A Pedi for the Lady...

And Other Engaging Calculus Activities

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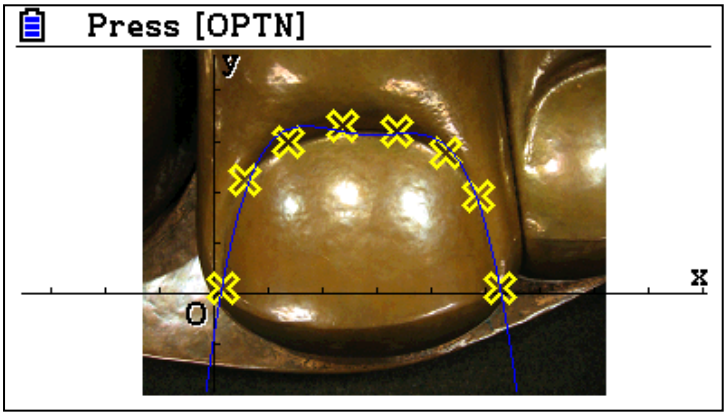
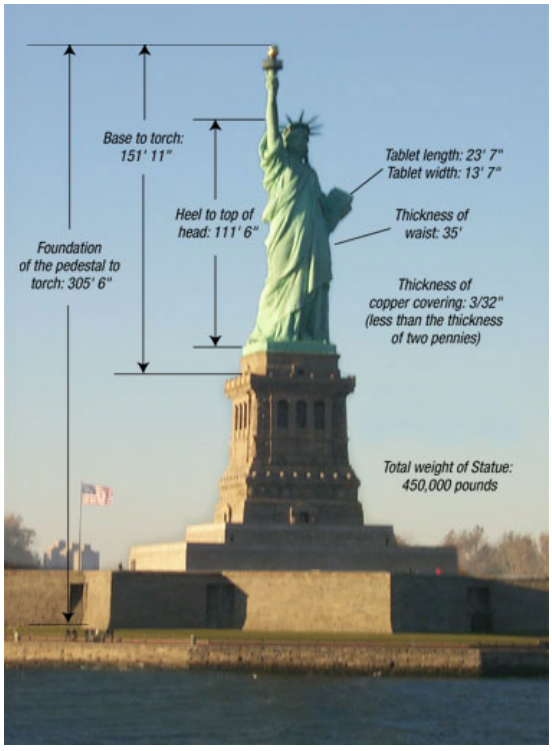
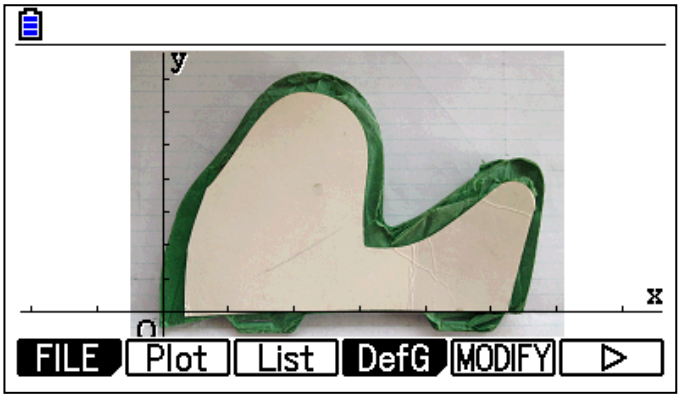
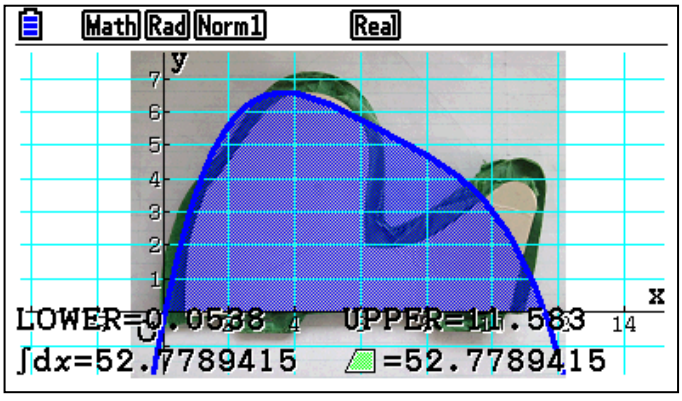
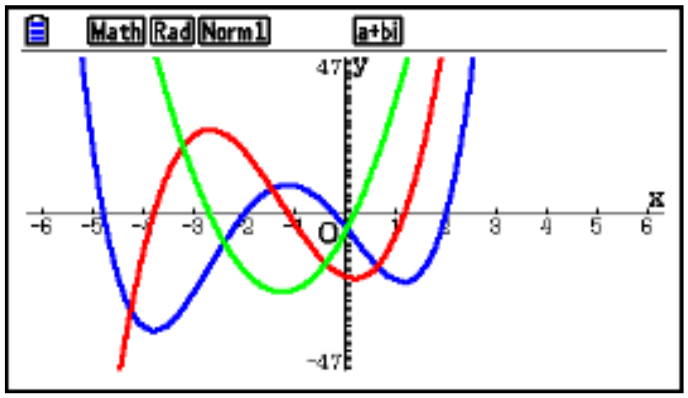
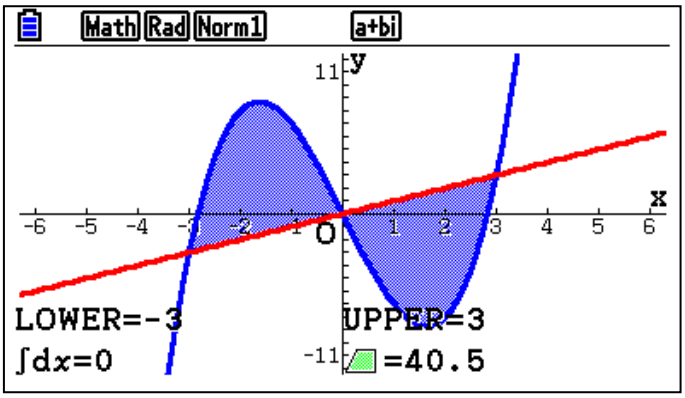






