

Teaching Problem Solving Using Cognitively Guided Instruction to ELL Students

National Council of Teachers of Mathematics
Annual Meeting and Exposition
San Francisco, CA, April 15, 2016

Angela Sencibaugh, Ed.D.
Valley Park School District
msencibaugh@vp.k12.mo.us

Joseph Sencibaugh, Ph.D.
Webster University
jsencibaugh77@webster.edu

Jennifer Bond, M.A.
Ferguson-Florissant School
District

jbond@fergflor.org

Learning Objectives

- Participants will be able to identify cognitive strategies for improving the mathematical problem-solving abilities of ELL students.
 - Identify common challenges for English language learners in solving math problems.
 - Learn to teach students how to utilize cognitively guided instruction and self-regulated strategy development for engaging in problem solving.
 - Learn the steps for implementing learning strategies to solve math problems.
 - Learn to assist students in monitoring and reflecting on the problem-solving process.

Questions for Discussion

1. What are specific math difficulties students in primary grades encounter?
2. What are your suggestions for improving the math achievement of primary students based on your knowledge, experience or math curricula?

Sample Problem

- Solve the following problem:
 - “At the County Fair, there are 36 children in line to ride the roller coaster. The roller coaster has 10 cars. Each car holds 4 children. How many children can sit 3 to a car, and how many have to sit 4 to a car?”
 - Solve the problem.

ELLs and Math Word Problems

“Word problems in mathematics often pose a challenge because they require that students read and comprehend the text of the problem, identify the question that needs to be answered, and finally create and solve a numerical equation.”

ELLs and Math Word Problems

“By explicitly teaching English in math class, teachers can help remove the roadblock that often prevents ELLs from making sense of a math word problem and, thus, from solving it.

The challenges that vocabulary, grammar, and syntax pose to ELLs can become English language development opportunities in math class.”

Mathematics Instruction for ELLs

1. Focus on students' mathematical reasoning, not accuracy in using language.
2. Focus on mathematical practices, not language as single words or definitions.
3. Recognize the complexity of language in mathematics classrooms and support students in engaging in this complexity.
4. Treat everyday and home languages as resources, not obstacles.

Research on Scholarship

- Cognitive development occurs faster in students who are actively engaged in their learning; therefore, mentally active children have faster cognitive development than passive children (Kamii & Rummelsburg, 2008).
- Mathematics instruction typically focuses on memorizing facts and information and test taking tips, which increases even more as the schools these students are enrolled in do not make adequate yearly progress (Spilde, 2013).
- Vygotsky's (1978) idea that young children's mathematical learnings begin before they attend formal schooling directly coincides with what Carpenter et al. (2000) use as the basis of their theory of cognitively guided math instruction (Spilde, 2013).

Conceptual Understanding

- The comprehension of mathematical concepts, operations, and relations.
- Refers to an integrated and functional grasp of mathematical ideas.
 - Students understand why a mathematical idea is important and the context in which is it useful.
 - They organize their knowledge into a coherent whole, which enables them to learn new ideas by connecting those ideas to what they already know.

Cognitively Guided Math Instruction

- *Cognitively Guided Instruction* is an inquiry-based approach to teaching mathematics that was developed at the Wisconsin Center for Education Research (Carpenter et al, 1999).
- Provides teachers with knowledge about the developmental stages of children's mathematical reasoning.
- Enables teachers to plan mathematics instruction based on their students' understanding and guide them toward greater mathematical reasoning and concept mastery.

Teacher Beliefs on Cognitively Guided Math Instruction

1. Children's learning should be considered as they make instructional decisions.
2. Children have informal knowledge that enables them to solve problems without instruction.
3. The teacher's role is to build a learning environment where children can construct their own knowledge rather than where the teacher is a transmitter of knowledge.
4. The learning of procedural skills does not have to come before children can solve problems.

