

Secondary Strategies that Sustain Sense-Making

NCTM April 15, 2016

Session #328

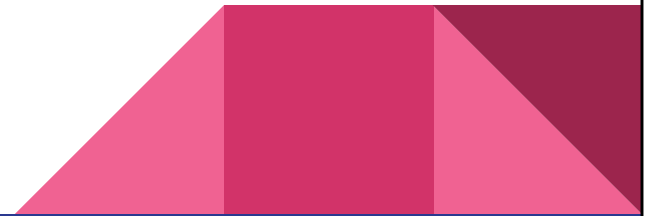
9:30 - 10:30

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Essential Question

Which elementary mathematics strategies can sustain sense-making at the secondary level?

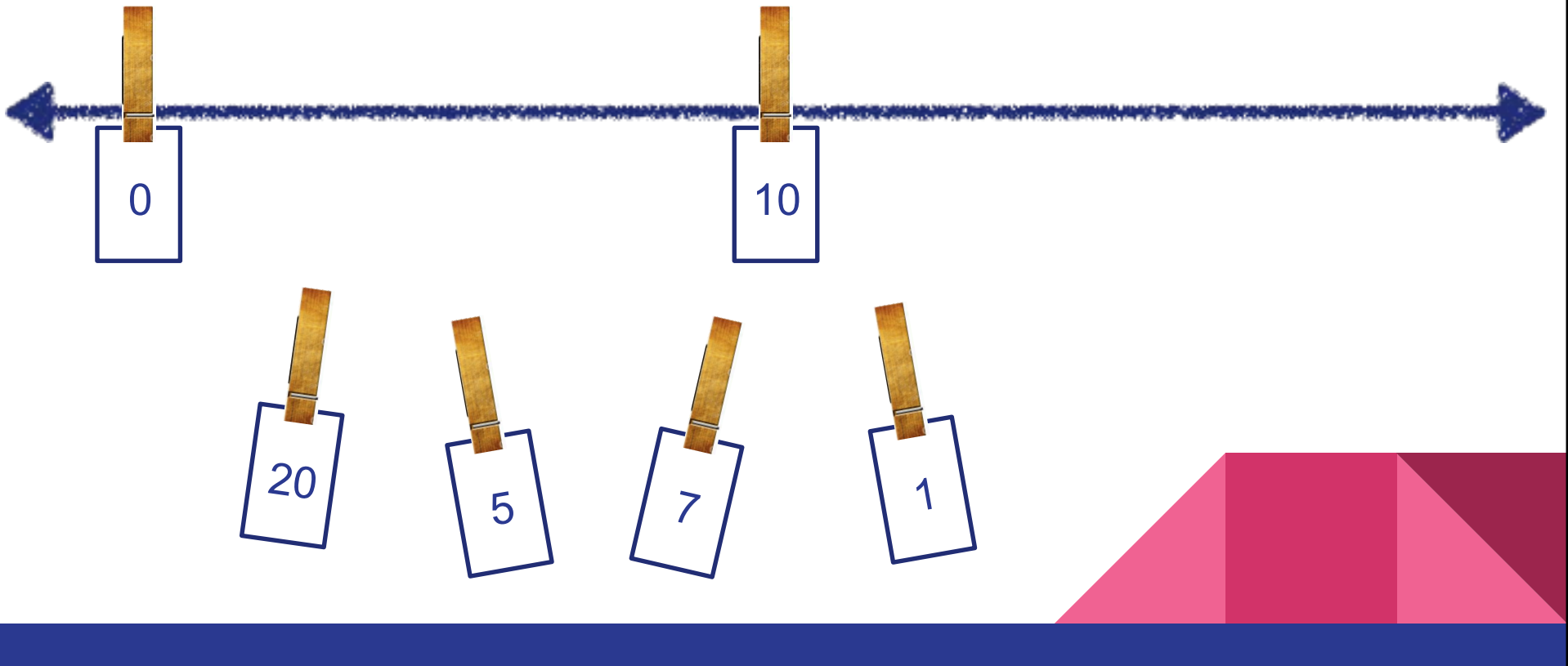


STRATEGIES

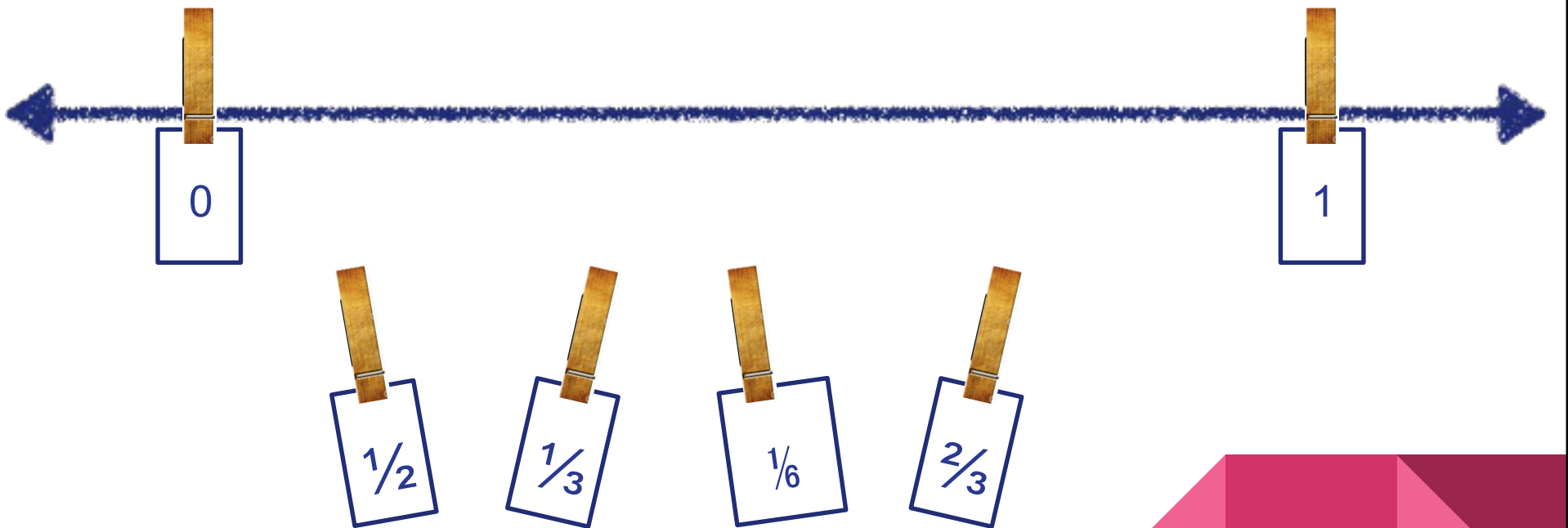
- NUMBER LINES
- FACT FAMILY TRIANGLES
- TAPE/STRIP DIAGRAM (BAR MODELS)
- DIAGRAMMING - SOLVING EQUATIONS
- AREA MODEL
- MANIPULATIVES
- RULE OF FOUR