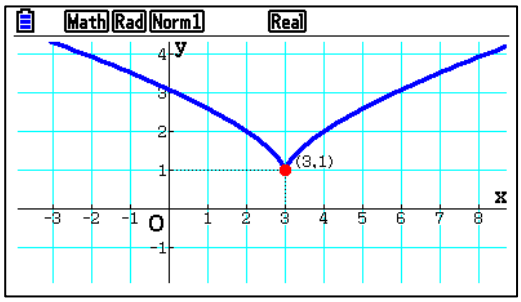


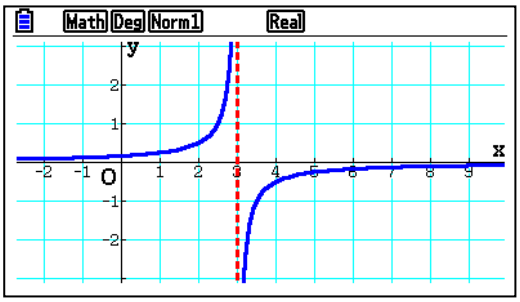


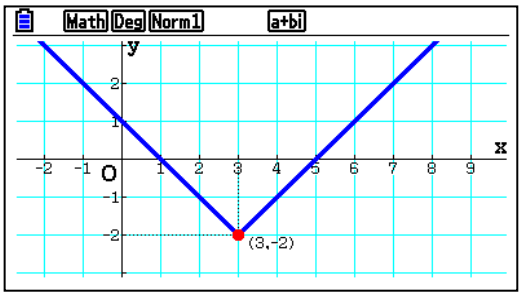


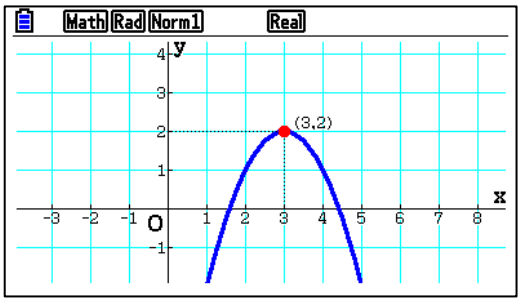
Sorting Cards Activity **Cut the cards out and sort according to each activity.**

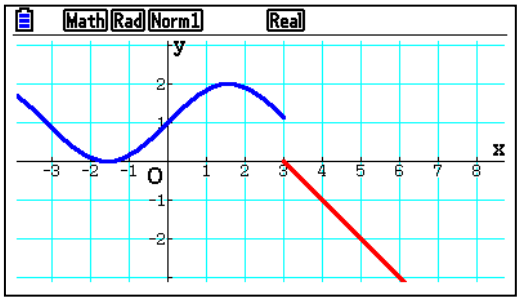
I. Continuous vs. Discontinuous at x=3
II. Limits that exist and that do NOT exist at x=3
III. Derivatives that exist and that do NOT exist at x=3

