Harnessing the Power of Transformations to Transform Your Teaching

Selena Oswalt   selena.oswalt@greatminds.org

Wendy Denbesten wendy.denbesten@greatminds.org

The foundation of transformations prepares students for higher level mathematics.

The connection between algebra and geometry starts in Grade 5.

- Reflections
- Rotations

Students come to understand transformations as functions.

Defining congruence and similarity in terms of motions grounds an abstract concept in tactile experiences.

Non-Rig

- Dilations

Congruence and similarity are extended beyond polygons.

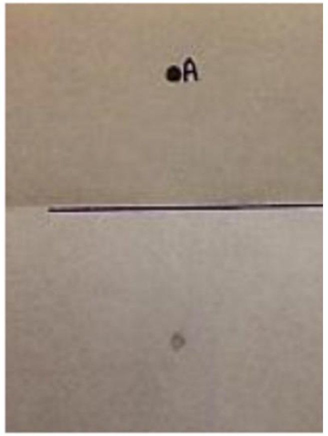
eureka-math.org

EUREKA MATH

Grade 6	Grade 7	Grade 8	High School
Solve real-world and mathematical problems involving area, surface area, and volume.	**Draw, construct, and describe geometrical figures and describe the relationships between them.**	**Understand congruence and similarity using physical models, transparencies, or geometry software.**	
6.G.1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes. . . .	7.G.1. Solve problems involving scale drawings of geometric figures. . . . 7.G.2. Draw . . . geometric shapes with given conditions. . . .	8.G.2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations. . . .	**Understand congruence in terms of rigid motions**
		8.G.4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations. . . .	**Understand similarity in terms of similarity transformations**
		8.G.1. Verify experimentally the properties of rotations, reflections, and translations. . . .	G-SRT.1. Verify experimentally the properties of dilations given by a center and a scale factor. . . .

Reflections

- Grade 5: Draw symmetric figures using distance and angle measure from the line of symmetry.



3. Use the plane below to complete the following tasks.

   a. Draw a line u whose rule is, y *is equal to* $x + \frac{1}{4}$.

   b. Construct a figure with a total of 6 points, all on the same side of the line.

   c. Record the coordinates of each point, in the order in which they were drawn, in Table A.

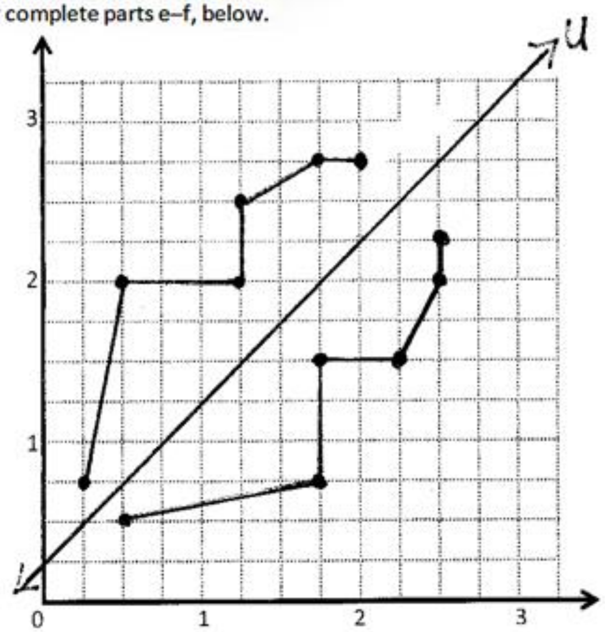
   d. Swap your paper with a neighbor and have him or her complete parts e–f, below.

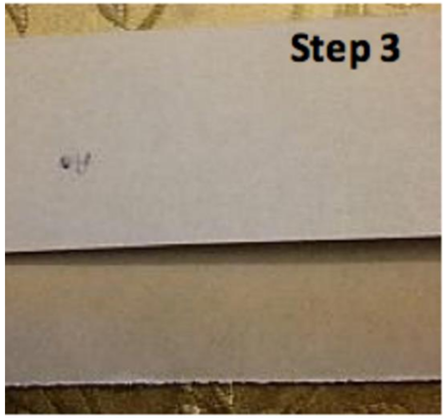
Table A

(x, y)
$(\frac{1}{4}, \frac{3}{4})$
$(\frac{2}{4}, 2)$
$(1\frac{1}{4}, 2)$
$(1\frac{1}{4}, 2\frac{2}{4})$
$(1\frac{3}{4}, 2\frac{3}{4})$
$(2, 2\frac{3}{4})$

Table B

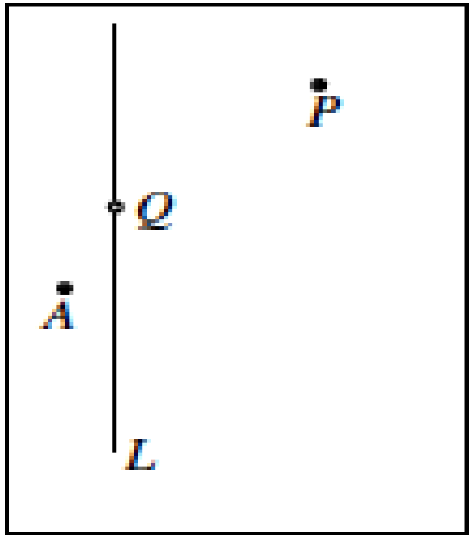
(x, y)
$(\frac{2}{4}, \frac{2}{4})$
$(1\frac{3}{4}, \frac{3}{4})$
$(1\frac{3}{4}, 1\frac{2}{4})$
$(2\frac{1}{4}, 1\frac{2}{4})$
$(2\frac{2}{4}, 2)$
$(2\frac{2}{4}, 2\frac{1}{4})$

