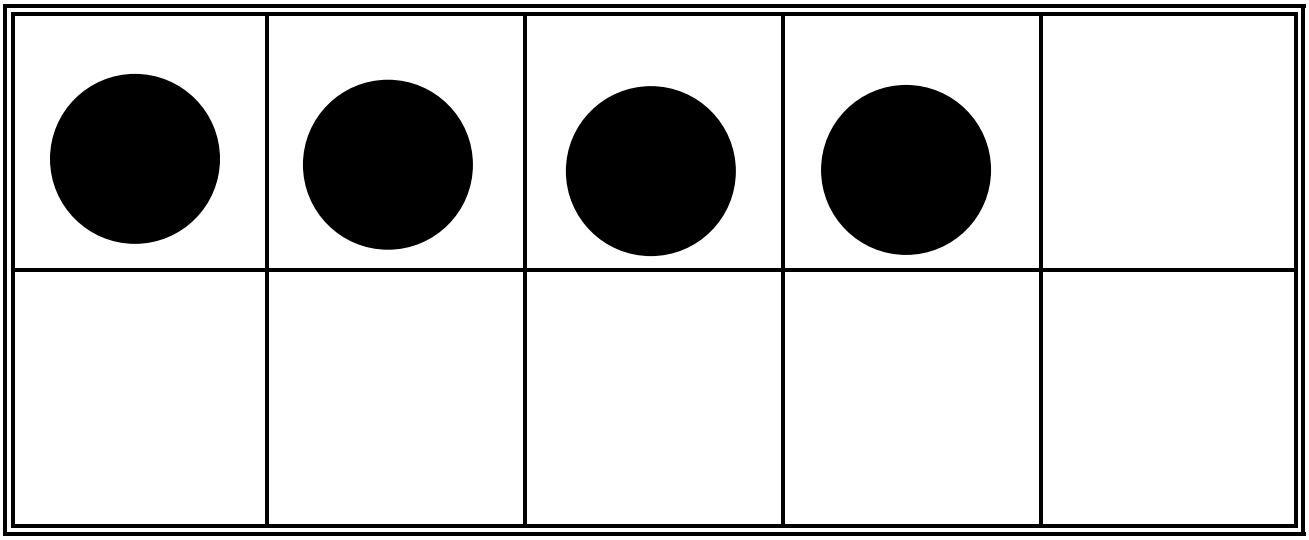


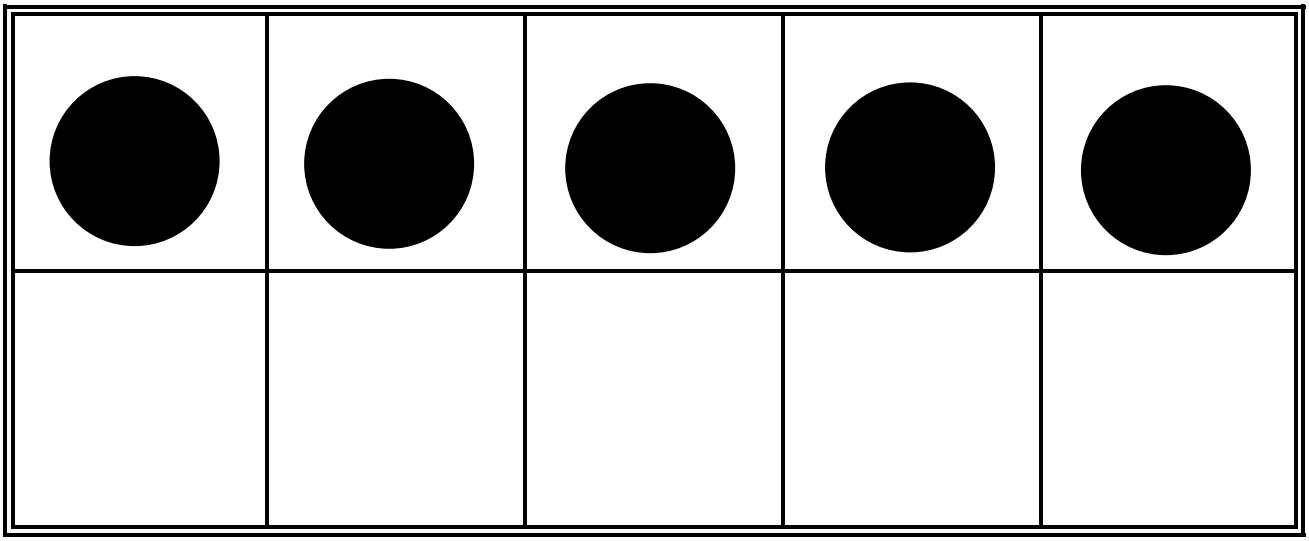


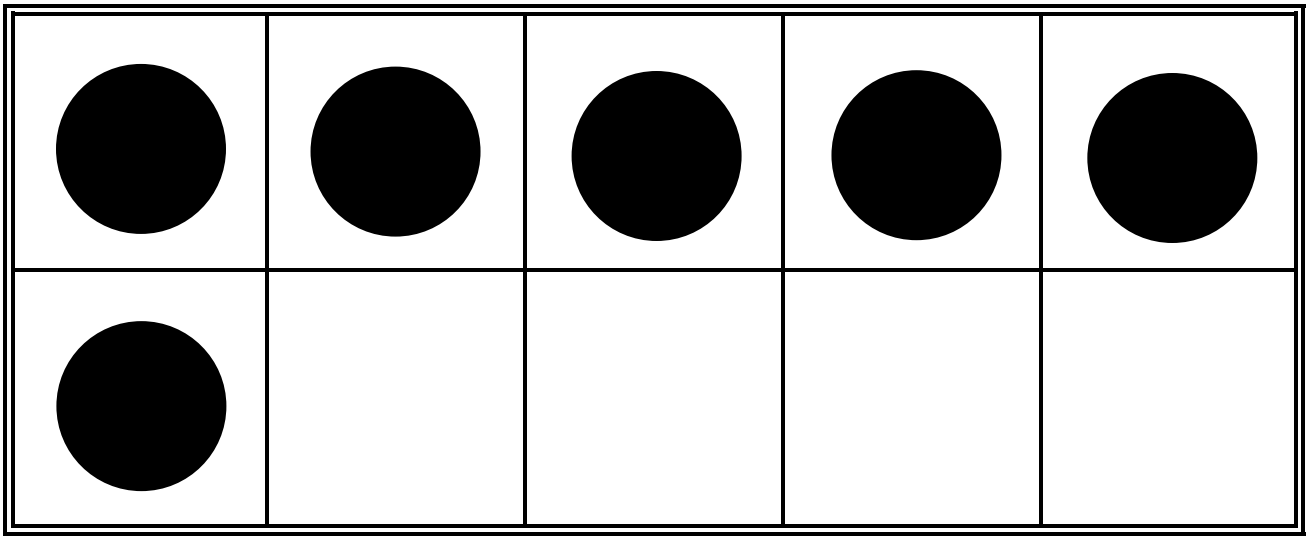


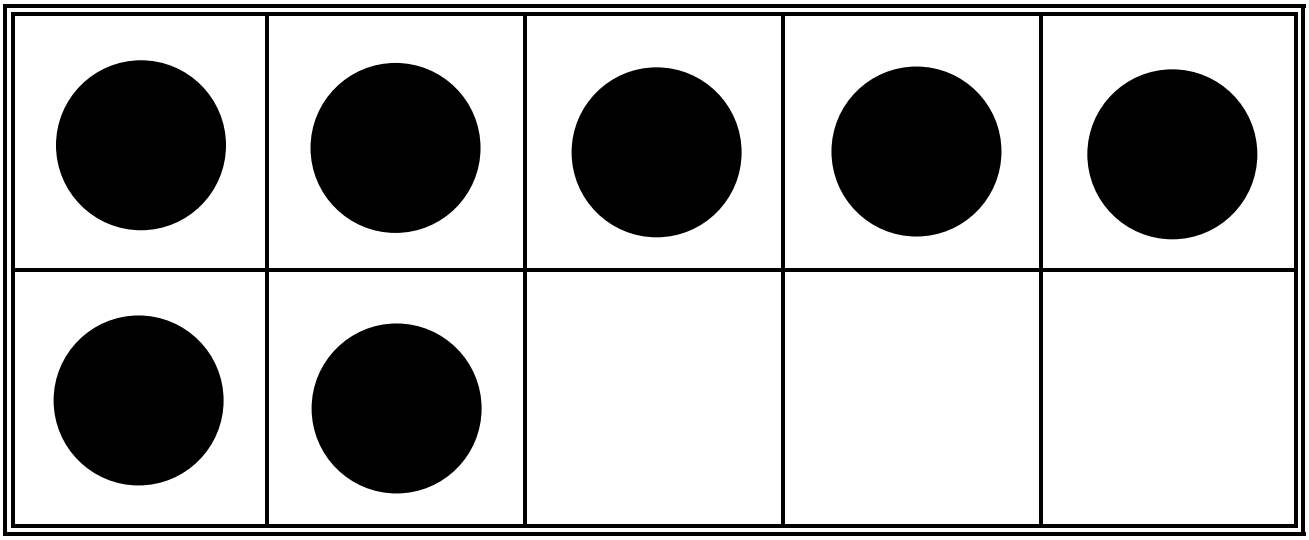


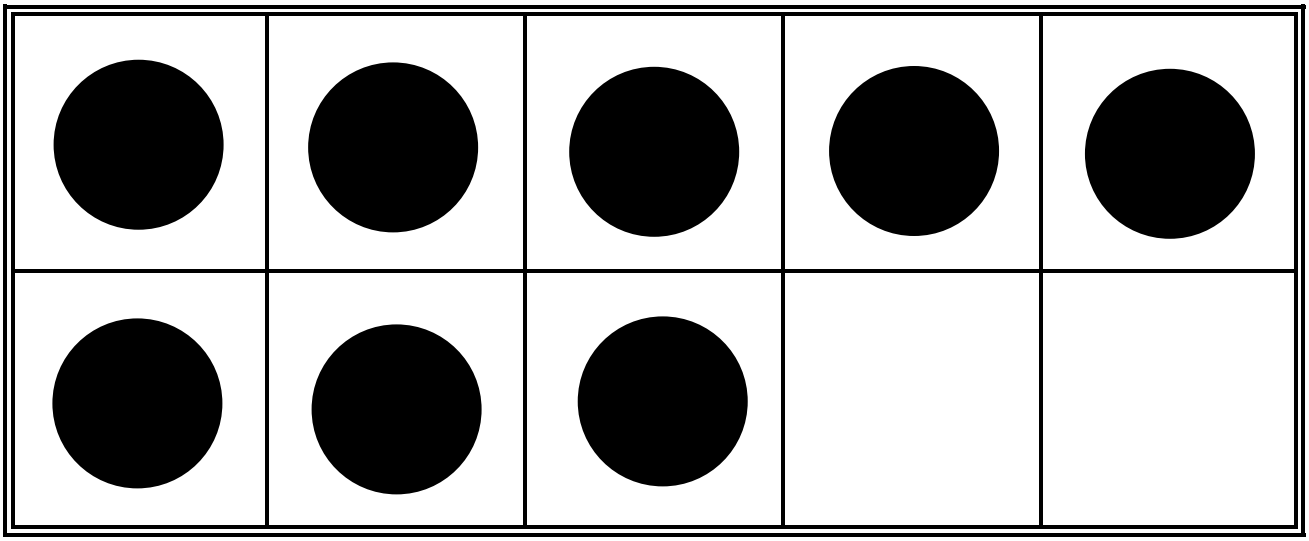


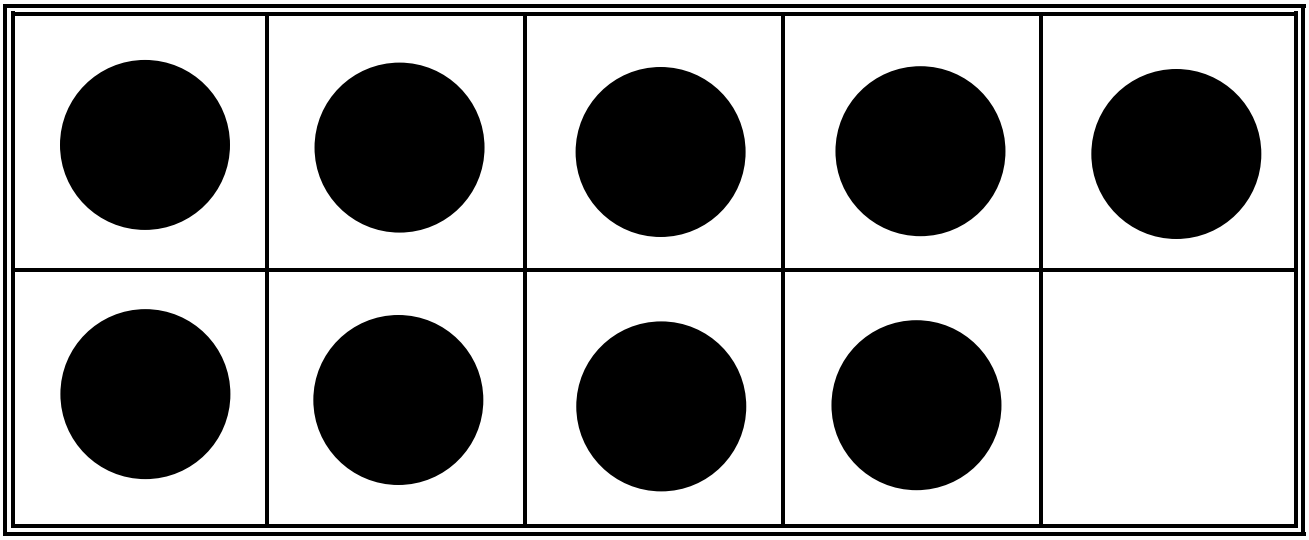


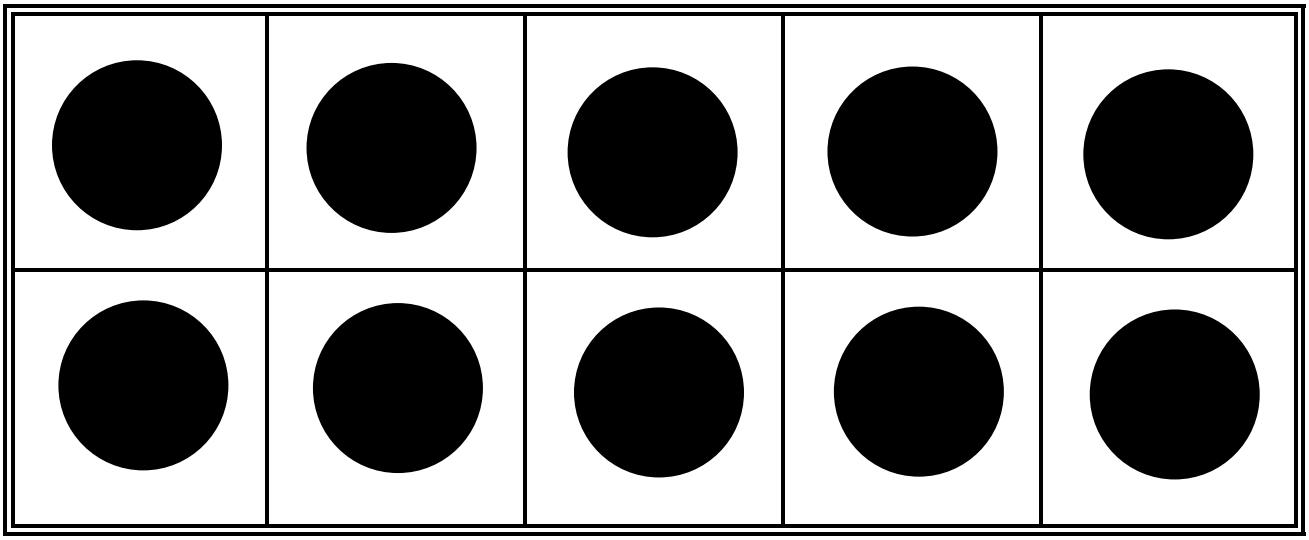