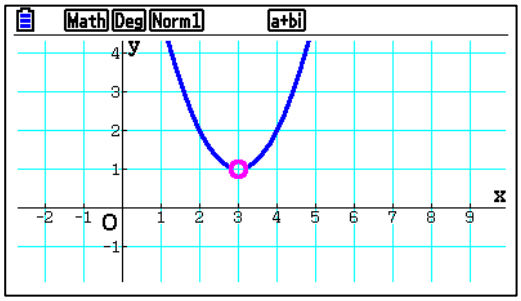


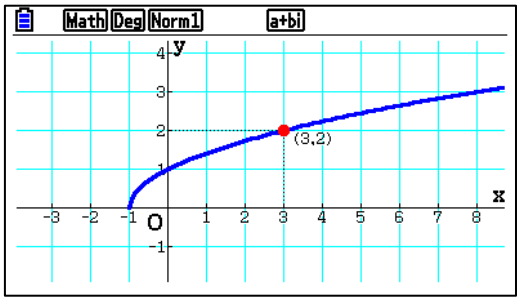


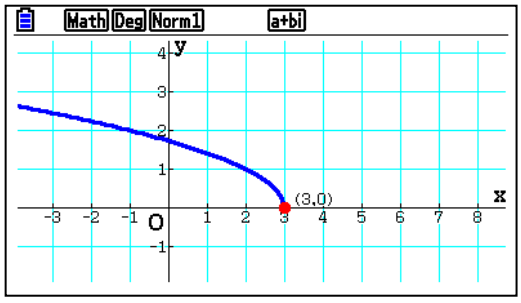


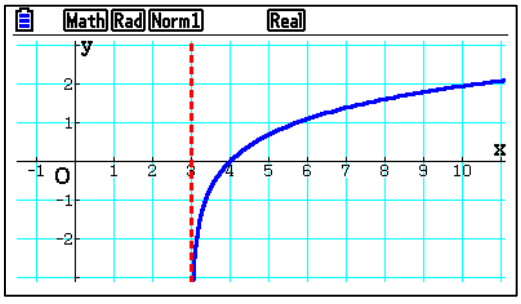


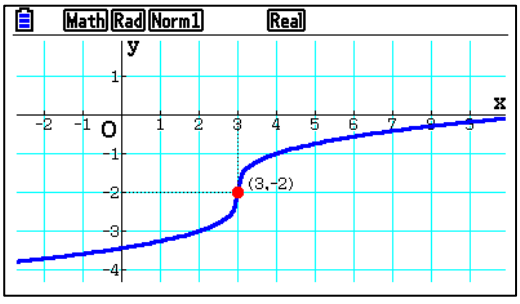






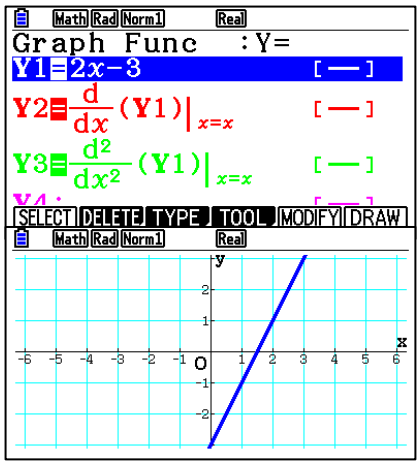






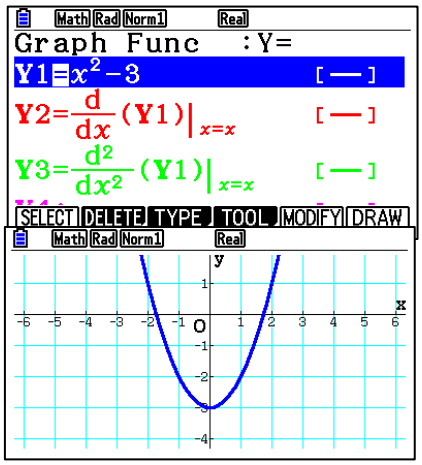
Discovering Derivative Rules

The derivative of a function represents the behavior of the slope of the function at each point along its domain. Given Y, graph Y' and Y" in order to discover methods of formulating derivatives analytically. Below each graph, write the equation you *think* would produce the graph of the derivative and then check by graphing it as your Y4 graph. Repeat the process for Y".

1. 

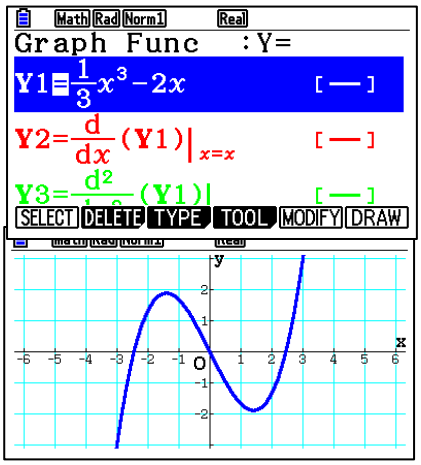
$y' =$ _____

$y'' =$ _____

2. 

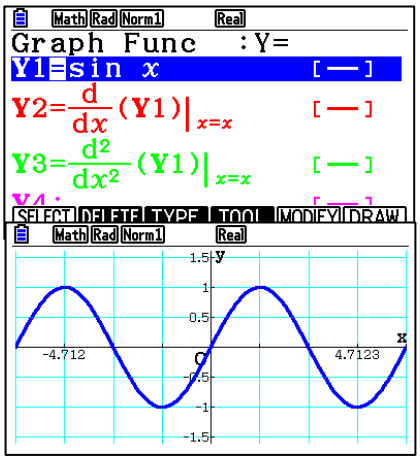
$y' =$ _____

$y'' =$ _____

3. 

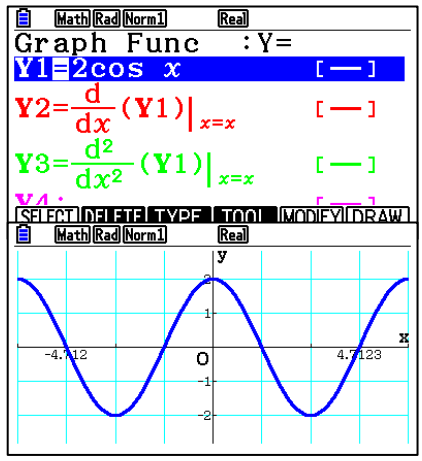
$y' =$ _____

$y'' =$ _____

4. 

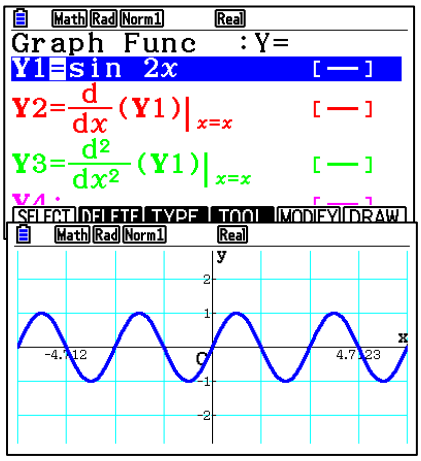
$y' =$ _____

$y'' =$ _____

5. 

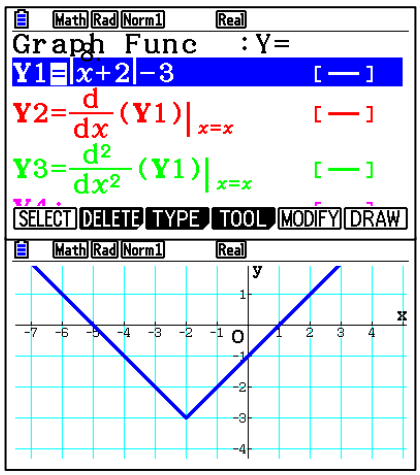
$y' =$ _____

$y'' =$ _____

6. 