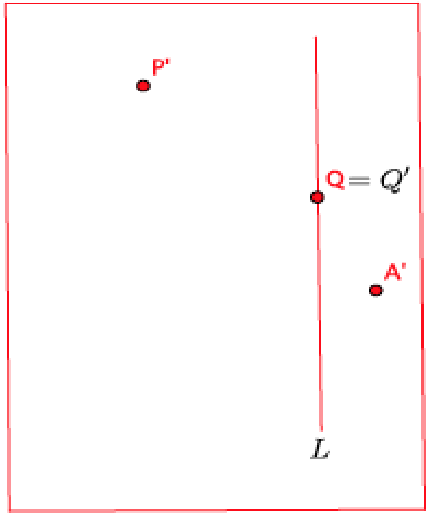


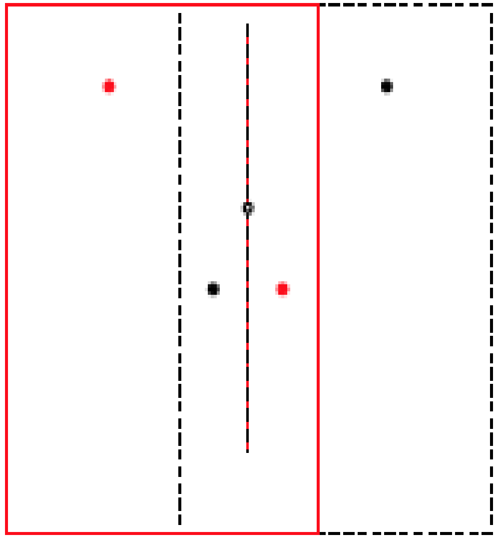


Reflections

- Grade 8: Definition of reflection and basic properties





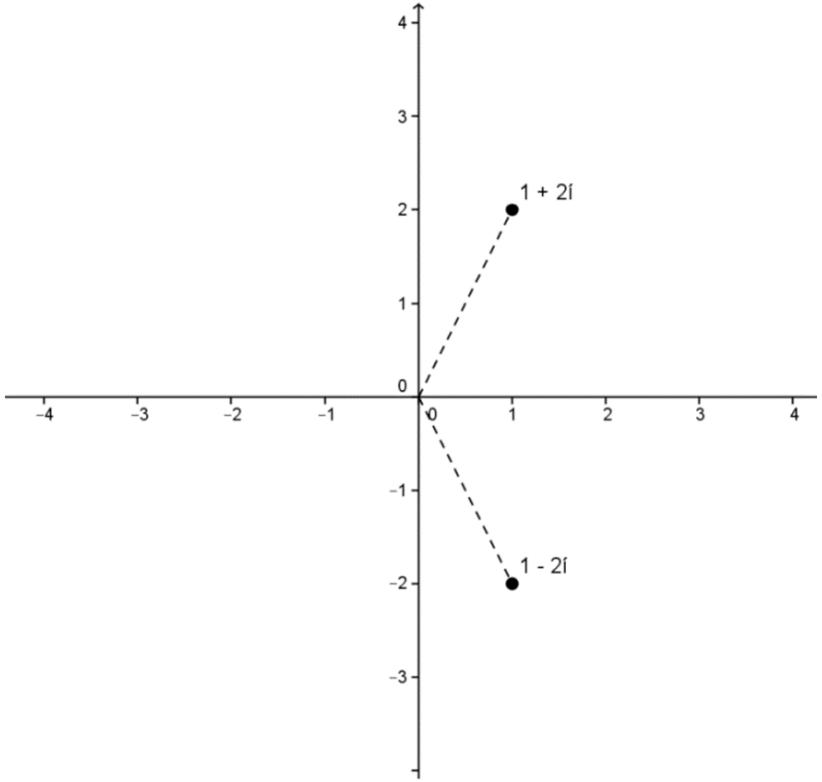


eureka-math.org

Reflections

- Precalculus: The geometric effect of complex arithmetic

A complex conjugate is a reflection of the complex number across the real axis.



Translations

- Grade 5: Generate two number patterns from given rules, plot the points, and analyze the patterns.

Line ℓ

Rule: y is 2 more than *x*

x	y	(x, y)
1	3	(1,3)
5	7	(5,7)
10	12	(10,12)
15	17	(15,17)

Line m

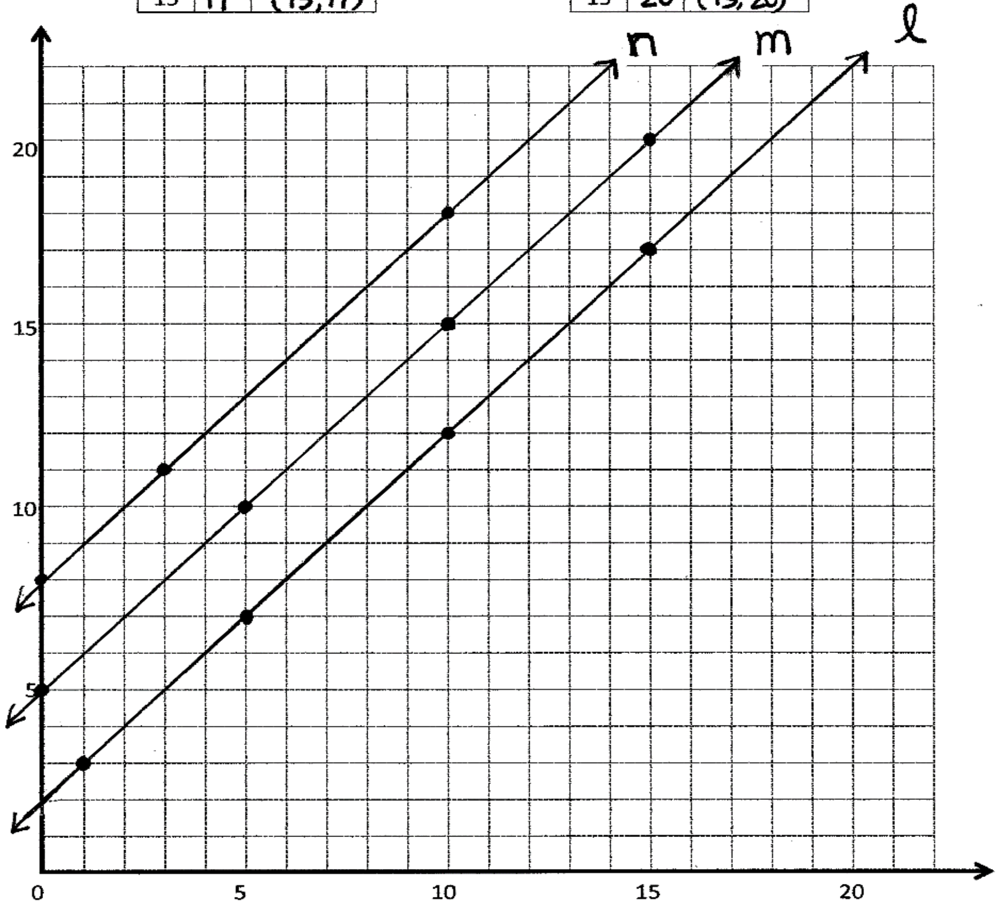
Rule: y is 5 more than *x*

x	y	(x, y)
0	5	(0,5)
5	10	(5,10)
10	15	(10,15)
15	20	(15,20)