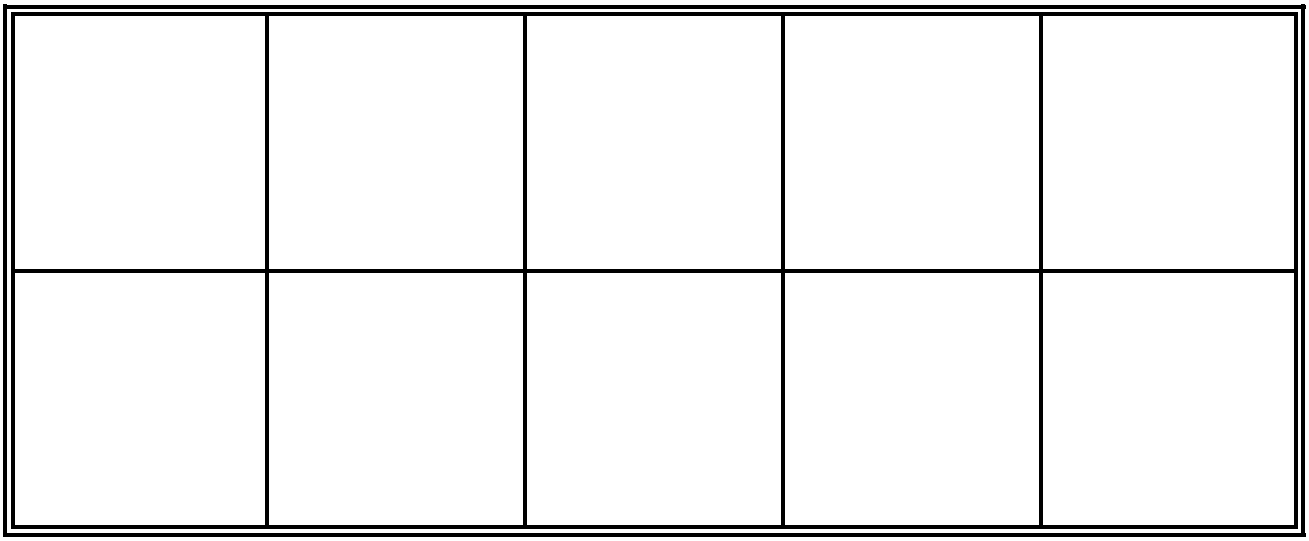


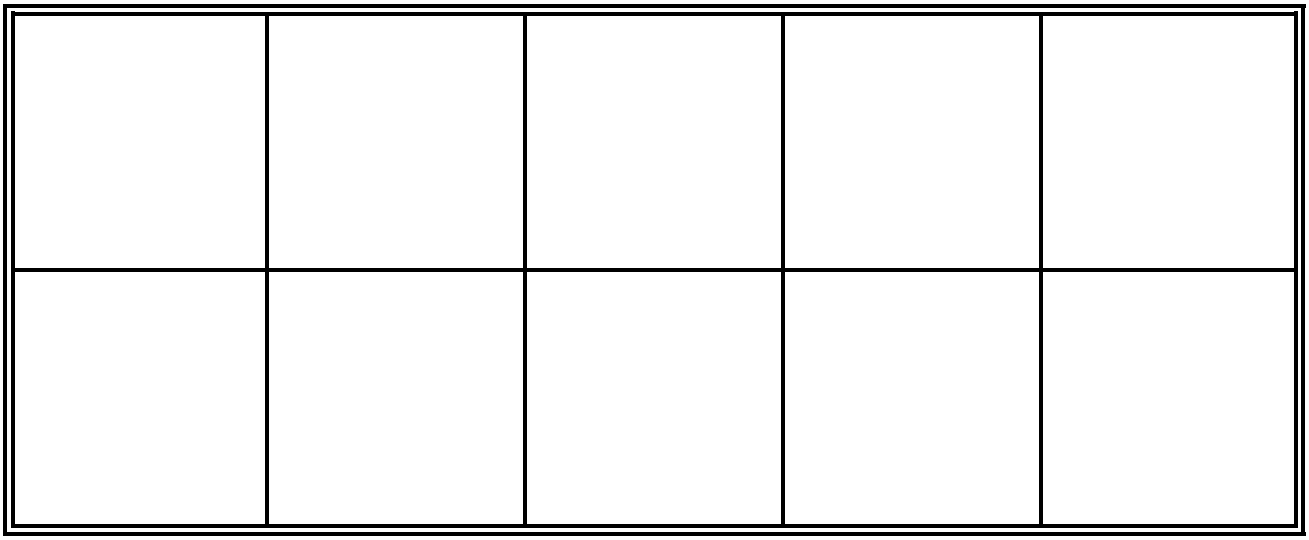












Number Ten Frames 0 to 20

Polka-Dot Border in Red

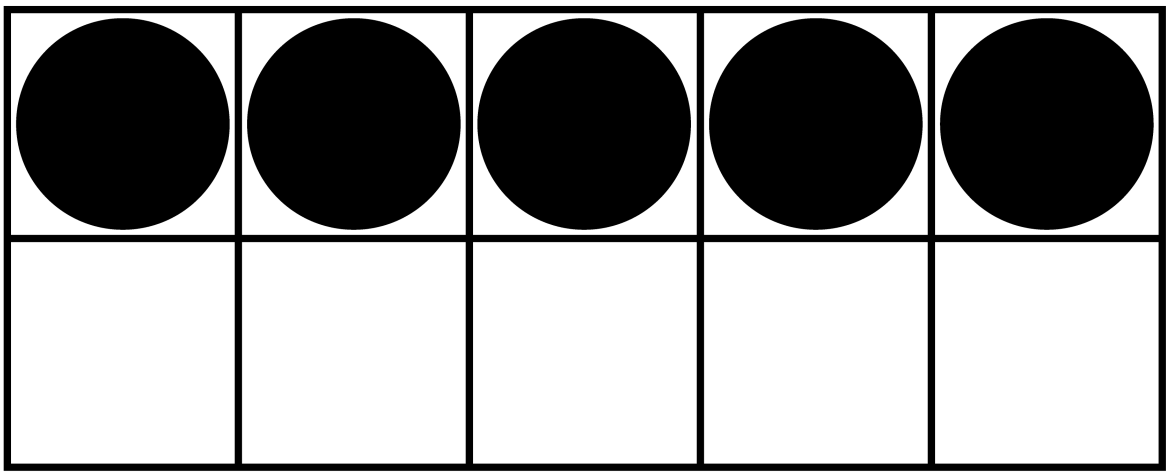
Jennifer Tilton
www.kindertrips.blogspot.com

Graphics From the Pond
http://frompond.blogspot.com