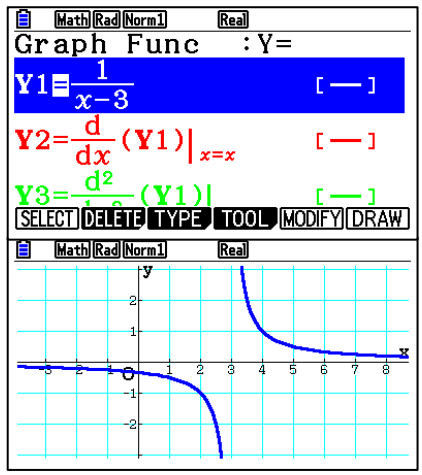
$y' =$ _____

$y'' =$ _____

7. 

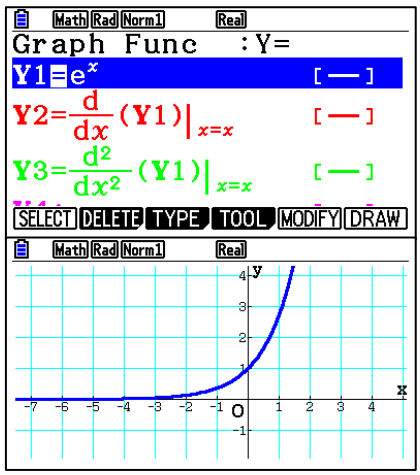
$y' =$ _____

$y'' =$ _____

8. 

$y' =$ _____

$y'' =$ _____

9. 

$y' =$ _____

$y'' =$ _____

Comparing characteristics of f, f', and f"

Using a graphing calculator, produce the graph of $f(x) = x^4 + 5x^3 - 3x^2 - 20x - 4$. Adjust the view window so that all of the critical values are visible. Now graph the first derivative. If you graph vertical lines at the points where the first derivative is zero, what do you notice? _____

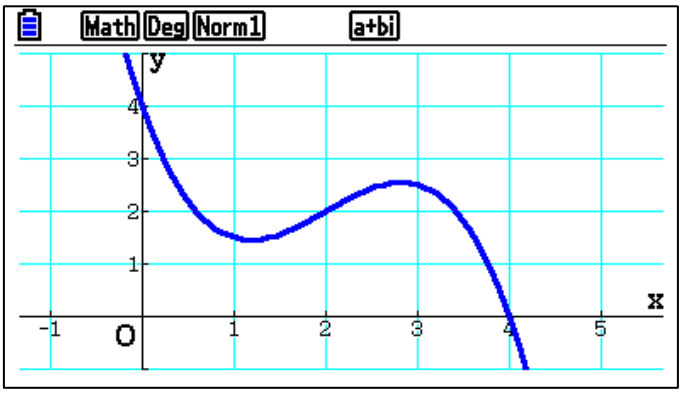
Return to the graph entry screen and graph the second derivative with the function. Graph vertical lines where the second derivative is zero. Do you notice any relationship between the two graphs?

_____

Lastly, graph the first derivative and the second derivative. Graph vertical lines where the second derivative has a value of zero. What do you notice? _____

Using some "ideas" about derivative graphs, <u>model</u> a possible first and second derivative graph using a Wikki Stix that could be produced by the following functions. Use a different color Wikki Stix for the first and the second. Once completed, compare with your partner.

1.



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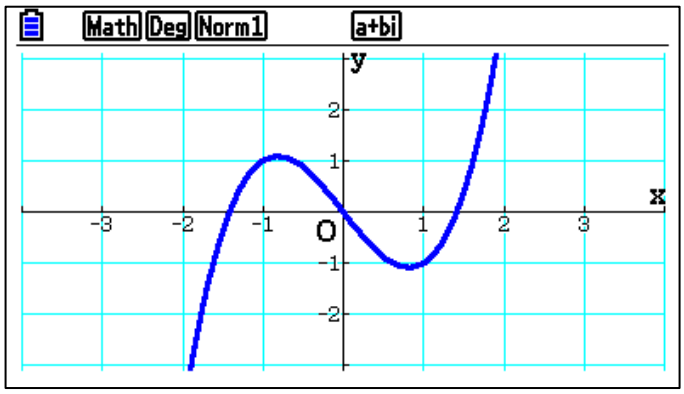
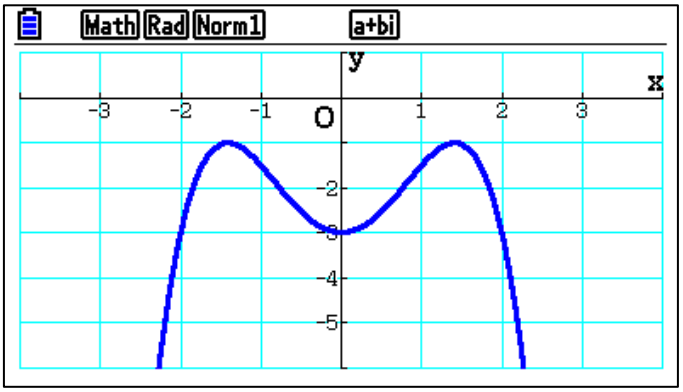
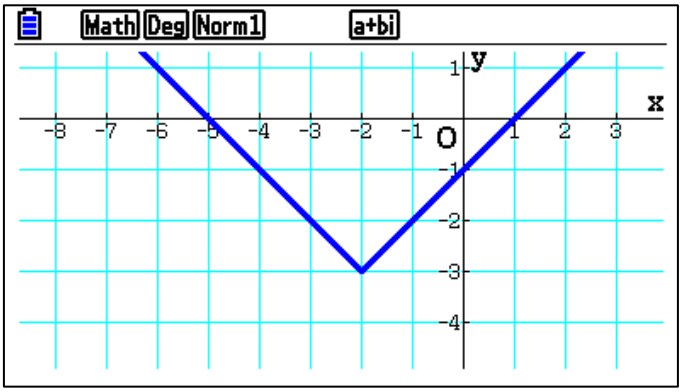