Line n

y is 8 more than X

(0, 8)
(3, 11)
(10, 18)



eureka-math.org

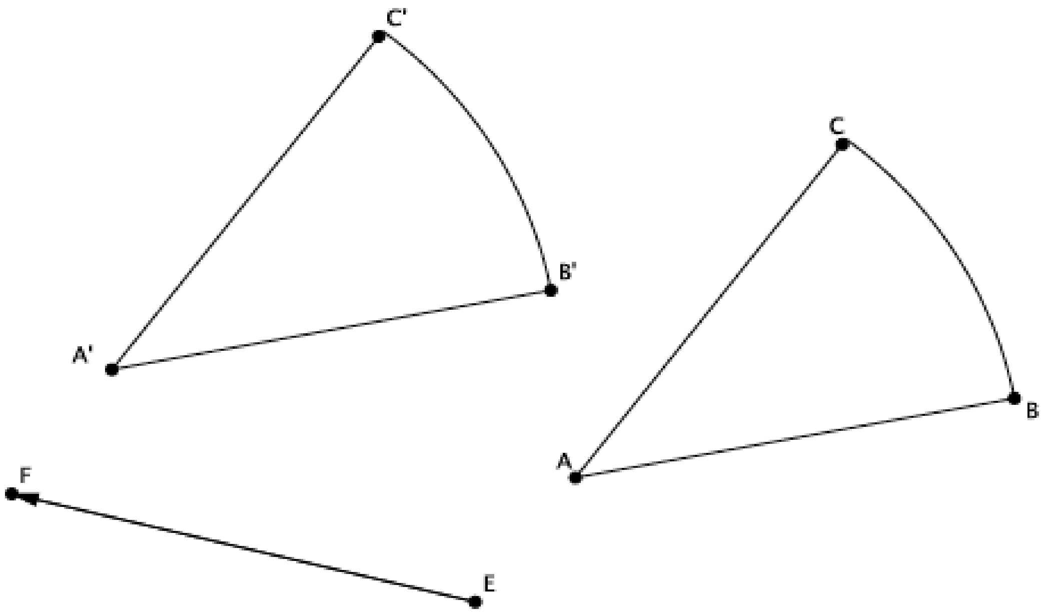
EUREKA MATH

Translations

- Grade 8: Sequencing translations

Sample exercise: Translate the curved shape ABC along the given vector. Label the image.

- The symbol $F(P)$ denotes a specific single point, unambiguously.
- The point $F(P)$ is called the image of P by F. Sometimes the image of P by F is denoted simply as P' (read “P prime”).
- The transformation F is sometimes said to *move* the point P to the point $F(P)$.
- Translation(A) is the point A$'$.

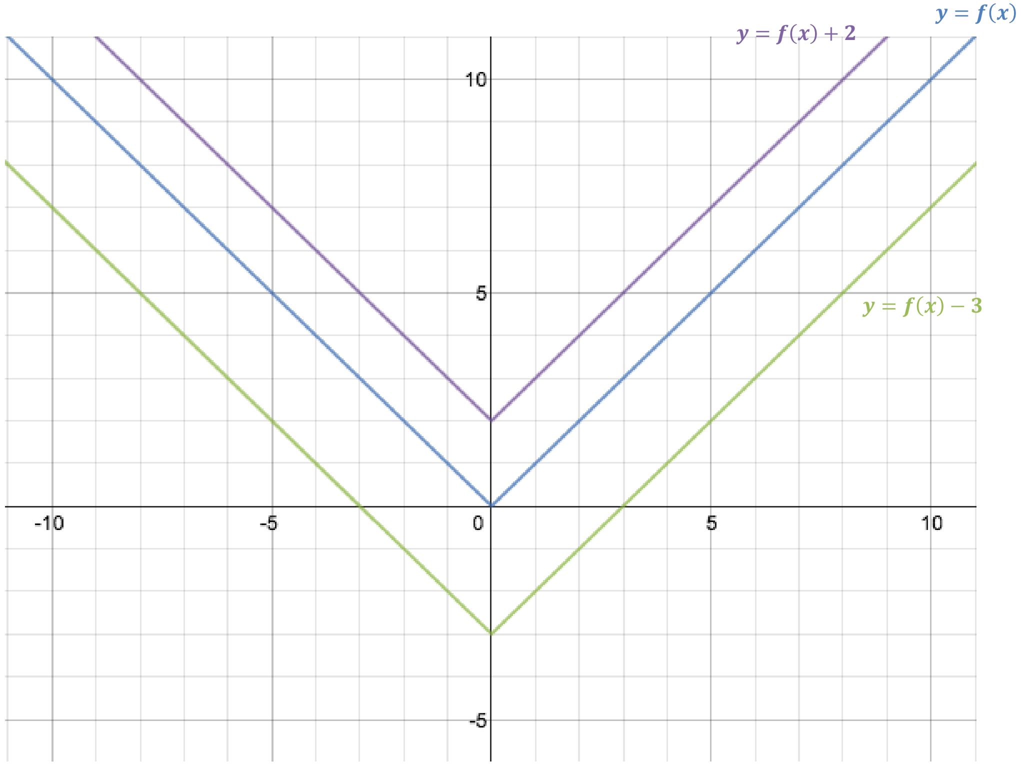


eureka-math.org

Translations

- Algebra I: Translating graphs of functions

x	$f(x) = \lvert x \rvert$	$g(x) = f(x) - 3$	$h(x) = f(x) + 2$
-3	3	0	5
-2	2	-1	4
-1	1	-2	3
0	0	-3	2
1	1	-2	3
2	2	-1	4
3	3	0	5