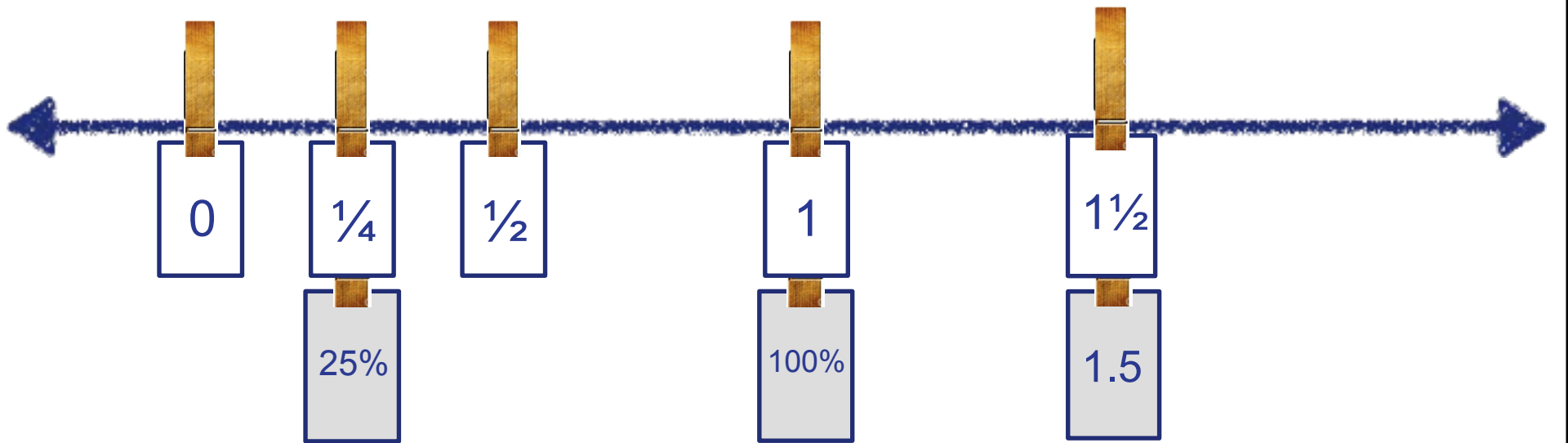
Number Lines and Ordering Counting Numbers



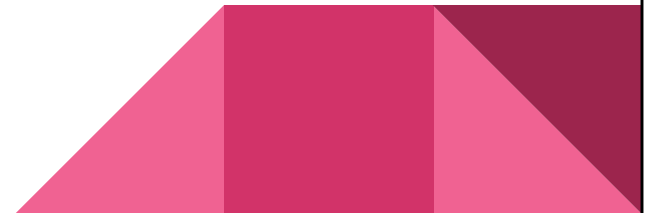
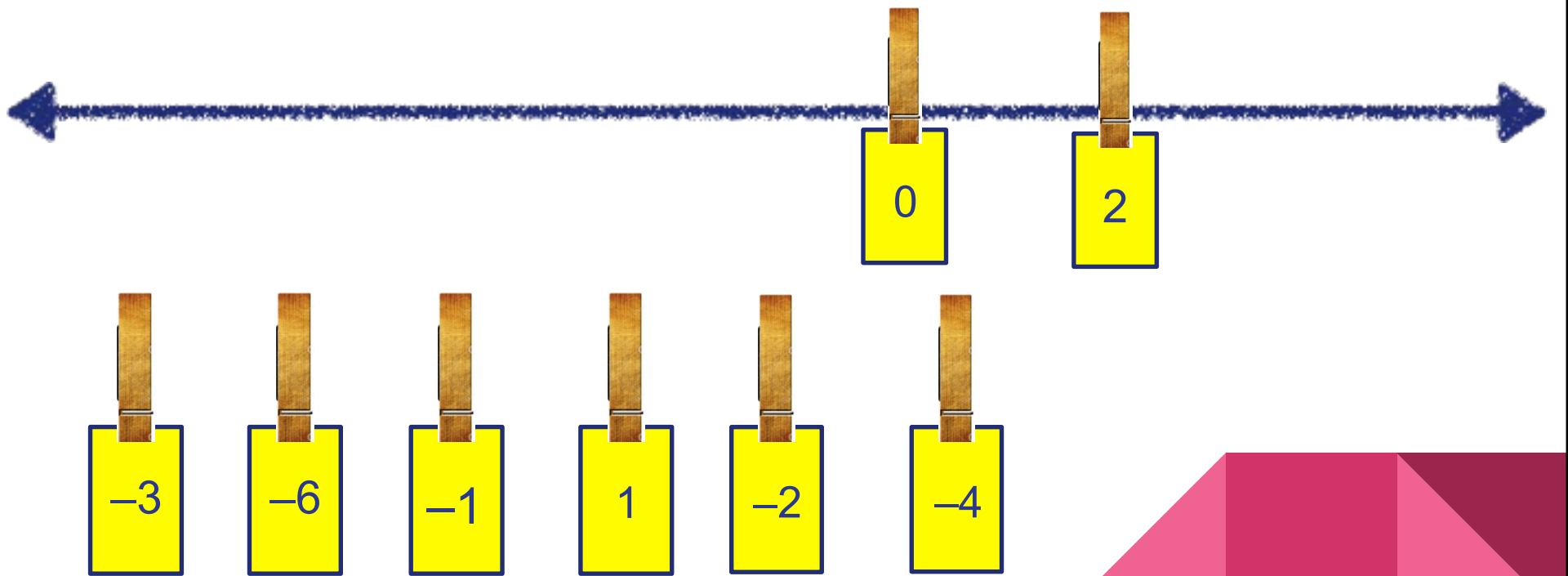
Number Lines and Rational Numbers (Positive Fractions)