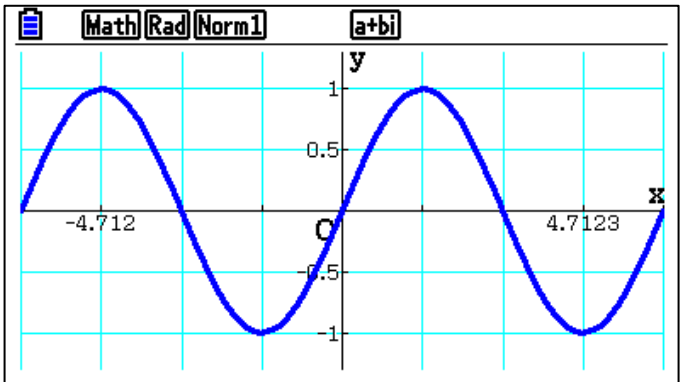
3.



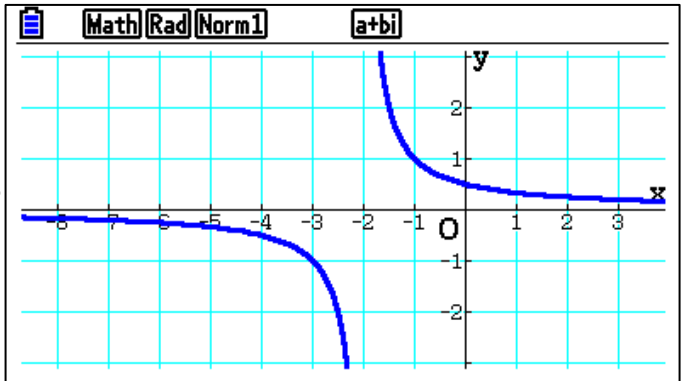
4.



5.

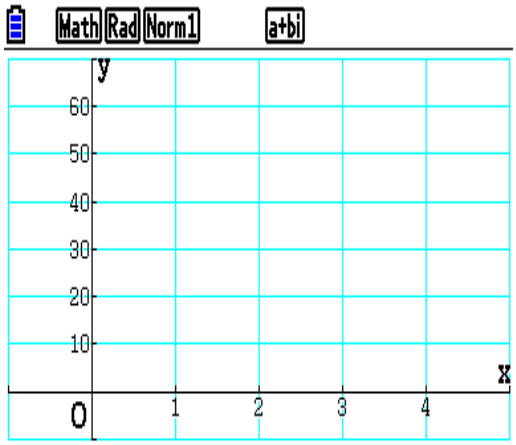


6.



Did you know that D=*RxT* is connected to velocity, integration, and the area under a curve? Let's see how this simple concept is considered to be *CALCULUS!*

1. Texas Pete is on I-20 moving west at the rate of 60 mph. At 1:00, he presses the cruise control on the car and then lets the car travel at this *steady* rate for 3 hours.



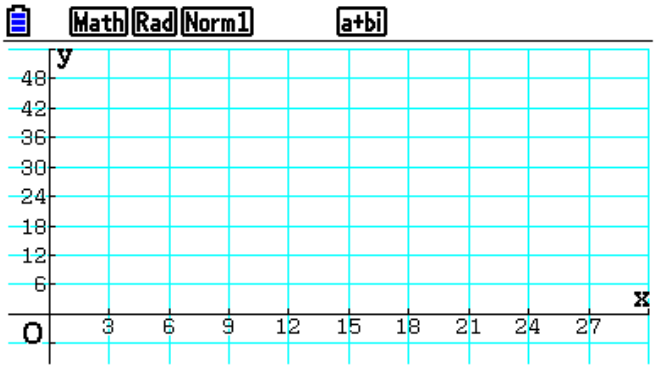
a) Draw a graph for this problem representing the 3 hours from 1:00.

b) Find the area trapped between the graph and the x-axis.

c) Describe the meaning of the area in context to this problem.

d) Find the slope of the graph and describe its meaning with respect to the problem.

2. Professor Ben Dover is going from home to work. It is a straight path from his home to work with no stops along the way. It takes him 6 minutes to get to 48 mph. He then travels at 48 mph for 18 minutes and then it takes him another 3 minutes to come to a stop.



a) Draw the graph of this problem.