Background Paper: www.allfulloflove.etsy.com

five

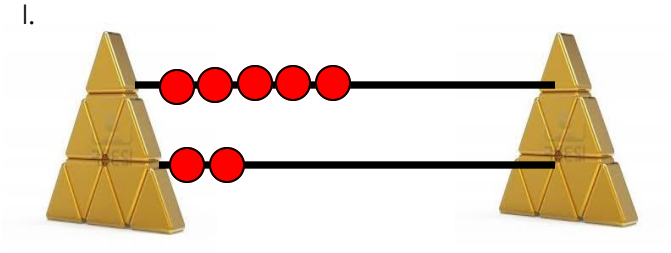
5



Name _____ Date _____

Recording Sheet Directions: Teacher would give a number to the students to represent on the rekenrek with two pushes. Students draw they way they would represent that number. Then students would write math statements to match. Have students verbally share with a partner their work/how they know what they know.

I.



5 + 3 = 8 8 = 3 + 5

There are a variety of sheets attached to this. Choose the one that works best for your students.

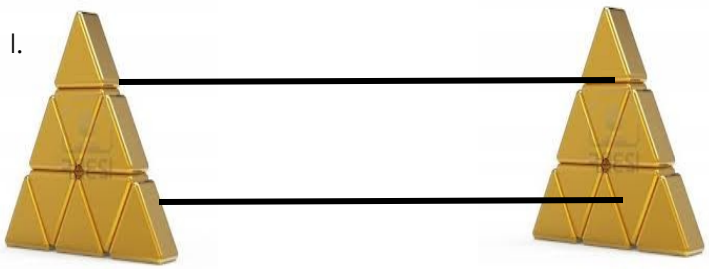
The rekenrek that *YOUR* students use may look like this or this—what makes sense to them?

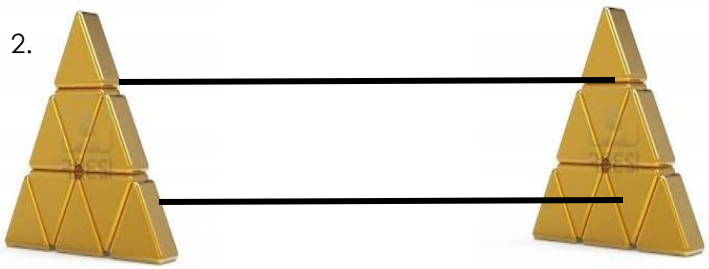
Scaffold?

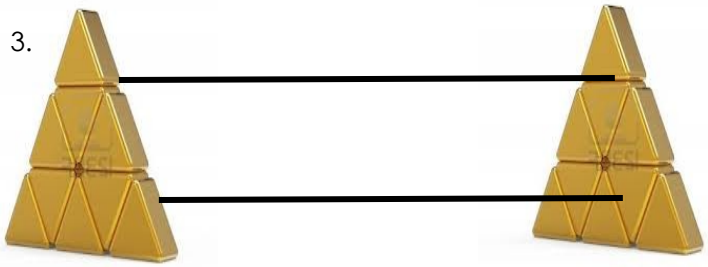
Start with just drawing?
Then labeling?
Then math statements?