eureka-math.org

EUREKA MATH

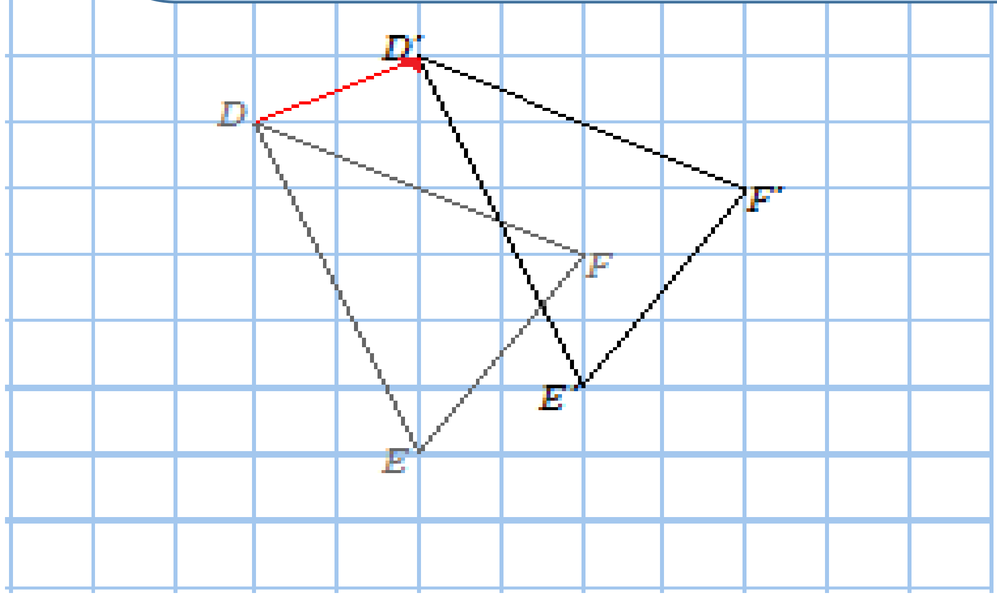
Translations

- Geometry: Translation by const

How does translation T along vector AB affect the points in the plane?

For any point P on the line AB, $T_{\overrightarrow{AB}}(P)$ is the point Q on $\overleftrightarrow{AB}$ so that $\overrightarrow{PQ}$ has the same length and the same direction as $\overrightarrow{AB}$. For any point P not on $\overleftrightarrow{AB}$, $T_{\overrightarrow{AB}}(P)$ is the point Q so that points P, Q, A, and B form a parallelogram.

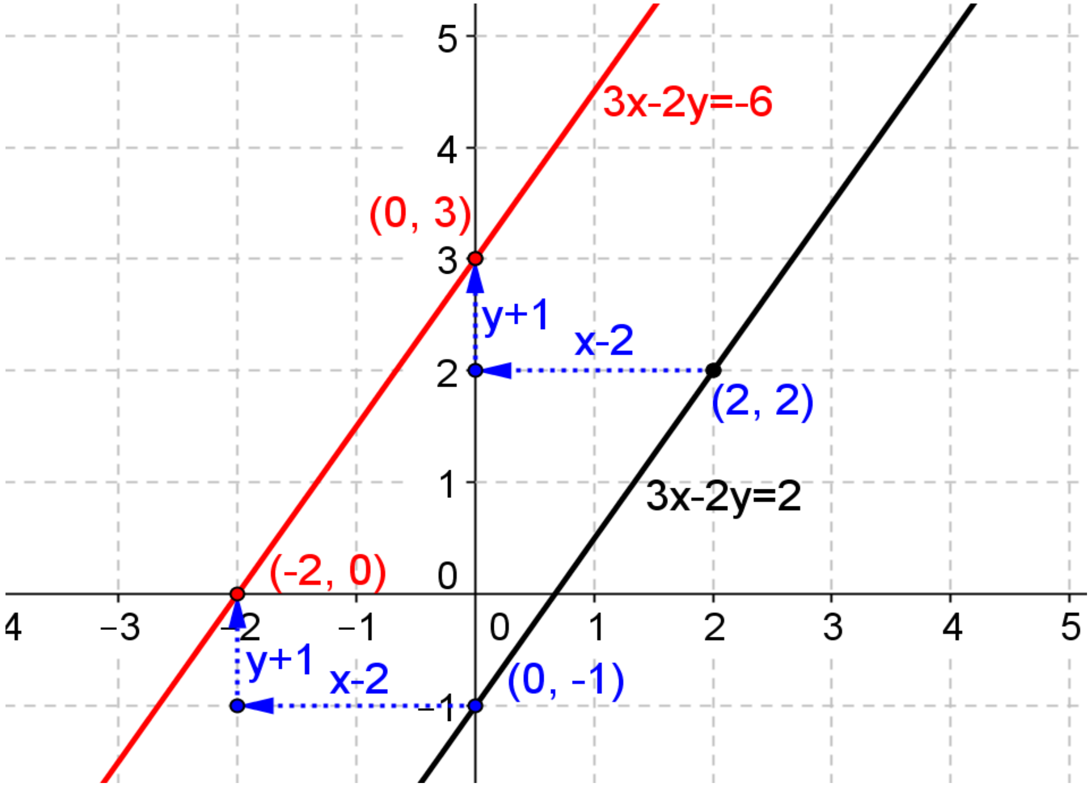
Sample exercise: Translate the figure 1 unit up and 2 units right. Draw the vector that defines the translation.



Translation

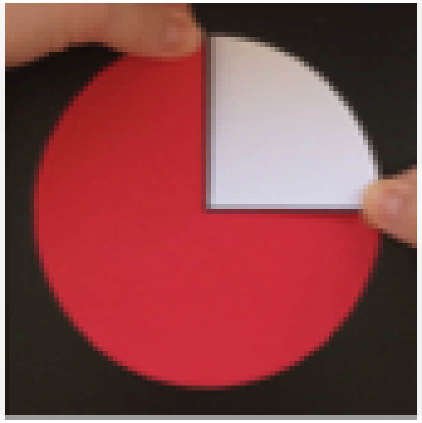
- Precalculus: Vectors and translation maps

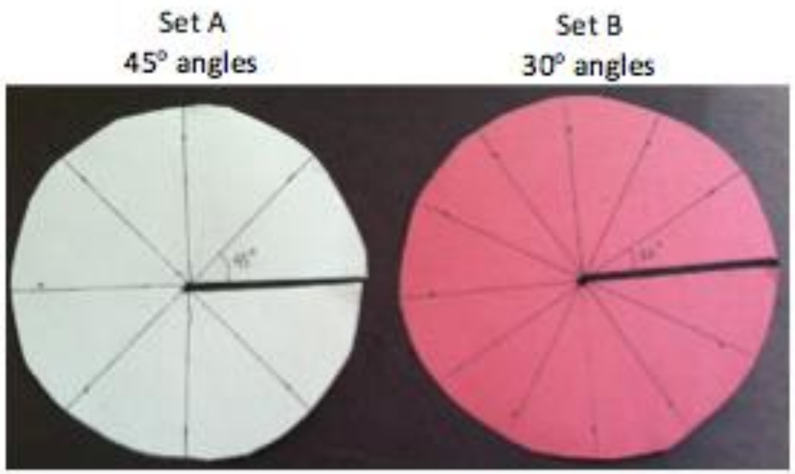
Given the vector $\mathbf{v} = \langle -2, 1 \rangle$, find the image of the line $3x - 2y = 2$ under the translation map $T_{\mathbf{v}}$. Graph the original line and its image, and explain the geometric effect of the map $T_{\mathbf{v}}$.



