Unpacking Division: Understanding the Structure

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Teaching for Understanding

Standards

- Common Core Content Standards
  - Grade 3: Understand properties of multiplication and the relationship between multiplication and division.
  - Grades 5: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.
- Standards for Practice:
  - Model with mathematics.
  - Look for and make use of structure.

391 ÷ 23 = ?

- What does this mean?
- Can you put it into context?
- What language do we use to talk about division?
- Does the language change when we write it this way?

$$23\overline{)391}$$

Multiplication

- 8 x 6 = 48
  - 8 groups
  - 6 in each group
  - 48 in all

- A x *B* means *A* groups with *B* in each group.

Modeling Contextual Problems

- Use unit cubes to model the action:
  - I have **18** cookies. I will give my **three** friends the same amount until there are no cookies left. How many cookies will each person get?
  - Write an equation for this word problem.
- Use unit cubes to model the action:
  - I have **18** cookies. I will give my friends **three** cookies each until there are none left. How many friends can I give cookies to?
  - Write an equation for this word problem.
- How are these alike? How are they different?

Division: Fair Shares (Partitive)

- I have **18** cookies. I want to share them equally with **three** friends. How many cookies should each of my friends get?

- **3 groups of __?__ is 18**

- **How many in each group?**

Division: Equal Groups (Measurement)

- I have **18** cookies. I will give my friends **three** cookies **each** until there are none left. How many friends can I give cookies to?

- **__?__ groups of 3 is 18**

- **How many groups?**

- Start with 18, take out groups of 3. How many groups?
- OR Start with 18. How many times can you subtract 3?

How many groups?
How many in each group?

What does 12 ÷ 3 mean?

- Fair shares: How can 12 be shared fairly among 3?
  - 3 groups with __?__ in each group is 12

- Equal groups: If 12 are put into groups of 3, how many groups will there be?
  - __?__ groups with 3 in each group is 12

How many groups?
How many in each group?

Model with Base Ten units:

- 48 children are put on 4 teams. How many on each team?

- 48 children are put into groups of 4. How many groups?

Algorithm for Division of Whole Numbers

Use Base Ten blocks to model:

- 248 apples put into 8 bags. How many in each bag?
- If you drove 567 miles at a constant speed and if it took you 9 hours, how fast were you going?
- 152 cookies to be shared with the class. Each student gets 4 cookies. How many students are there?

Fractions as Division

3 tomatoes shared equally among 4 salads.

How much tomato is in each salad?

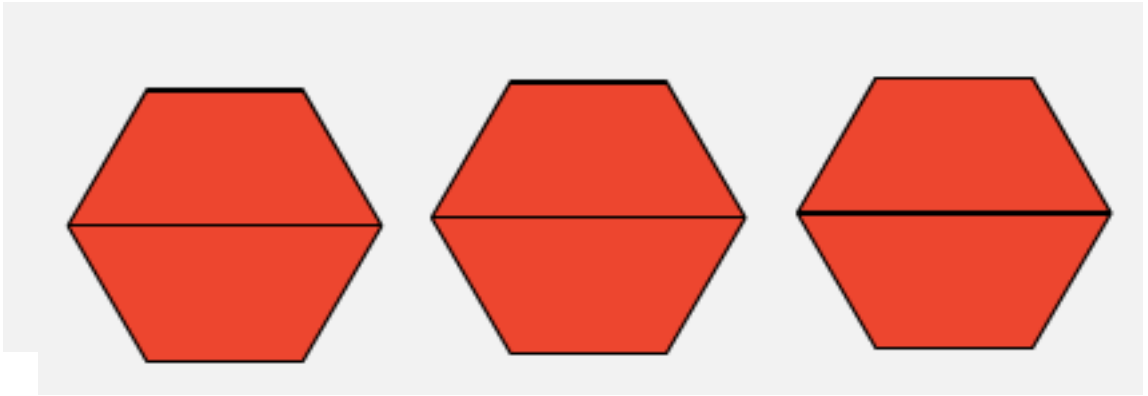
6 miles of highway to be repaved in 4 days.

How much has to be completed each day?

One meaning of $\frac{3}{4}$ is 3 divided by 4.