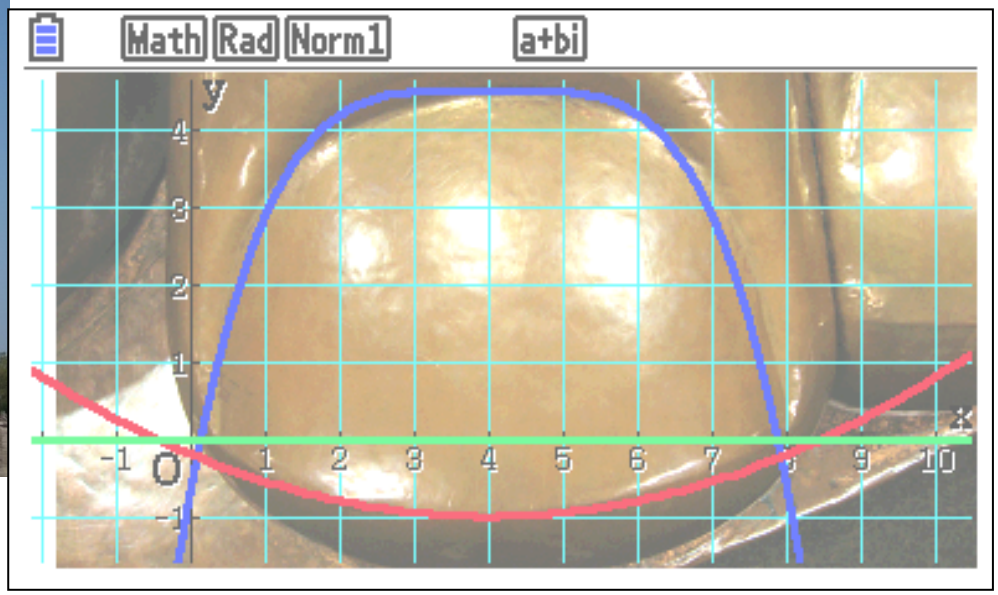
b) Find the area under the curve and state the meaning of this area as related to this problem.

c) Find the slope of the first six minutes and the last three minutes. What do these numbers represent?

3. After a hike in Muir Woods, Sasquatch is filling his hot tub with hot water using his on-demand water heater. As time passes, the hot water is dispersed more slowly. The table below states the rates at which the heater fills the hot tub at 20 minute intervals. If the hot tub initially had 10 gallons in it, how many gallons are in it at the end of 120 minutes?

Time min	0	20	40	60	80	100	120
Rate gal/min	50	46	46	40	35	30	20

A Pedi For the Lady



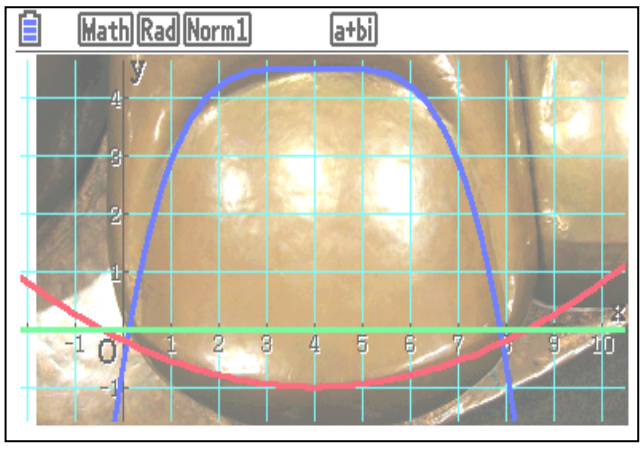
1. Estimate the area in square inches trapped between the curves outlining the shape of Lady Liberty's big toe nail by counting squares. Each unit in the picture is equivalent to about .45 feet. Round final answers to tenths place.

Area ≈ _____ square units

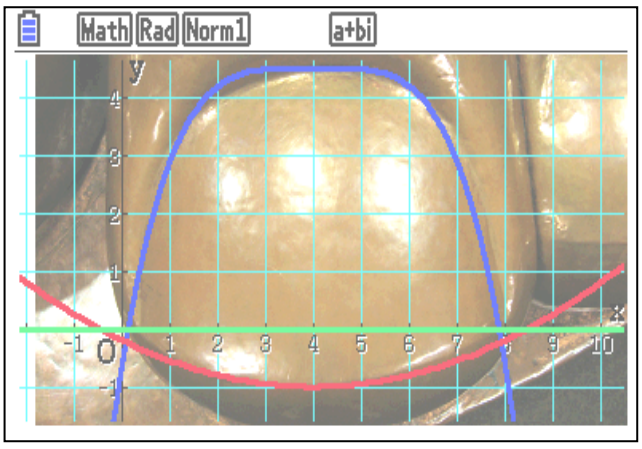
Area ≈ _____ square feet

2. Color/Highlight the appropriate area by each of the following Riemann Sum Methods using **4** equal sub-intervals.

Right-Hand



Left-Hand



3. Estimate the area by each of the following methods using 4 equal subdivisions.

Right-Hand= $\int_0^8 Y1\,dx \approx$ _____ sq. **units**

Left-Hand= $\int_0^8 Y1\,dx \approx$ _____ sq. **units**

Right-Hand= $\int_0^8 Y1\,dx \approx$ _____ sq. **feet**

Left-Hand= $\int_0^8 Y1\,dx \approx$ _____ sq. **feet**

A Pedi For the Lady **continued…….**

4. What is the significance of the 0 and 8 on the integral to this problem?_____

What could be done to make this method of estimating more accurate? _____

What *width* would each rectangle approach if an infinite number of rectangles were "squeezed" in between 0 and 8? _____

5. Find **Quartic** equations modeling the **toenail's** shape from above and below of the Statue of Liberty's big toenail. Enter the coordinates where x-values are integers into the regression feature of your calculator. Round values to the thousandth's place.

Upper Curve: Y1= _____ Lower Curve: Y2= _____

(Usually I do not give the values. I have students estimate values from the graph in order to have slightly different answers.)

x	y
0	-.2
1	2.8
2	4.2
3	4.4
4	4.5
5	4.4
6	4.2
7	3
8	-.2

x	y
0	-.1
1	-.6
2	-.8
3	-.9
4	-1
5	-.9
6	-.8
7	-.6
8	-.1

6. Enter an equation that represents the trapped area using the equations from step 5. Find this area by ***integrating*** between the x-coordinates of the intersection points. The x-coordinate of the intersection points provide the lower, a, and upper, b, values for the integral below.

a=_____ & b=_____ Area: $\int_a^b \left[(Y1) - (Y2) \right] dx \approx$_____

7. An ounce of polish cost about **$25** per ounce. It will cover approximately **20** square feet. Use this information to come up with a reasonable estimate for painting the toenail on the largest toe. Then give an estimate for painting all ten toenails. Provide justification and work for your estimate.