Rotations

- Grade 4: Use a Circular Protractor to Understand a 1-degree Angle as 1/360 of a Turn. Explore Benchmark Angles Using the Protractor.
    - http://greatminds.net/maps/math/video-gallery/g4m4-tb-l5-1





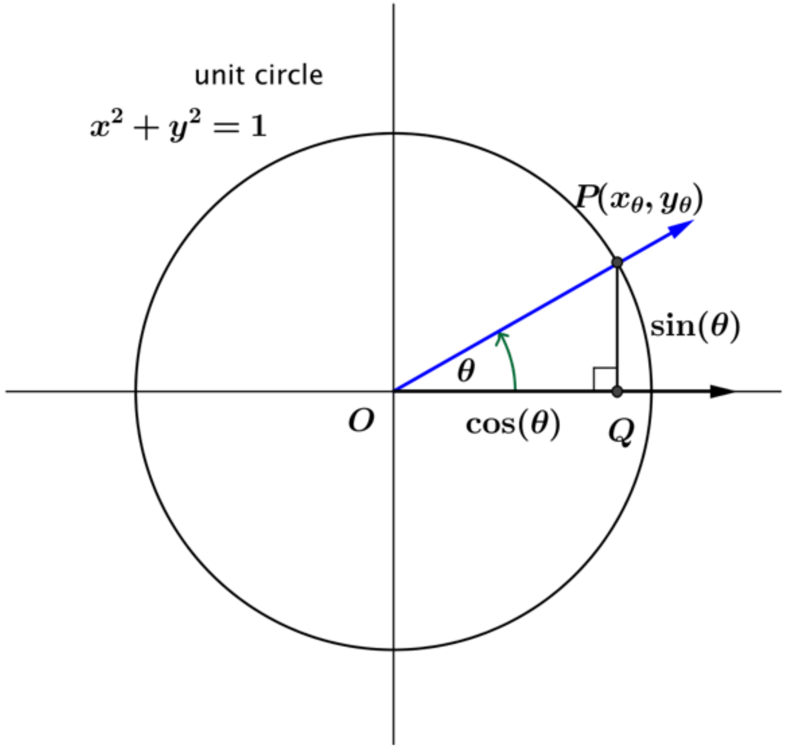
Rotations

- Geometry: Congruence

- Students devise formal methods of proof by the direct use of transformations.
- Two figures are said to be congruent if there is a sequence of rigid motions that takes one figure onto the other.
- Each rigid motion must be specified. For example, a rotation must have a point and angle specified.
- Students develop an understanding of traditional triangle congruence criteria.

Rotations

- Algebra II: Trigonometry



SINE FUNCTION (description): The *sine function*, $\sin: \mathbb{R} \to \mathbb{R}$, can be defined as follows: Let θ be any real number. In the Cartesian plane, rotate the initial ray by θ radians about the origin. Intersect the resulting terminal ray with the unit circle to get a point (x_θ, y_θ). The value of $\sin(\theta)$ is y_θ.

COSINE FUNCTION (description): The *cosine function*, $\cos: \mathbb{R} \to \mathbb{R}$, can be defined as follows: Let θ be any real number. In the Cartesian plane, rotate the initial ray by θ radians about the origin. Intersect the resulting terminal ray with the unit circle to get a point (x_θ, y_θ). The value of $\cos(\theta)$ is x_θ.

Rotations

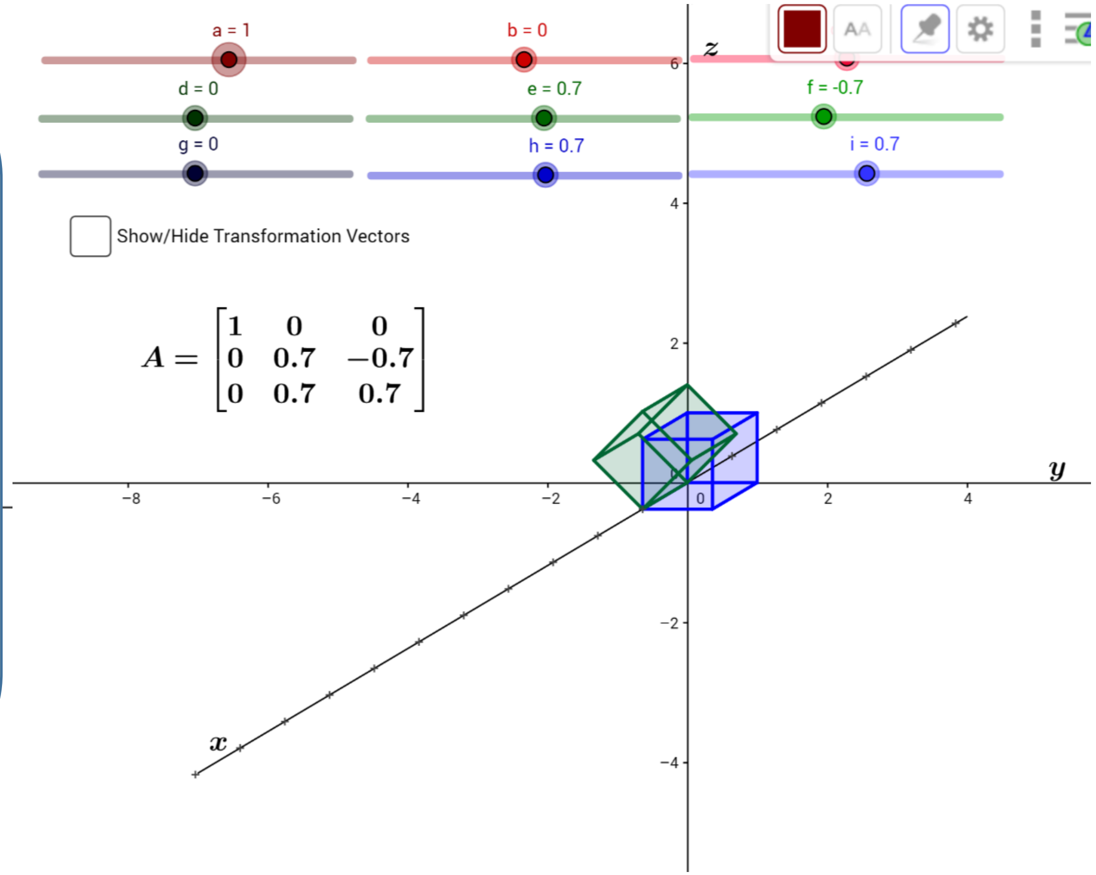
- Precalculus: Linear transformations applied to cubes