Division with Fractions

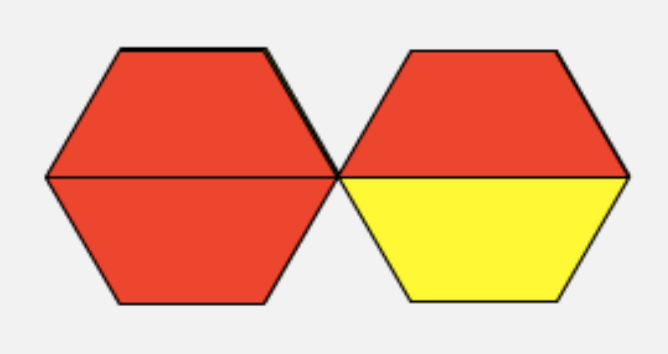
- What does $3 \div \frac{1}{2}$ mean?
- Hint: How many groups?
- Use Pattern blocks to model.



Division with Fractions

- What does $\frac{3}{4} \div 3$ mean?
- Hint: How many in each group?
- We need both meanings to make sense of division with fractions.
- Model with pattern blocks

  or fraction circles.



Division with Fractions

At the end of the day, a bakery had one-half of a loaf of French bread left. The three employees split it up, with each taking home the same amount. How much of a loaf did each employee take home?

- What is the operation?
- How can you use pattern blocks or fraction circles to model this?

Division with Fractions

- What does $\frac{2}{3} \div \frac{1}{3}$ mean?
- Hint: How many groups?
- Can you write a contextual problem?
- Model the action.

Division with Fractions

- What does $\frac{1}{3} \div \frac{2}{3}$ mean?
- Hint: How many groups?
- Can you write a contextual problem?
- Model the action.

Contextual Problem

A highway crew can resurface $\frac{2}{3}$ of a mile of highway a day.

There are $2\frac{1}{3}$ miles in the project.

How long will the job take them?

- What is the operation?
- How can you model this?

Developing the algorithm: Division

- How can you think about $A \div B$?
- A is __?__ groups of B.
- A is B groups with ____?____ in each group.

An algorithm

- What does this mean? $\frac{2}{3} \div \frac{1}{6}$
- Can you model it?
- Divide fractions by finding common denominators:

$$\frac{2}{3} \div \frac{1}{6} = \frac{4}{6} \div \frac{1}{6} = \frac{?}{?}$$

Common Denominator Algorithm

- Will it always work? Try these:

$$\frac{3}{4} \div \frac{1}{2} \qquad \frac{1}{5} \div \frac{2}{3} \qquad \frac{1}{2} \div \frac{5}{6}$$

Developing the algorithm: Division

$$1 \div \frac{1}{2} = 2$$

$$1 \div \frac{1}{3} = 3$$

$$1 \div \frac{2}{3} = \frac{3}{2}$$

$$1 \div \frac{3}{4} = \frac{4}{3}$$

$$1 \div \frac{3}{5} = \frac{5}{3}$$

$$1 \div \frac{4}{5} = \frac{5}{4}$$

What is the pattern?

Can you develop an algorithm?

Developing the algorithm: Division

$$A \div B = \frac{A}{B}$$

$$\frac{A}{B} = \frac{A \bullet \frac{1}{B}}{B \bullet \frac{1}{B}} = \frac{A \bullet \frac{1}{B}}{1} = A \bullet \frac{1}{B}$$

$$A \div B = A \bullet \frac{1}{B}$$

Developing the algorithm: Division

$$\frac{A}{B} \div \frac{C}{D} = \frac{A \bullet D}{B \bullet D} \div \frac{B \bullet C}{B \bullet D} = \frac{AD \div BC}{BD \div BD}$$

$$AD \div BC = \frac{AD}{BC} = \frac{A \bullet D}{B \bullet C} = \frac{A}{B} \bullet \frac{D}{C}$$

$$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \bullet \frac{D}{C}$$

ALGORITHM: Multiply by the reciprocal of the divisor

Teaching for conceptual understanding

- Teachers' abilities to **identify the main mathematical concepts** that are important for their students to learn, and to **understand those concepts** are critical.
- Provide opportunities for students to build on their existing knowledge.
- Introduce symbols and procedures ***after*** students understand the concepts these symbols and procedures are meant to represent.

Phillipp, R. (2000). *Unpacking a conceptual lesson: The case of dividing fractions*. Center for Research in Mathematics and Science Education, San Diego State University: San Diego, CA.

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Please complete the form at your table and take it to ETA's booth to thank them for the materials.

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