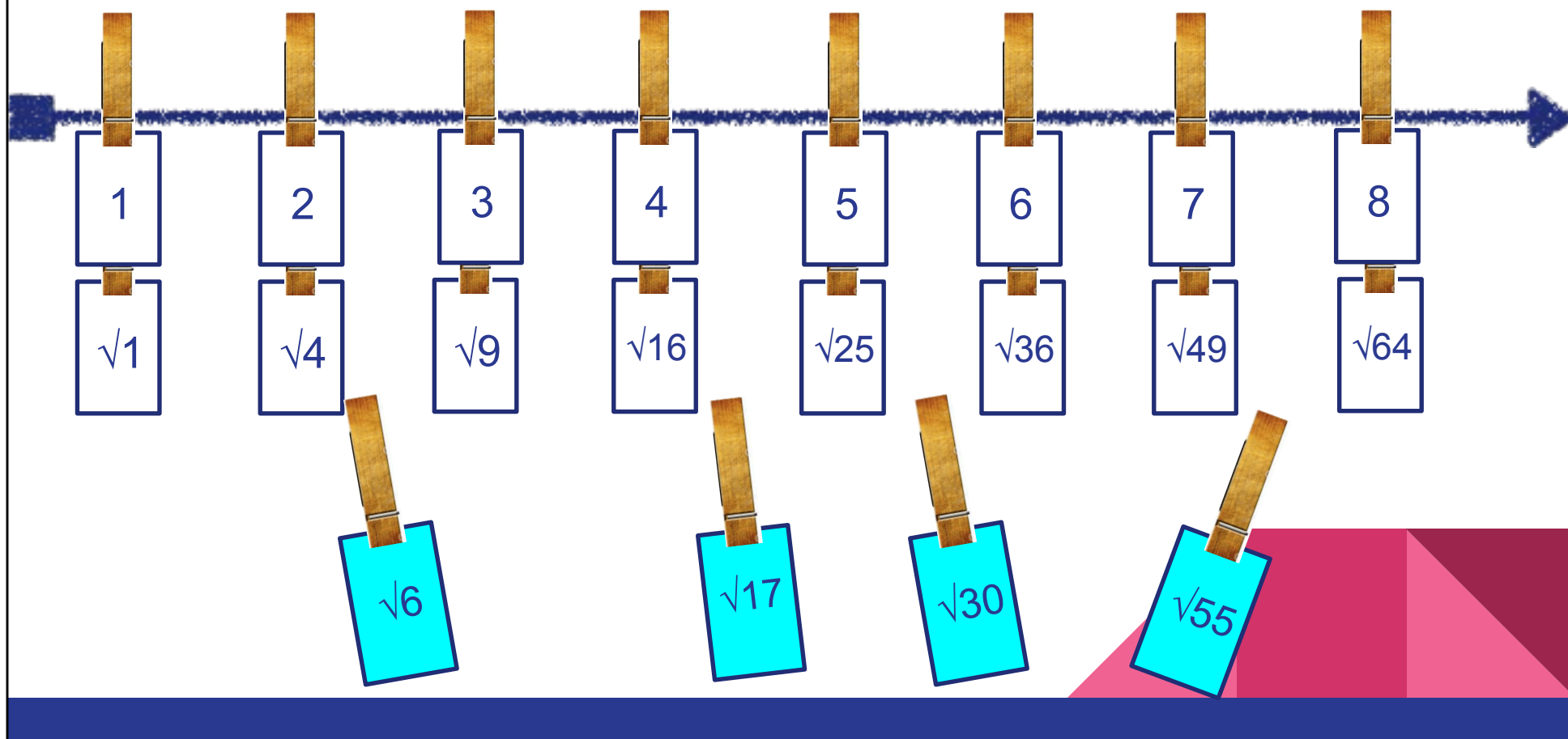
Positive Rational Numbers (Fractions, Decimals, Percents)



The Number Line and Integers



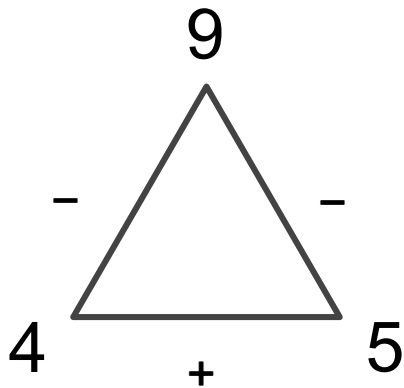
The Number Line, Perfect Squares and Estimating Square Roots



Fact Family Triangles

Create a fact triangle to show the relationship between:

4, 5, 9

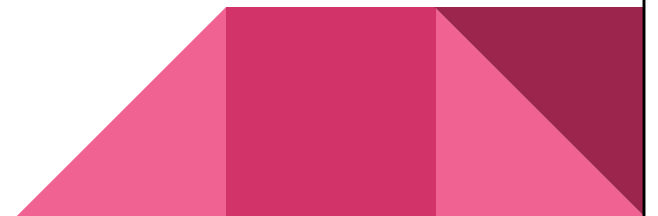


$$4 + 5 = 9$$

$$5 + 4 = 9$$

$$9 - 5 = 4$$

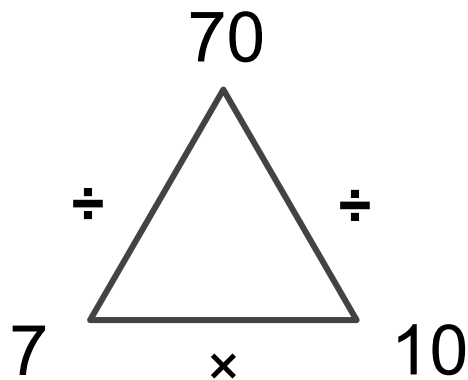
$$9 - 4 = 5$$



Fact Family Triangles

Create a fact triangle to show the relationship between:

7, 10, 70

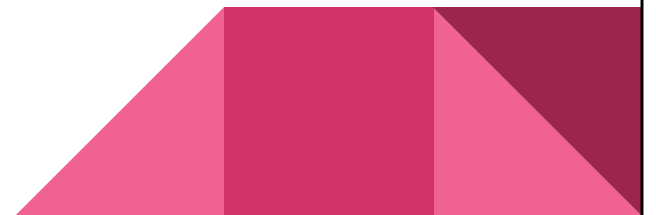


$$7 \cdot 10 = 70$$

$$10 \cdot 7 = 70$$

$$70 \div 10 = 7$$

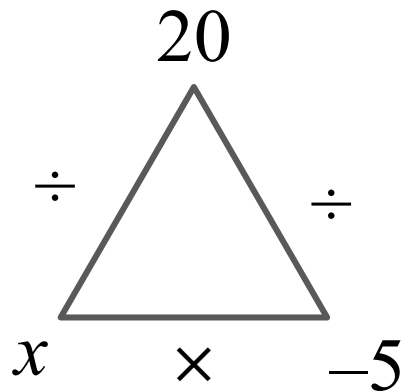
$$70 \div 7 = 10$$



Fact Family Triangles

Solve the equation by using a fact triangle model.