Sample exercise:

Make a prediction: What would be the geometric effect of the transformation

$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(45°) & -sin(45°) \\ 0 & sin(45°) & cos(45°) \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

on the unit cube? Use the GeoGebra demo to test your conjecture.



eureka-math.org

Dilations

- Grade 5: Generate two number patterns from given rules, plot the points, and analyze the patterns.

Line p

Rule: y is x times 2

x	y	(x, y)
0	0	(0,0)
1	2	(1,2)
3	6	(3,6)
4	8	(4,8)

Line q

Rule: y is x times 3

x	y	(x, y)
0	0	(0,0)
1	3	(1,3)
3	9	(3,9)
4	12	(4,12)

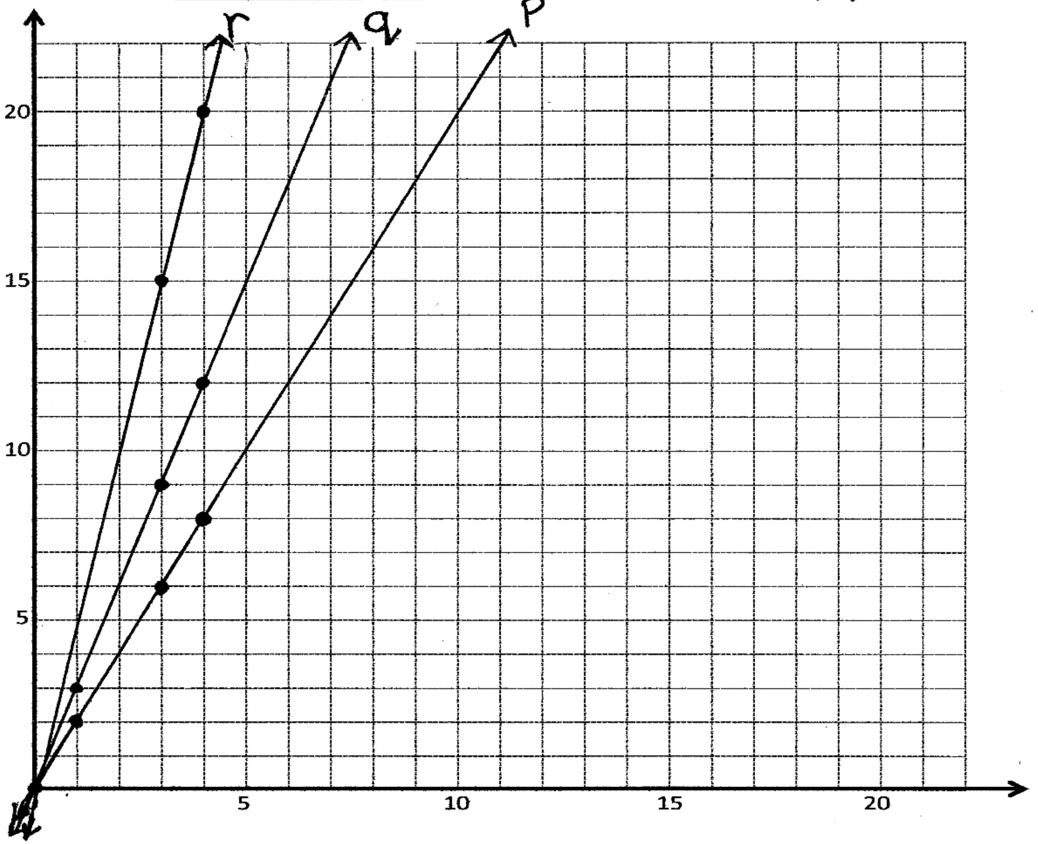
Line r

y is X times 5

(0, 0)

(3, 15)

(4, 20)

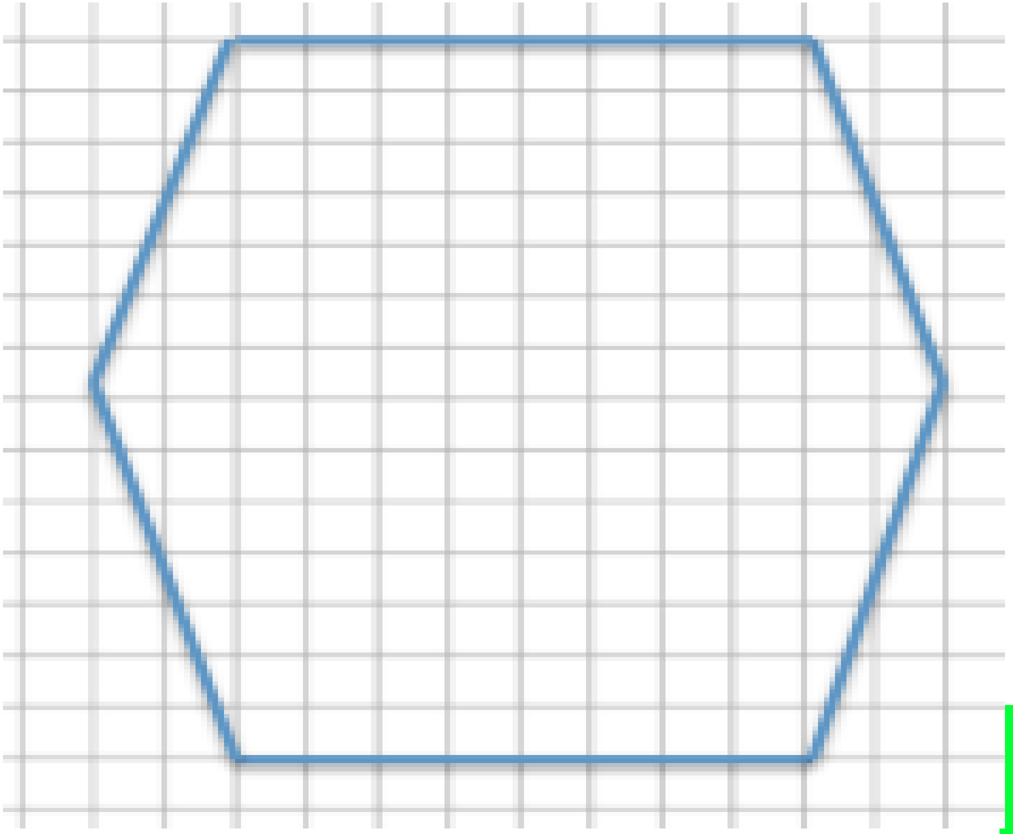


eureka-math.org

Dilations

- Grade 7: Scale drawings

Sample exercise:
Create a scale drawing of the original drawing given below using a horizontal scale factor of 80% and a vertical scale factor of 175%.