8. Research and the state 5 interesting facts about the Statue of Liberty that you did not already know.

1.

2.

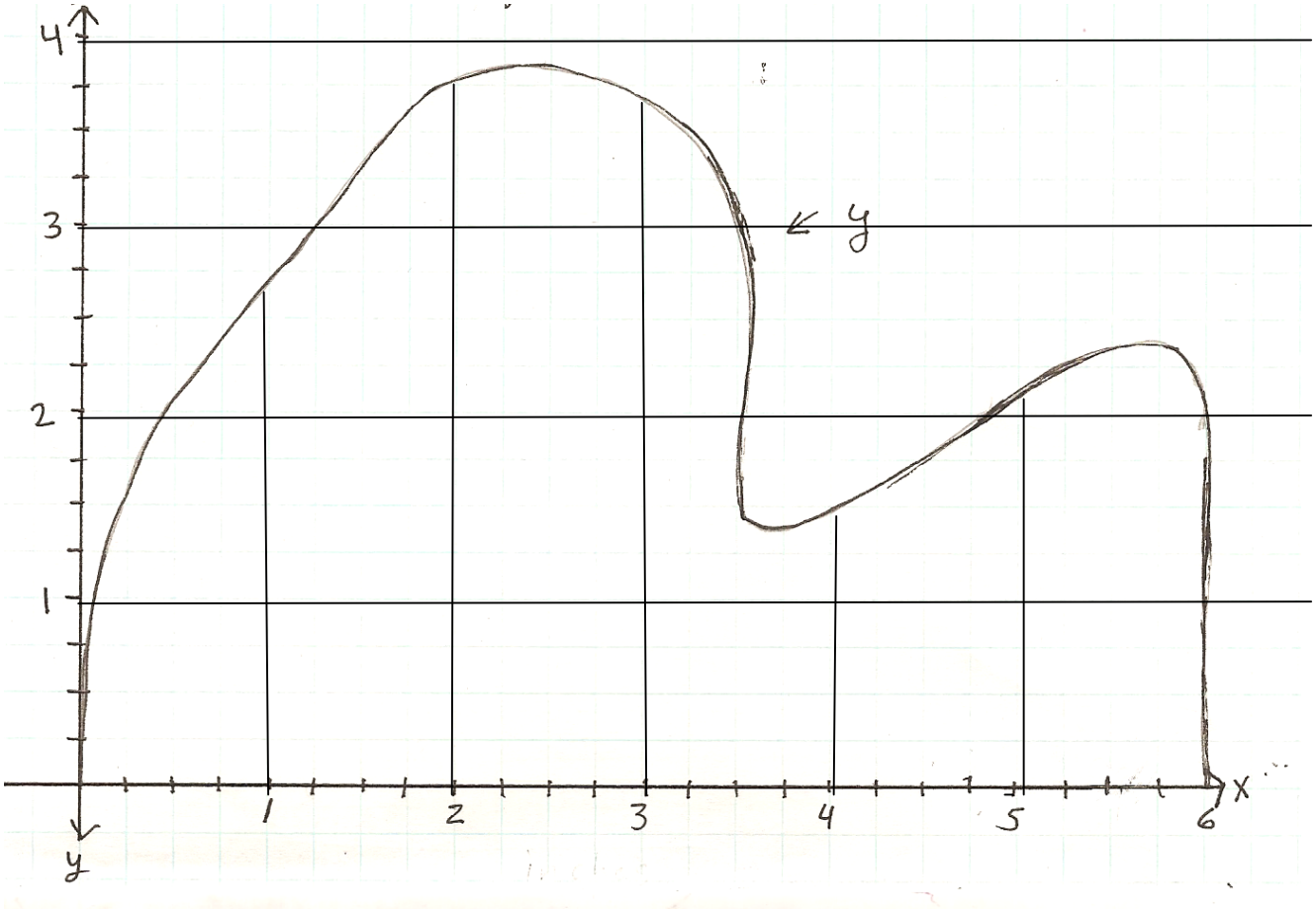
3.

4.

5

Slicing a Mushroom: Finding Area and Volume Activity

1. Estimate the area, in square inches, trapped between the curve and the x-axis by counting squares. Area_____



2. Lightly color the **area** covered by using the **Midpoint** Riemann Sum Method with 6 sub-intervals, each 1-inch long. Use a different color for each rectangle. Estimate and record the area covered by this method.

3. Find the *polynomial* that best fits the mushroom's shape by using the regression feature of your calculator. To increase accuracy, use the points from the integer and midpoint values on the graph.

Y1=_____

4. Graph the equation from step 3.

Find the area by "integrating" between the roots of Y1: **Area** = $\int_a^b Y1\,dx$ =_____

What do the roots represent in context to this graph? _____

Find the area by using this integral: **Area**: $\int_0^6 Y1\,dx$ =_____

What is the significance of 0 and 6 on this integral? _____

5. Compare the calculations for the area in step **2** and 4. What could be done to make these estimates better?

6. Now…. Imagine the shape being spun about the *x-axis* in such a way that it forms a ***solid***. Find the **volume** of this solid using the same 6 divisions as in step 2. Each of these divisions forms cylinder-like slices with the height of the graph at each midpoint representing the radius of a cylinder.