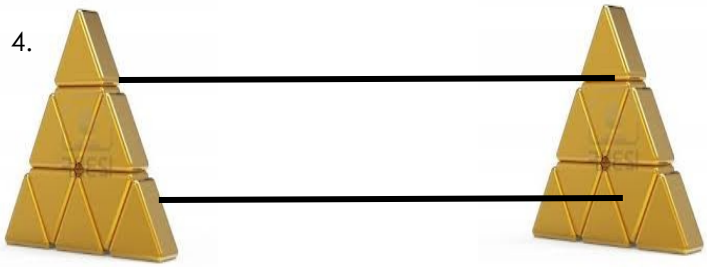
Name _____ Date _____

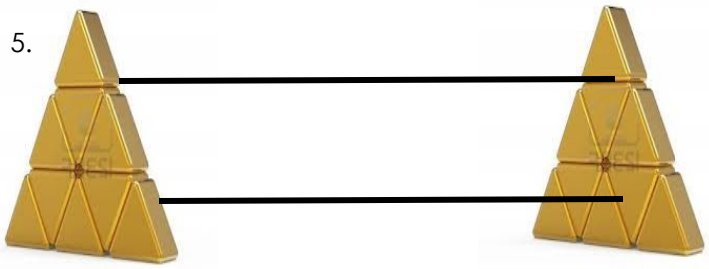
Recording Sheet Directions:

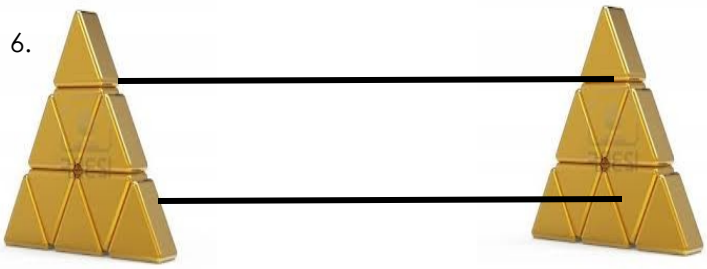
1. 

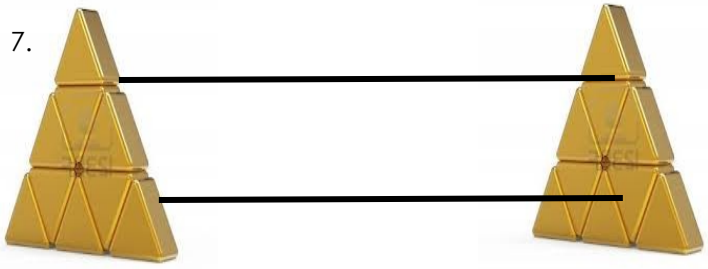
2. 

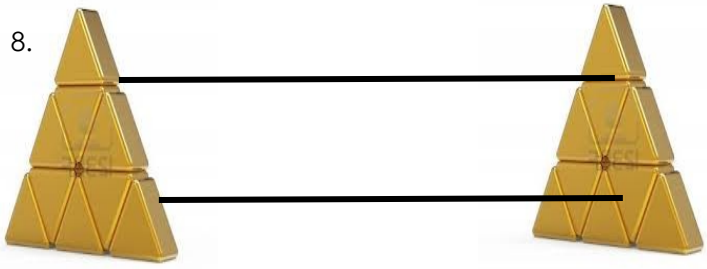
3. 

4. 

5. 

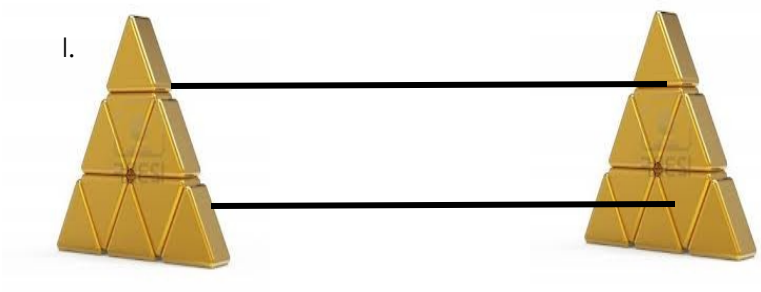
6. 

7. 

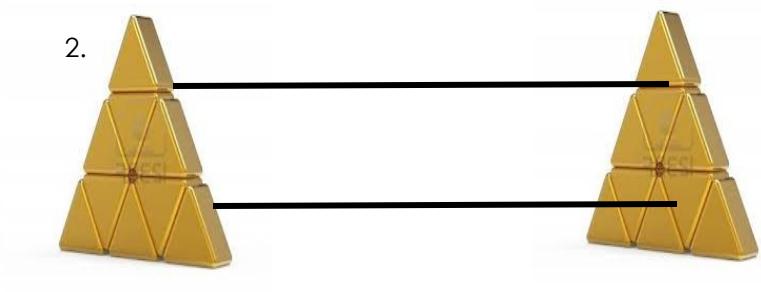
8. 

Name _____ Date _____

Recording Sheet Directions:

1. 

_____ _____

2. 