Dilations

- Grade 8: Linear functions and proportionality

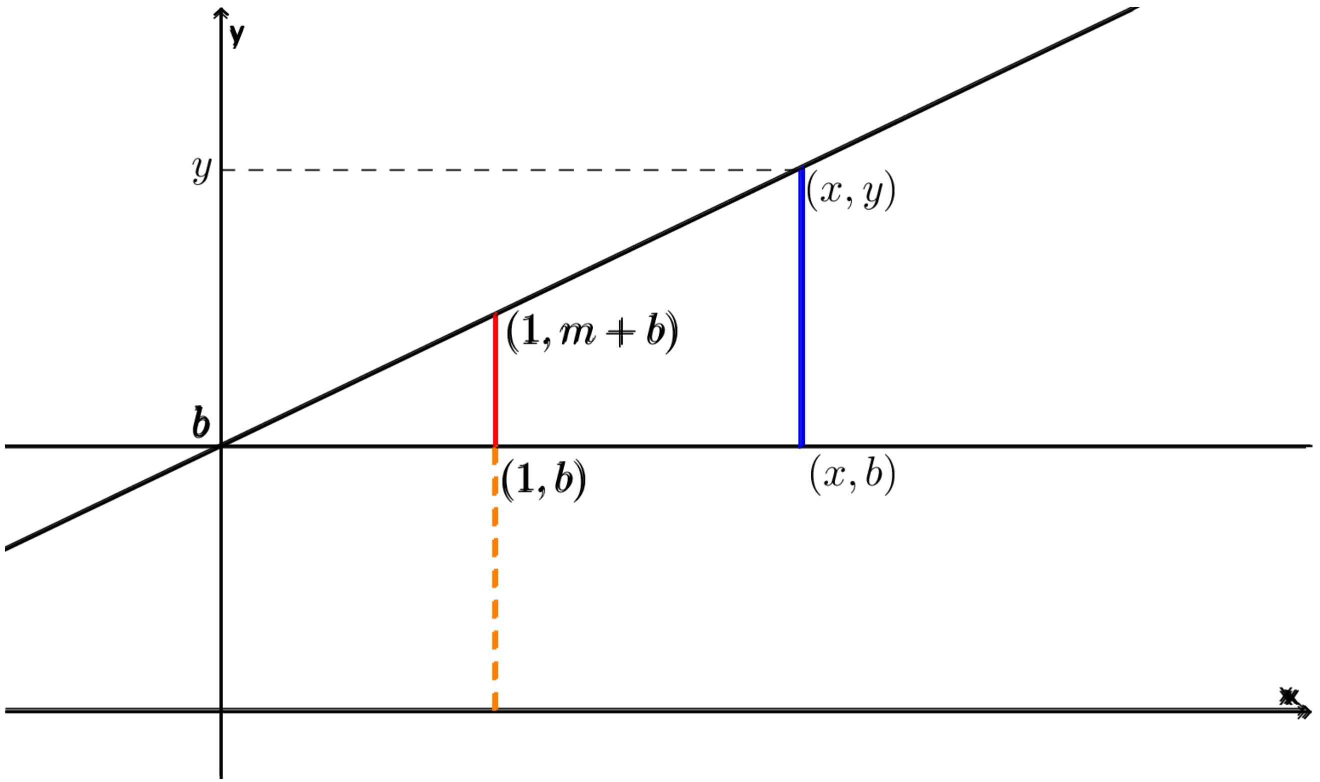
By similar triangles,

$$\frac{y-b}{m+b-b} = \frac{x}{1}.$$

$$\frac{y-b}{m} = \frac{x}{1}$$

$$\implies \quad y - b = mx$$

$$\implies \quad y = mx + b$$



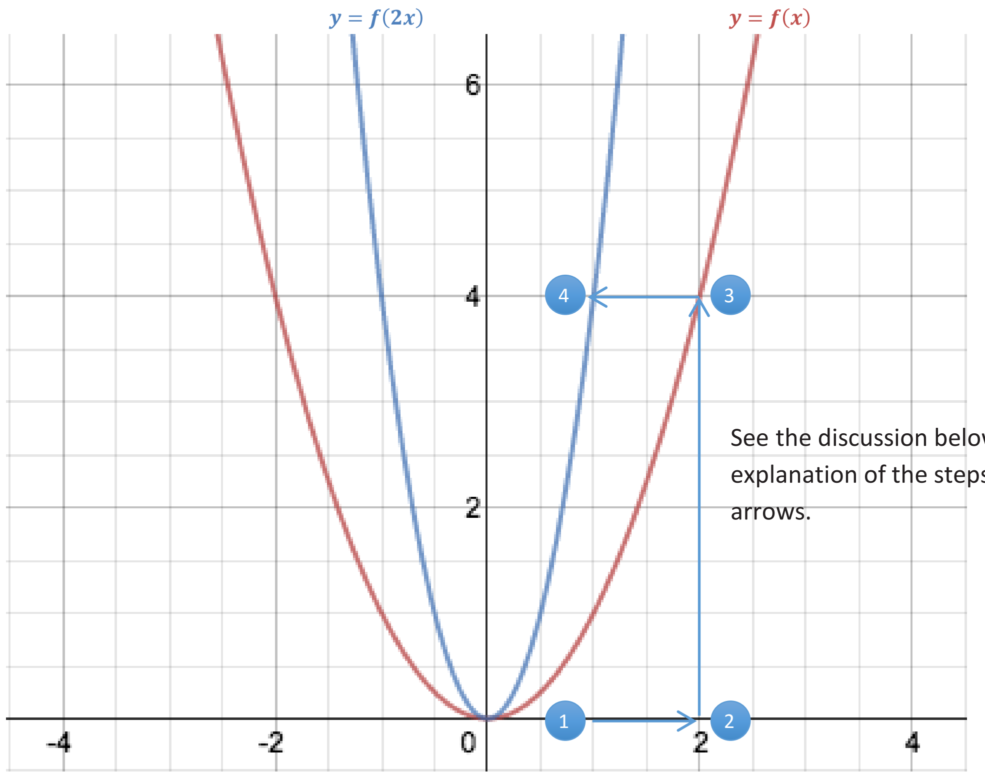
eureka-math.org

EUREKA MATH

Dilations

- Algebra I: Horizontal scaling by a scale factor of k

x	$f(x) = x^2$	$g(x) = f(2x)$
-3	9	36
-2	4	16
-1	1	4
0	0	0
1	1	4
2	4	16
3	9	36



See the discussion below for an explanation of the steps and arrows.