$$20 \div x = -5$$



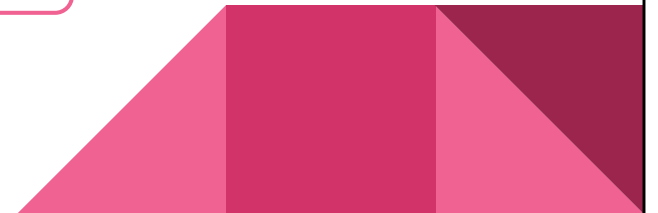
$$x \cdot -5 = 20$$

$$-5 \cdot x = 20$$

$$20 \div x = -5$$

$$20 \div -5 = x$$

$$x = -4$$



FROM PARCC Grade 7 PBA

Solve the equation $\frac{2}{3}(x - 6) = 6$

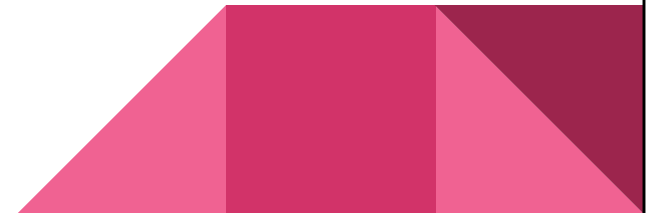
$$\begin{array}{c} 6 \\ \div \quad \triangle \quad \div \\ \frac{2}{3} \quad \cdot \quad (x - 6) \end{array}$$

$$6 \div \frac{2}{3} = x - 6$$

$$9 = x - 6$$

$$x = 9 + 6$$

$$x = 15$$



Grade 7 PARCC PBA Test

The amount of money Jamie earns is proportional to the number of hours she works. Jamie earns \$62.50 working 5 hours.

Create an equation that models the relationship between m , the amount of money Jamie earns, in dollars, and h , the number of hours she works.

Drag and drop the appropriate number and variables into each box.

12.05

12.50

57.50

67.50

m

h

$$\boxed{} = \boxed{} \cdot \boxed{}$$

Bar Models (Strip/ Tape Diagrams)

The amount of money Jamie earns is proportional to the number of hours she works. Jamie earns \$62.50 working 5 hours.

Write and solve an equation to show how much money Jamie makes per hour.

$$5 \cdot x = 62.50$$

$$x \cdot 5 = 62.50$$

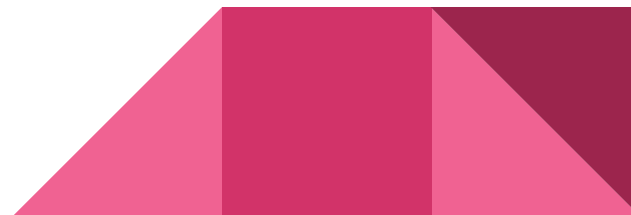
$$62.50 \div x = 5$$

$$62.50 \div 5 = x$$

Jamie



Jamie earns \$12.50 per hour.



Grade 7 PARCC PBA Test

The amount of money Jamie earns is proportional to the number of hours she works. Jamie earns \$62.50 working 5 hours.

Create an equation that models the relationship between m , the amount of money Jamie earns, in dollars, and h , the number of hours she works.

Drag and drop the appropriate number and variables into each box.

12.05 12.50 57.50 67.50 m h

$$\boxed{m} = \boxed{12.50} \cdot \boxed{h}$$

PARCC Grade 7 EOY Test

Devon exercised the same amount of time each day for 5 days last week.

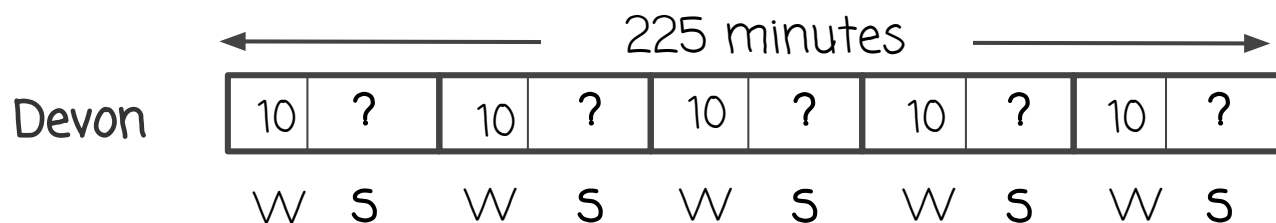
- His exercise included walking and swimming.
- Each day he exercised, he walked 10 minutes.
- He exercised for a total of 225 minutes last week.

What is the number of minutes Devon swam each of the 5 days last week?
Enter your answer in the box.

 minutes

Bar Models (Strip/Tape Diagrams)

Devon exercised the same amount of time each day for 5 days last week. He swam and walked every day. He walked 10 minutes each day. How many minutes each day did he swim? He exercised for a total of 225 minutes last week.



Devon swam
for 35 minutes
each day.

$$(10 + x) \cdot 5 = 225$$

$$225 - 10(5) = 5x$$

$$\frac{225 - 10(5)}{5} = x$$

$$10(5) + 5x = 225$$

$$225 - 5x = 10(5)$$


PARCC Grade 7 EOY Test

Devon exercised the same amount of time each day for 5 days last week.

- His exercise included walking and swimming.
- Each day he exercised, he walked 10 minutes.
- He exercised for a total of 225 minutes last week.

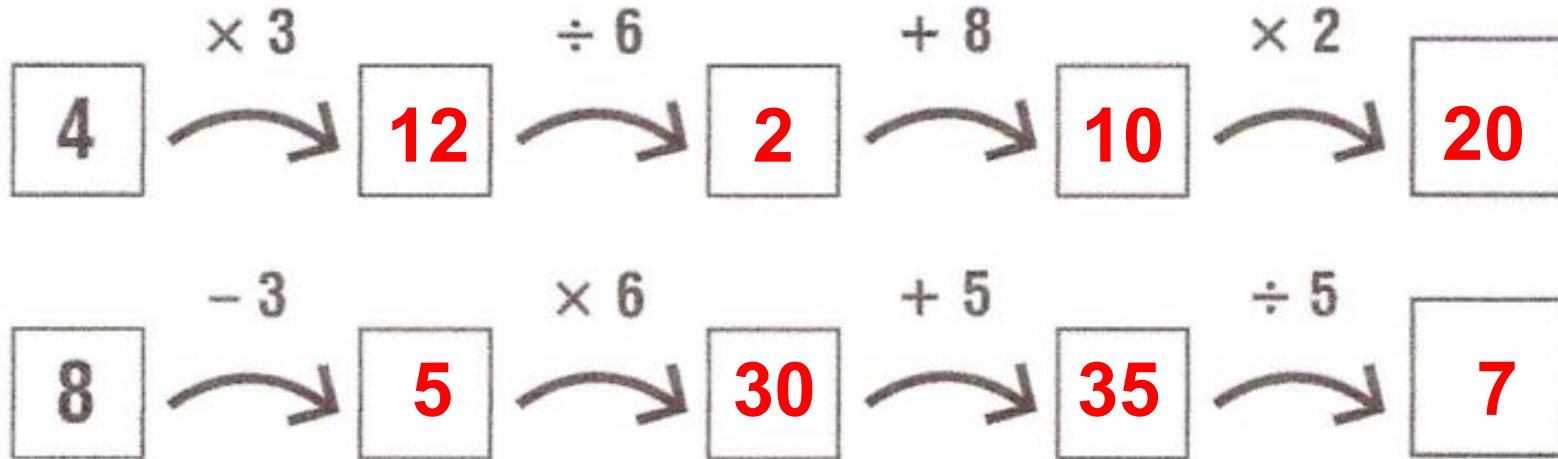
What is the number of minutes Devon swam each of the 5 days last week?
Enter your answer in the box.