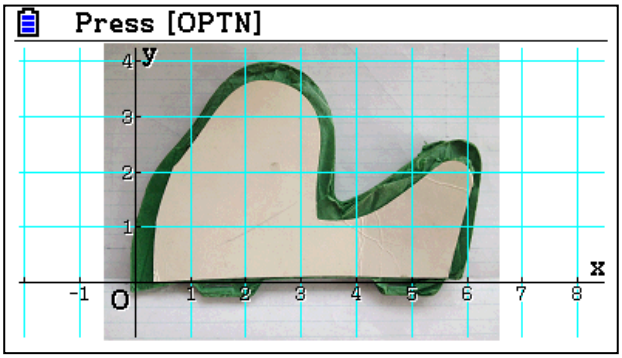
Volume of *any* cylinder: V=_____

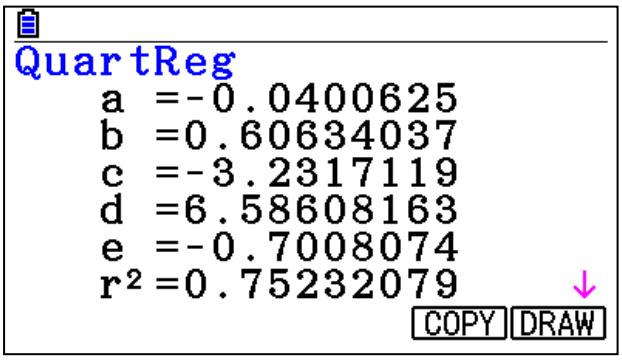
Volume of this figure estimated by using 6 cylindrical slices: V= _____

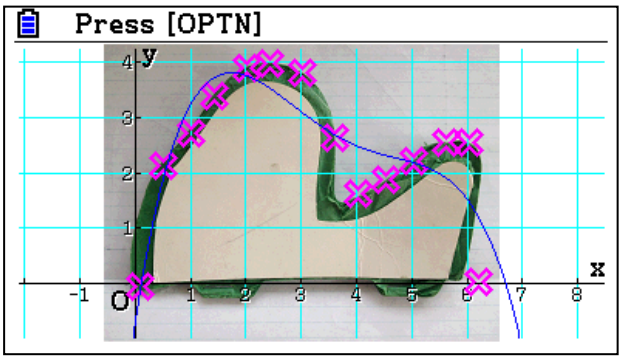
7. Using the equation from step 3, find the volume of this solid by "integrating" between the x-intercepts.

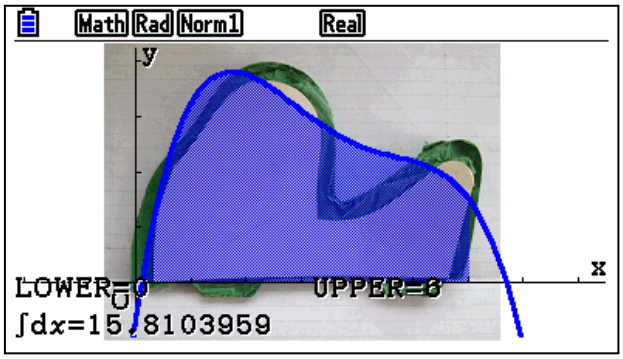
Volume= $\pi\int_0^6 (Y1)^2\,dx$ = _____ What does $Y1$ represent in this expression? _____

Using the Casio Picture Converter, any picture can be imported, and then many explorations can be done.









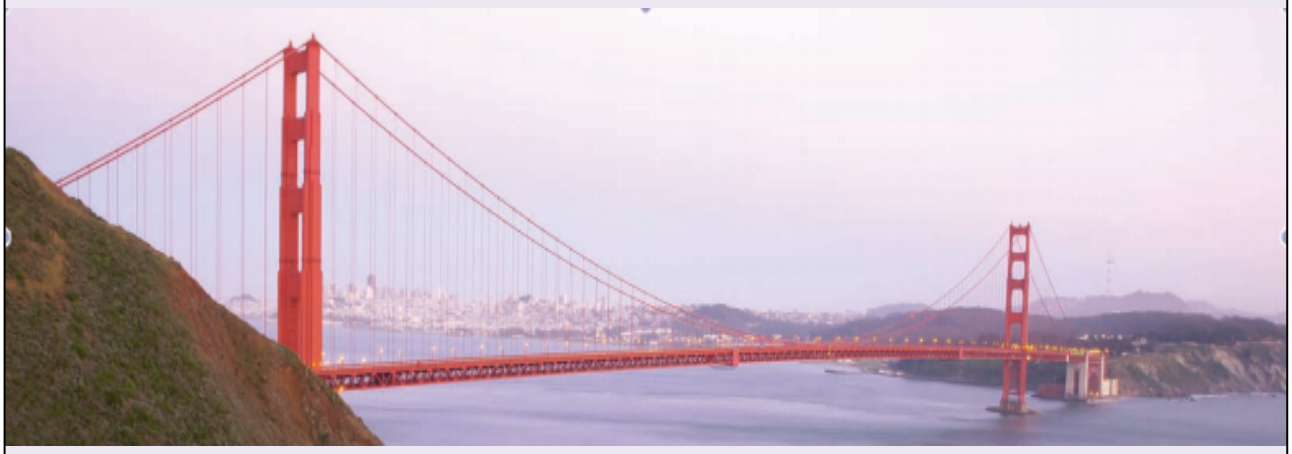
8. How does the area found using this method compare to the estimates found in steps **2** and **4**?

9. Estimate the volume by substituting the quartic equation into $V = \pi\int_0^6 (Y1)^2\,dx$ and calculating the integral.

(Based on the derived equation for Y1, the volume is approximately 145.651 cubic inches.)

10. How does this compare to your answer in question 6? _____

What calculus can we take back to our students from San Francisco?



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