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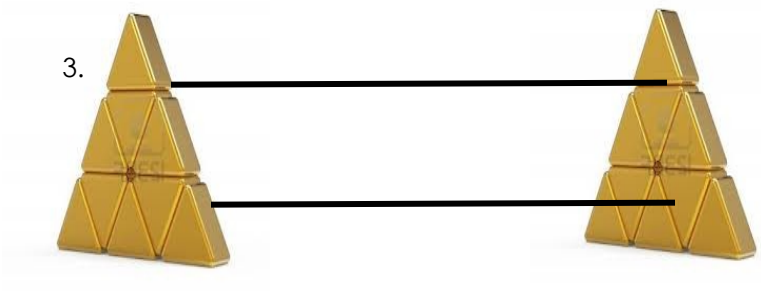
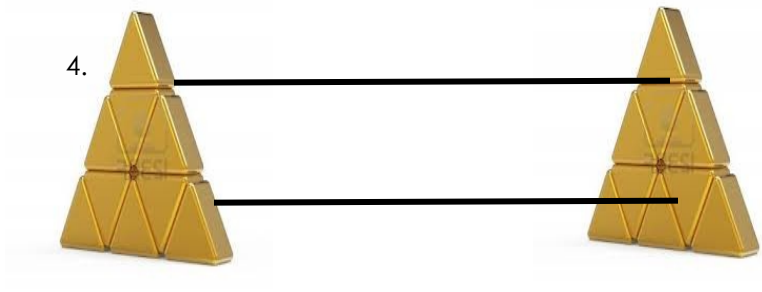
3. 

_____ _____

4. 

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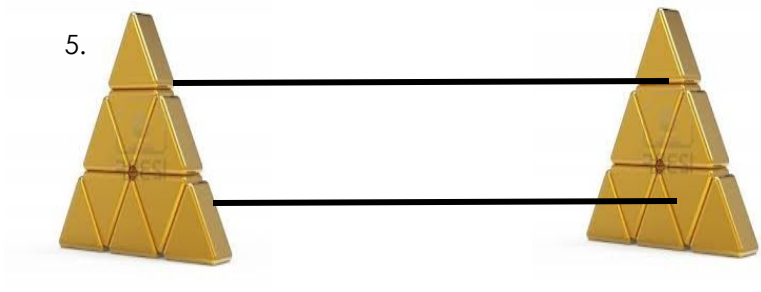
5. 

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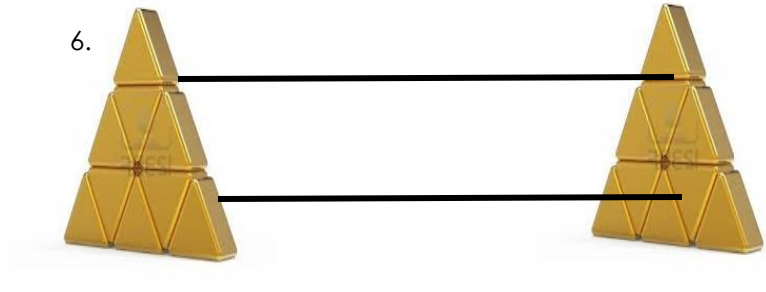
6. 

_____ _____

Name _____ Date _____

Recording Sheet Directions:

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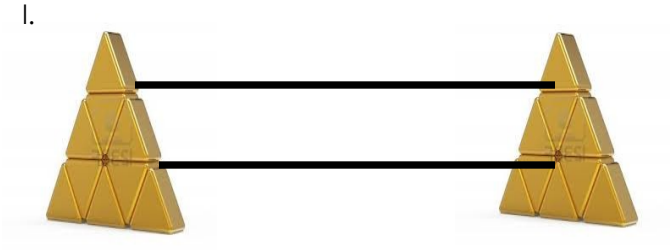
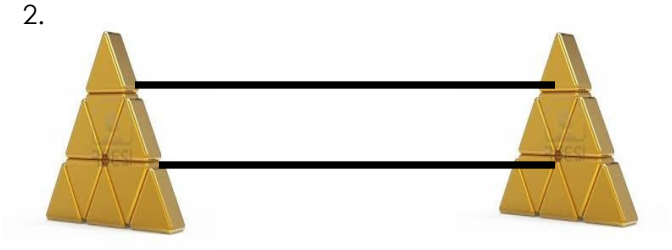


_____ _____

2.



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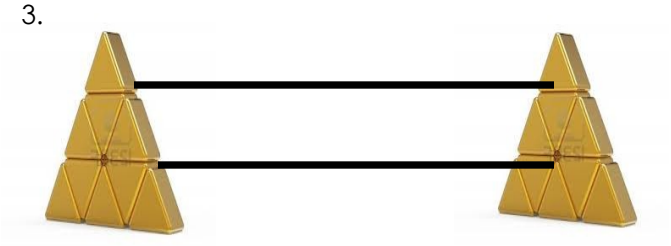
3.



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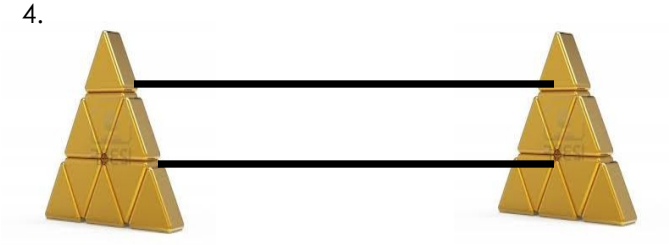

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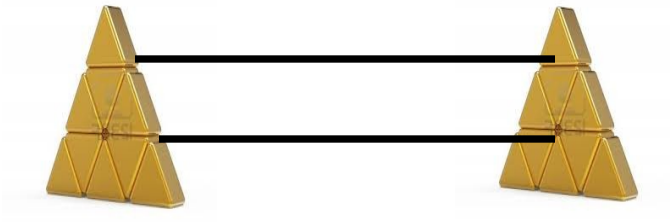
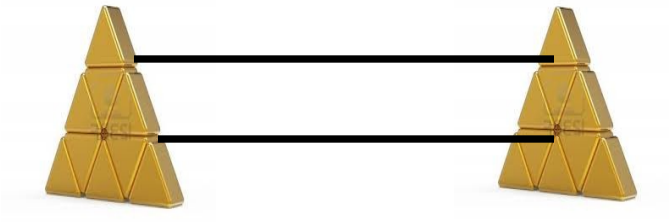


_____ _____

6.