minutes



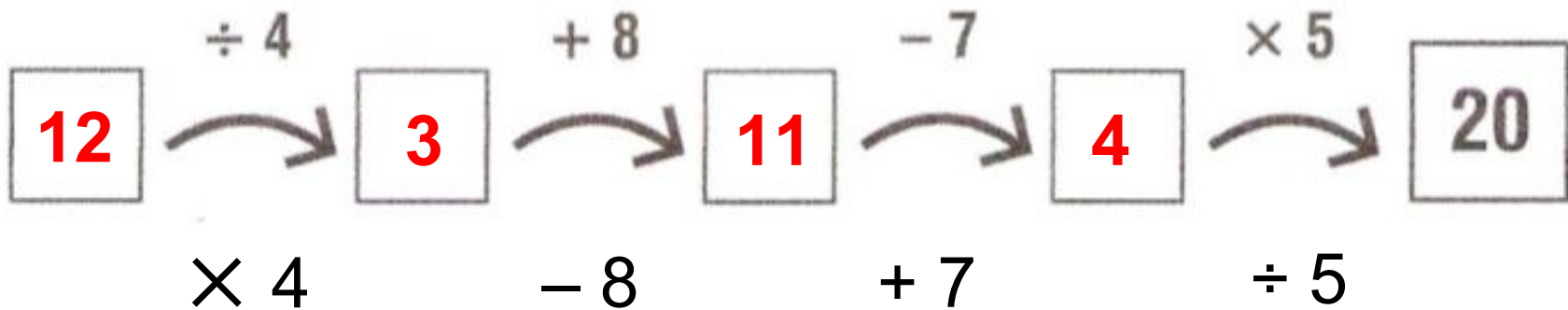
DIAGRAMMING

COMPLETE THE OPERATIONS SHOWN IN THE DIAGRAMS.



DIAGRAMMING

What was the input in the diagram shown below?

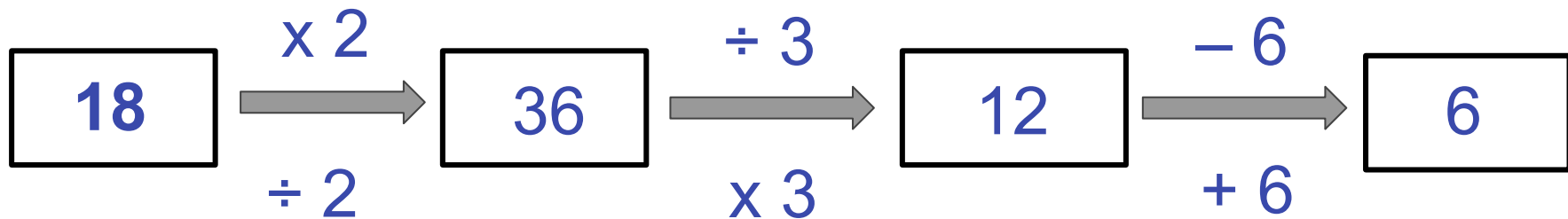


What equation could this diagram represent?

$$(n/4 + 8 - 7) \cdot 5 = 20$$

DIAGRAMMING

Solve the equation by diagramming. $\frac{2}{3}y - 6 = 6$



Grade 7 PARCC PBA Test

Jessica rented 1 video game and 3 movies for a total of \$11.50.

- The video game cost \$4.75 to rent.
- The movies cost the same amount each to rent.

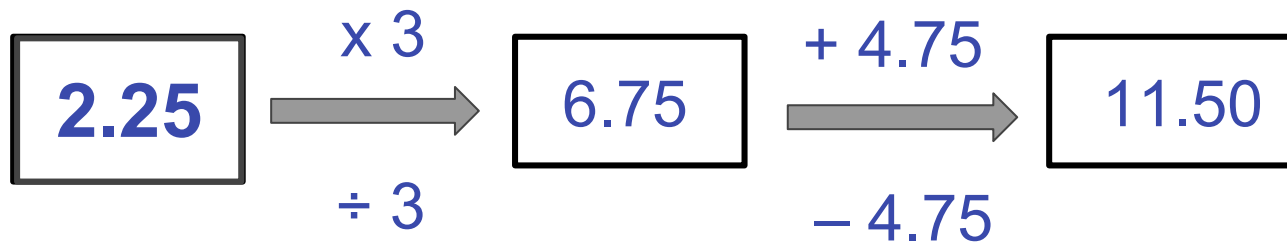
What amount did Jessica pay to rent each movie?

Enter your answer in the box.

\$

PROBLEM SOLVING USING DIAGRAMMING

Jessica rented one video game and three movies for a total of \$11.50. The video game cost \$4.75 to rent. Each movie cost the same amount to rent. What amount did Jessica pay to rent each movie?



Jessica paid \$2.25 to rent each movie.

Grade 7 PARCC PBA Test

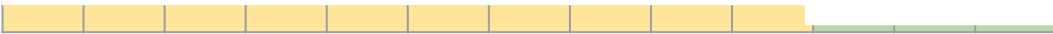
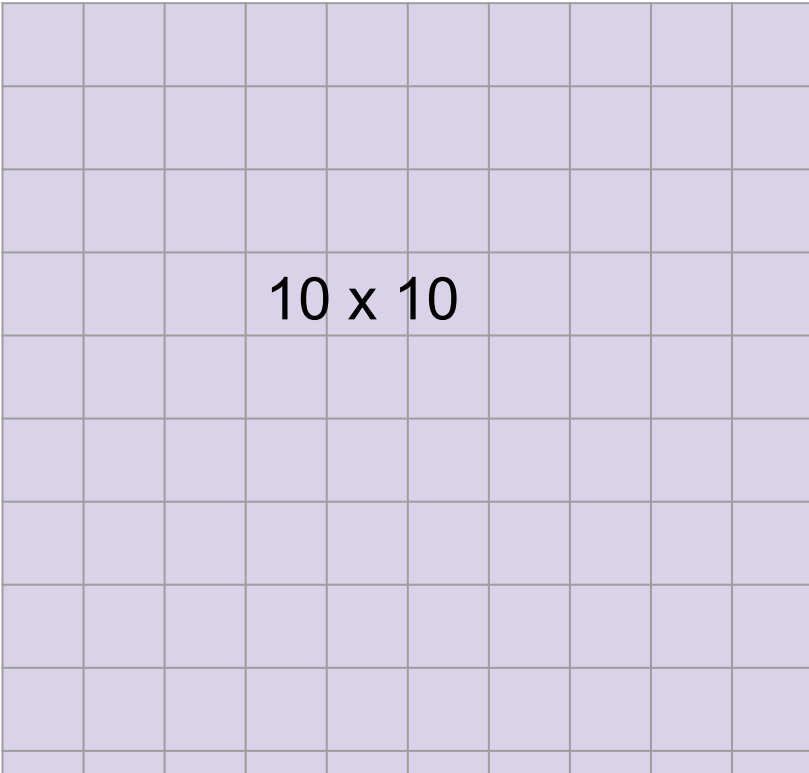
Jessica rented 1 video game and 3 movies for a total of \$11.50.