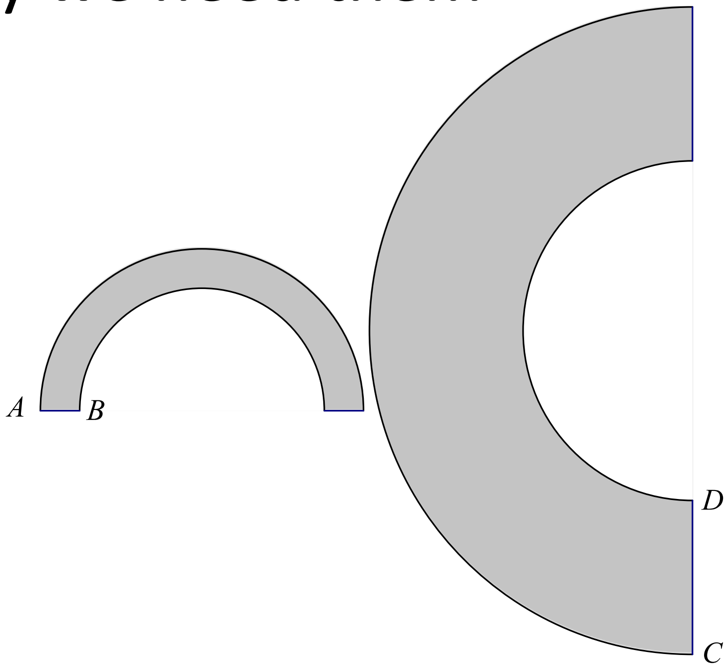
Dilations

Two figures are similar if there exists a similarity transformation that maps one figure onto the other.

A similarity transformation is a composition of a finite number of dilations or rigid motions.

- Geometry: Similarity transformations and why we need them

Sample exercise:
Show that no sequence of basic rigid motions and dilations takes the small figure to the large figure. Take measurements as needed.



eureka-math.org

Dilations

- Algebra II: Are all parabolas similar?

- **Students prove that all parabolas with the same distance between the focus and directrix are congruent to each other, and in particular, that they are congruent to a parabola with vertex at the origin, axis of symmetry along the y-axis, and equation of the form $y = \frac{1}{2p}x^2$.**
- **Students then prove that all parabolas are similar using transformations of functions.**

Sample exercise:

Are the two parabolas defined below similar or congruent or both? Justify your reasoning.

Parabola 1: The parabola with a focus of $(0, 2)$ and a directrix line of $y = -4$

Parabola 2: The parabola that is the graph of the equation $y = \frac{1}{6}x^2$

For parabola 1, the distance between the focus and directrix is 6 units. For parabola 2, the distance between the focus and directrix is only 3 units.

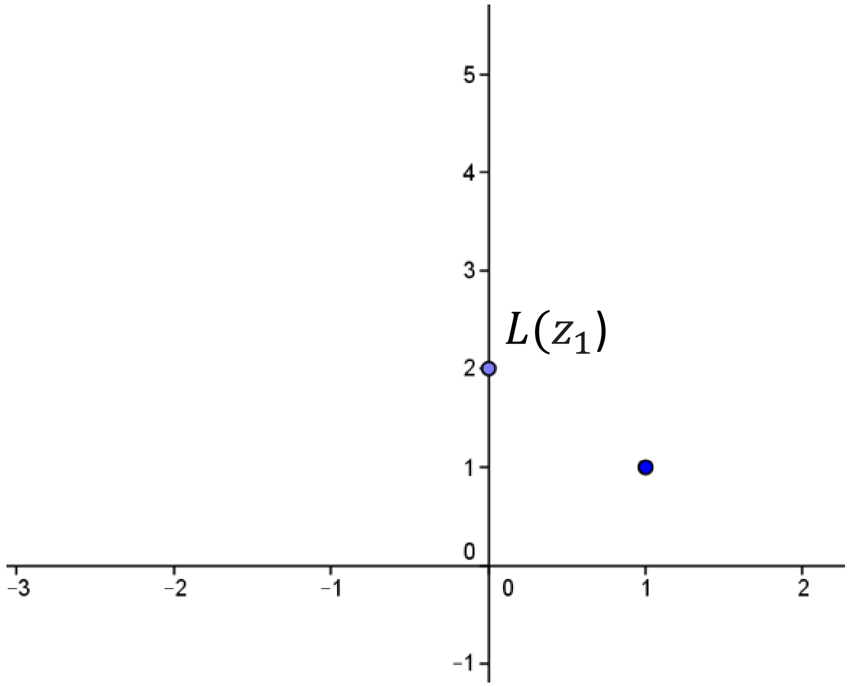
Dilations

- Precalculus: The geometric effect of complex arithmetic

Sample exercise:

Describe the geometric effect of $L(z) = (1 + i)z$ for $z_1 = 1 + i$.

Plot the images on graph paper, and describe the geometric effect in words.



Transformations are essential to:

- Understanding how objects move
- Graphing functions on a coordinate plane
- Making sense of congruence and similarity
- Assigning a geometric meaning to the complex number system
- Simplifying complicated calculations
- Working with matrices and vectors
- Harnessing the powerful connection between algebra and geometry

Online Support

SOCIAL MEDIA:

- Facebook.com/eurekamathofficial
- Pinterest.com/eurekamath0130
- Twitter.com/eureka_math

EUREKA MATH PROGRAM SUPPORT:

- EurekaProgramSupport@GreatMinds.org

EUREKA MATH CHAMPIONS NETWORK:

- Greatminds.net/maps/math/champions