_____ _____

Name _____ Date _____

Recording Sheet Directions:

1.

2.

3.

4.

5.

6.

7.

8.

Name _____ Date _____

Recording Sheet Directions:

1.

2.

3.

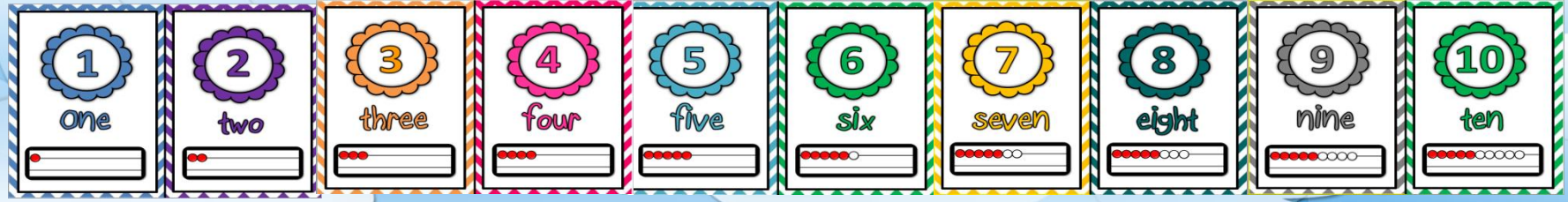
4.

5.

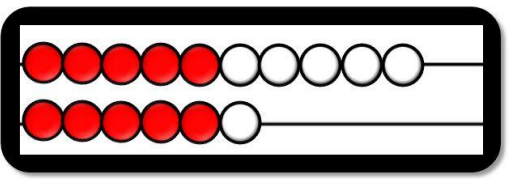
6.

1.

2.



1-20 Classroom Number line



With a Rekenrek

									0
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110

									0
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110

●	1	2	3	4	5	6	7	8	9	10	11	12
●	1	2	3	4	5	6	7	8	9	10	11	
●	1	2	3	4	5	6	7	8	9	10	●	1
●	1	2	3	4	5	6	7	8	9	●	1	2
●	1	2	3	4	5	6	7	8	●	1	2	3
●	1	2	3	4	5	6	7	●	1	2	3	4
●	1	2	3	4	5	6	●	1	2	3	4	5
●	1	2	3	4	5	●	1	2	3	4	5	6
●	1	2	3	4	●	1	2	3	4	5	6	7
●	1	2	3	●	1	2	3	4	5	6	7	8
●	1	2	●	1	2	3	4	5	6	7	8	9
●	1	●	1	2	3	4	5	6	7	8	9	10
	●	1	2	3	4	5	6	7	8	9	10	11
●	1	2	3	4	5	6	7	8	9	10	11	12

Quick Look Routine

*Additional Number Talks procedures can be found in Chapter 2, p. 16-20, slides on the Math Portal

Expected Time (5-15 min.)	Quick Look Display options: dot cards on ConnectED, printed cards from lesson, Additional Options: Number Talks PPT on portal, ten frames, Rekenreks
Subitizing is quickly knowing a quantity without counting. Quick Looks are a mental math activity that develops this critical skill by helping students decompose quantities in to parts in different ways increasing student's ability to think **flexibly**.	
Establish Math Talks Expectations	• MAC/CHAMPS • Refer to Turn and Talk Anchor Chart • Refer to Classroom Discourse Anchor Chart
Flash Quick Look Card for 3 seconds	• provide WAIT time • when students know how many, they show a quiet thumb-up • if a student is a "fasty", they can show you a second way by holding up another finger
Ask whole class to whispers their solution	• listen for consensus/errors
Record total (number of dots)	• avoid verbal or physical expressions that indicate agreement or disagreement
Students Turn and Talk	• ask shoulder or "peanut/butter and jelly partners to each have a turn to explain how they know the total number of dots • walk around and listen– this will help make decisions for whole class sharing • ask students to share how their partner's solution

Record Individual Student thinking
*Math Practices:
	• call on 3-4 students/partnerships
	• After each share, ask the class,
	  ◦ who else solved it "Toni's" way?
	  ◦ who solved it a different way?
	• allow students to interact with the Quick Look Card by drawing on it, point with a laser etc.
	• label each Strategy (1, 2, 3 etc. or by names)
	• record using number model starting with total number of dots (8=5+2+1)
	• if you restate the share solution, ask
	  ◦ Is this right? Is this how you saw it?

Remember– avoid celebrating thinking with "You are so smart!" and use specific praise such as "WOW! You persevered! or "Fantastic strategy! Etc.)	

Number Name Collection

Draw a Picture

Number

Tally Marks

Ten Frames

Rekenrek