- The video game cost \$4.75 to rent.
- The movies cost the same amount each to rent.

What amount did Jessica pay to rent each movie?

Enter your answer in the box.

\$



AREA MODEL:

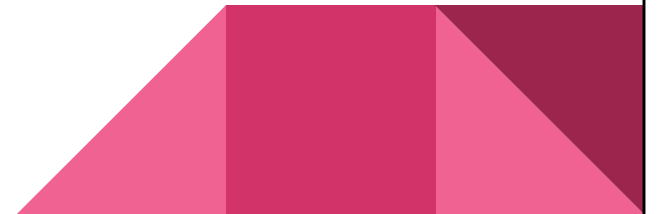
$$13 \times 12$$

$$= (10 + 3)(10 + 2)$$

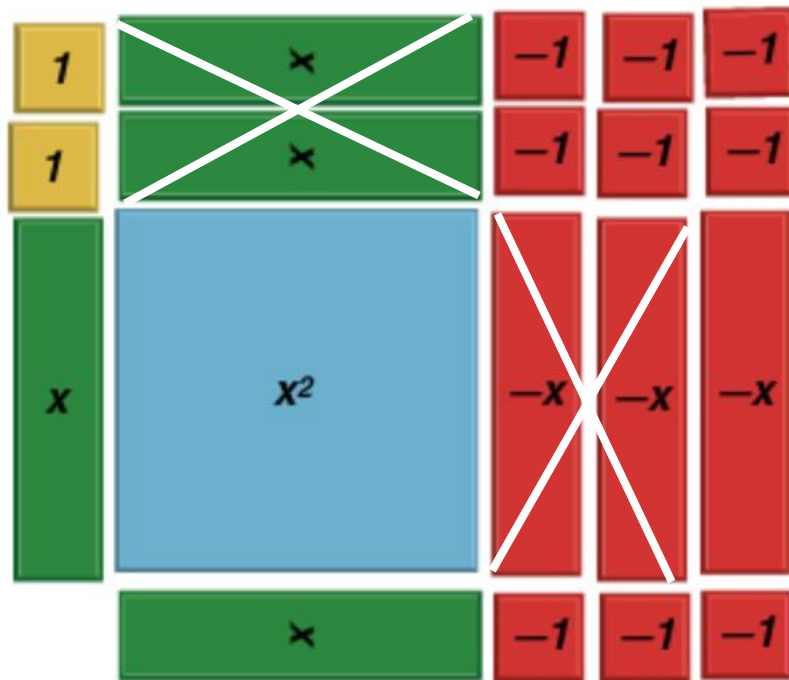
$$= 10 \cdot 10 + 10 \cdot 2 + 3 \cdot 10 + 2 \cdot 3$$

$$= 100 + 30 + 20 + 6$$

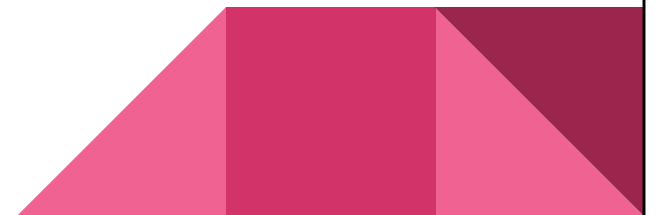
$$= 156$$



AREA MODEL (Algebra Tiles): $(x + 2)(x - 3)$



$$= x^2 - 1x - 6$$



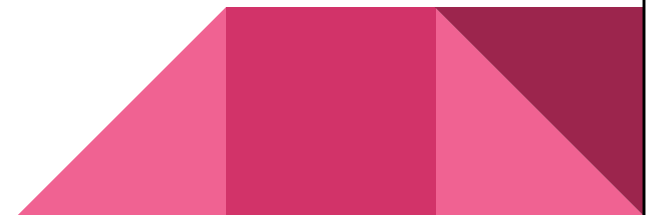
AREA MODEL (Box Strategy)

	x	2
x	x^2	$2x$
-3	$-3x$	-6

$$(x + 2)(x - 3)$$

$$= x^2 + 2x - 3x - 6$$

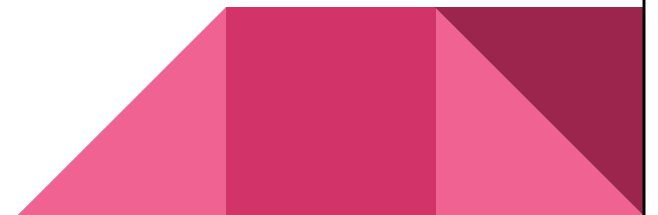
$$x^2 - x - 6$$



AREA MODEL (Box Strategy)

	$3x$	-2
$2x$		
-1		

$$(3x - 2)(2x - 1)$$
$$= 6x^2 - 3x - 4x + 2$$
$$6x^2 - 7x + 2$$



Factoring using Box Strategy $-12n^2 - 11n + 15$

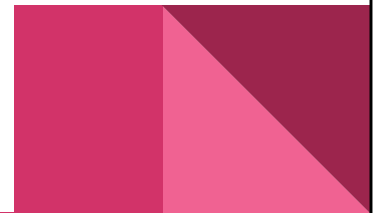
	$-3n$	-5
$4n$	$-12n^2$	$-20n$
-3	$9n$	$+15$

$$= (4n - 3)(-3n - 5)$$

$$-180n^2$$

$$-18n, 10n$$

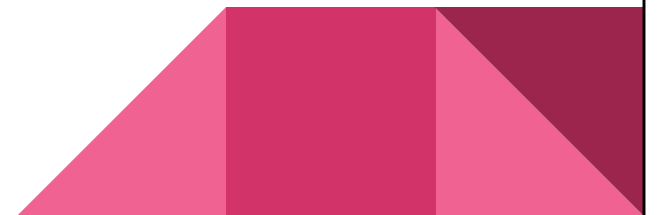
$$-20n, 9n$$



Algebra 1 PARCC PBA

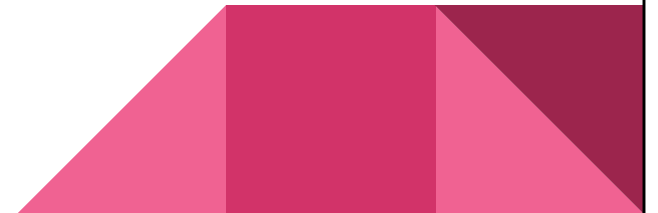
Which factorization can be used to reveal the zeros of the function $f(n) = -12n^2 - 11n + 15$?

- A. $f(n) = -n(12n + 11) + 15$
- B. $f(n) = (-4n + 3)(3n + 5)$
- C. $f(n) = -(4n + 3)(3n + 5)$
- D. $f(n) = (4n + 3)(-3n + 5)$



STRATEGIES

- NUMBER LINES
- FACT FAMILY TRIANGLES
- TAPE/STRIP DIAGRAM (BAR MODELS)
- DIAGRAMMING - SOLVING EQUATIONS
- AREA MODEL
- MANIPULATIVES
- RULE OF FOUR



Resources

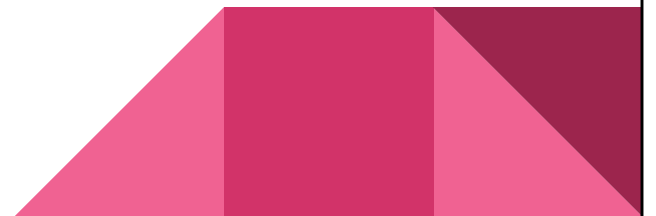
- ❖ Common Core State Standards, 2011
- ❖ www.mathplayground.com/thinkingblocks.html
- ❖ [McGraw-Hill Fact Triangle app](#)
- ❖ PARCC [Practice Test items](#)
- ❖ learner.org (area model)
- ❖ [glencoe virtual manipulatives](#)
- ❖ CCSSM Progressions document - [Ratio and Proportion](#)

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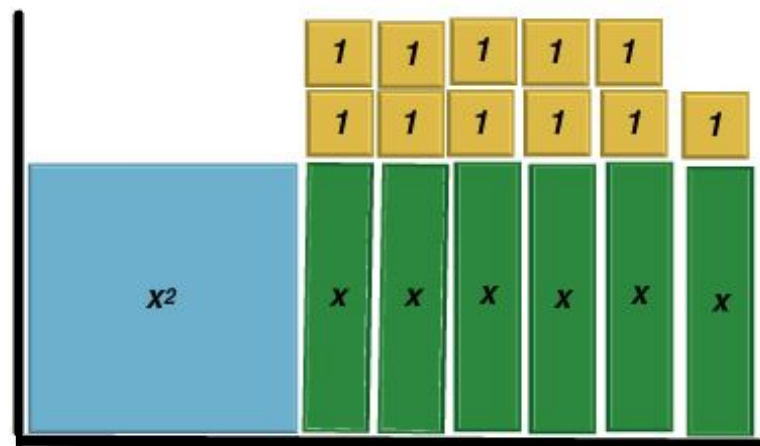
Completing the Square and Vertex Form

Making Sense of Quadratic Functions using
Algebra Tiles



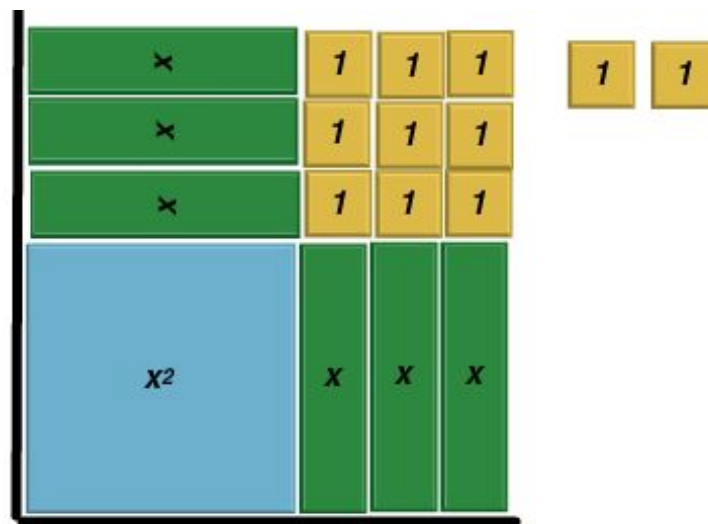
This expression has too many pieces for it to be a perfect square. How many extra ones do you have? Rewrite as an equivalent expression having a perfect square component.

$$x^2 + 6x + 11$$



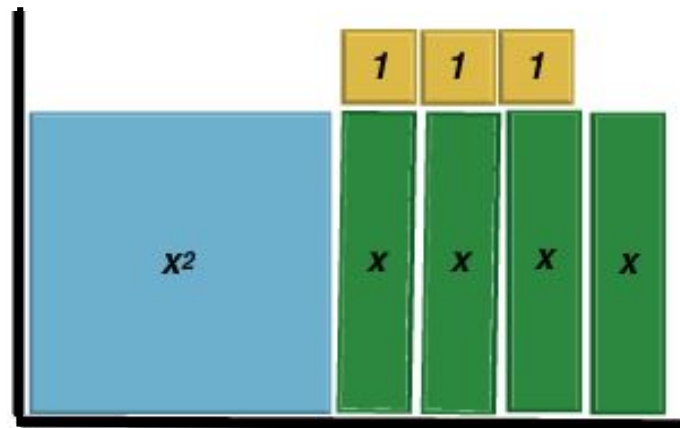
This expression has too many pieces for it to be a perfect square. How many extra ones do you have? Rewrite as an equivalent expression having a perfect square component.

$$x^2 + 6x + 11 = (x + 3)^2 + 2$$



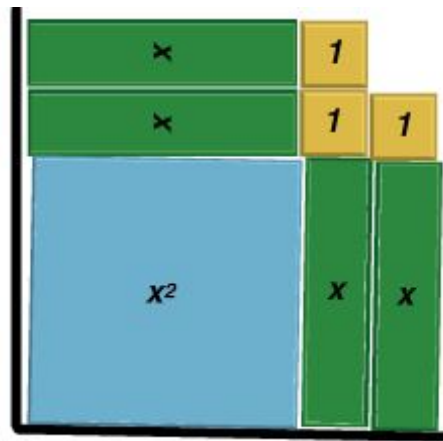
This expression is missing pieces to make it a perfect square. How many ones do you need? Rewrite as an equivalent expression having a perfect square component.

$$x^2 + 4x + 3$$



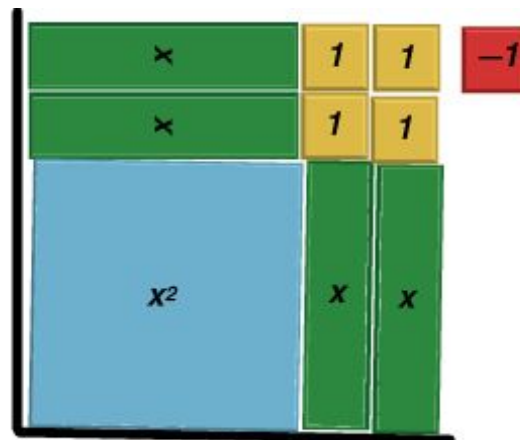
This expression is missing pieces to make it a perfect square. How many ones do you need? Rewrite as an equivalent expression having a perfect square component.

$$x^2 + 4x + 3$$



This expression is missing pieces to make it a perfect square. How many ones do you need? Rewrite as an equivalent expression having a perfect square component.

$$x^2 + 4x + 3 = (x + 2)^2 - 1$$



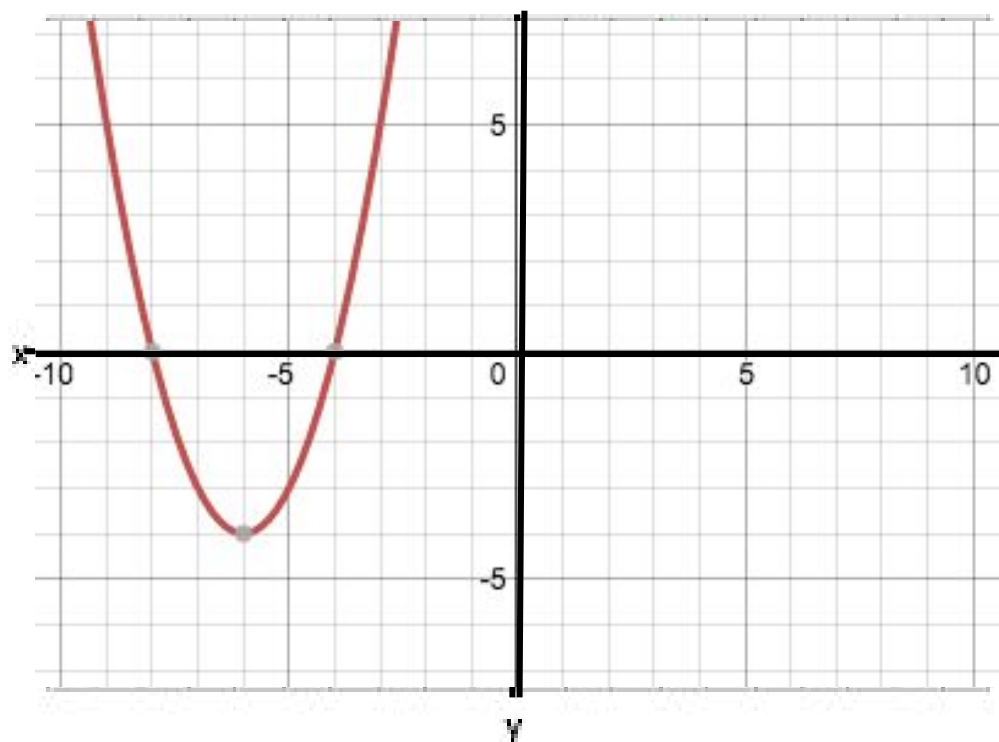
Write in vertex form. Graph the function.

$$y = x^2 + 12x + 32$$

$$y = x^2 + 12x + 36 - 36 + 32$$

$$y = (x + 6)^2 - 4$$

Vertex $(-6, -4)$



Algebra 1 PARCC PBA

The cost to manufacture x pairs of sunglasses can be represented by a function, $C(x)$.

Select from the drop-down menus to correctly complete the statement about function C .

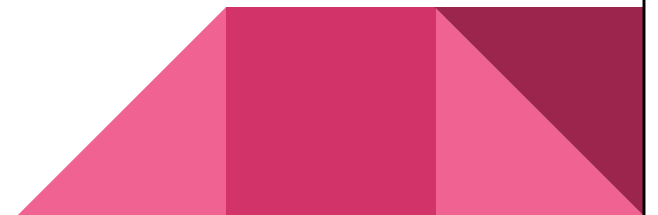
If $C(4) = 398$, then pairs of sunglasses cost \$.

0	4.00
1	398.00
4	1592.00
99.50	
398	

"RULE OF FOUR"

The cost to manufacture 3 pairs of sunglasses is \$298.50.

Assuming the cost to manufacture each pair of sunglasses is constant, determine the cost of manufacturing 4 pairs of sunglasses.

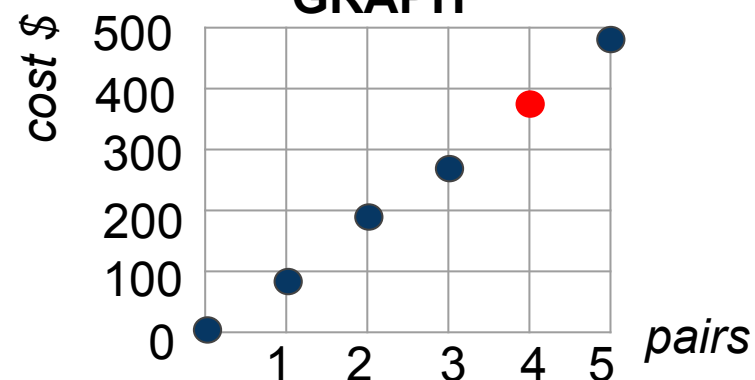


PROBLEM SOLVING USING THE "RULE OF FOUR"

TABLE

pairs (x)	0	1	2	3	4
cost C(x)	0	99.5	199	298.5	398

GRAPH



EQUATION

$$C(x) = 99.5x$$
$$C(4) = 99.5(4)$$
$$C(4) = \$398.00$$

CONTEXT

The cost to manufacture 3 pairs of sunglasses is \$298.50.

The cost per pair is \$99.50.

4 pairs of sunglasses cost \$398.00.

Algebra 1 PARCC PBA

The cost to manufacture x pairs of sunglasses can be represented by a function, $C(x)$.

Select from the drop-down menus to correctly complete the statement about function C .

If $C(4) = 398$, then pairs of sunglasses cost \$.

0	4.00
1	398.00
<input type="text" value="4"/>	1592.00
99.50	
398	

Grade 8 PARCC EOY Test

Two different proportional relationships are represented by the equation and the table.

Use the drop-down menus to complete the sentence comparing the rates of change of the proportional relationships.

Proportion A

$$y = 9x$$

Proportion B

x	y
0	0
3	34.5
5	57.5
8	92

The rate of change in Proportion A is

Choose...



Choose...



than the rate of change in Proportion B.