Developing Mathematical Reasoning Using Word Problems

- Focuses on student knowledge and encourages teachers to pose story problems that can be solved by any means chosen by the child.
 - Problem-posing and problem-solving become the focus of the mathematics class, rather than the traditional emphasis on memorization of facts and algorithms.
- An important aspect of applying this knowledge when teaching is sequencing word problems from easiest to most difficult.
- Such sequencing allows children to develop mathematical reasoning.

PROBLEM-SOLVING SITUATIONS

JOINING PROBLEMS		
Join: Result Unknown (JRU)	Join: Change Unknown (JCU)	Join: Start Unknown (JSU)
<p>♦ Grandmother had 5 strawberries. Grandfather gave her 8 more strawberries. How many strawberries does Grandmother have now?</p> <p align="center">$5 + 8 = \square$</p>	<p>♥ Grandmother had 5 strawberries. Grandfather gave her some more. Then Grandmother had 13 strawberries. How many strawberries did Grandfather give Grandmother?</p> <p align="center">$5 + \square = 13$</p>	<p>♠ Grandmother had some strawberries, Grandfather gave her 8 more. Then she had 13 strawberries. How many strawberries did Grandmother have before Grandfather gave her any?</p> <p align="center">$\square + 8 = 13$</p>
SEPARATING PROBLEMS		
Separate: Result Unknown (SRU)	Separate: Change Unknown (SCU)	Separate: Start Unknown (SSU)
<p>♦ Grandfather had 13 strawberries. He gave 5 strawberries to Grandmother. How many strawberries does Grandfather have left?</p> <p align="center">$13 - 5 = \square$</p>	<p>♥ Grandfather had 13 strawberries. He gave some to Grandmother. Now he has 5 strawberries left. How many strawberries did Grandfather give Grandmother?</p> <p align="center">$13 - \square = 5$</p>	<p>♠ Grandfather had some strawberries. He gave 5 to Grandmother. Now he has 8 strawberries left. How many strawberries did Grandfather have before he gave any to Grandmother?</p> <p align="center">$\square - 5 = 8$</p>
PART -PART -WHOLE PROBLEMS		
Part-Part-Whole: Whole Unknown (PPW:WU)	Part-Part-Whole: Part Unknown (PPW:PU)	
<p>♦ Grandmother has 5 big strawberries and 8 small strawberries. How many strawberries does Grandmother have altogether?</p> <p align="center">$5 + 8 = \square$</p>	<p>♥ Grandmother has 13 strawberries. Five are big and the rest are small. How many small strawberries does Grandmother have?</p> <p align="center">$13 - 5 = \square$ or $5 + \square = 13$</p>	
COMPARE PROBLEMS		
Comp. Difference Unknown	Comp. Quantity Unknown	Comp. Referent Unknown
<p>♦♥ Grandfather has 8 strawberries. Grandmother has 5 strawberries. How many more berries does Grandfather have than Grandmother?</p> <p align="center">$8 - 5 = \square$ or $5 + \square = 8$</p>	<p>♠ Grandmother has 5 strawberries. Grandfather has 3 more strawberries than Grandmother. How many strawberries does Grandfather have?</p> <p align="center">$5 + 3 = \square$</p>	<p>♠ Grandfather has 8 strawberries. He has 3 more strawberries than Grandmother. How many strawberries does Grandmother have?</p> <p align="center">$8 - 3 = \square$ or $\square + 3 = 8$</p>
MULTIPLICATION & DIVISION PROBLEMS		
Multiplication	Measurement Division	Partitive Division
<p>♦ Grandmother has 4 piles of strawberries. There are 3 strawberries in each pile. How many strawberries does Grandmother have?</p> <p align="center">$4 \times 3 = \square$</p>	<p>♦ Grandmother had 12 strawberries. She gave them to some children. She gave each child 3 strawberries. How many children were given strawberries?</p> <p align="center">$12 \div 3 = \square$</p>	<p>♦♥ Grandfather has 12 strawberries. He wants to give them to 3 children. If he gives the same number of strawberries to each child, how many strawberries will each child get?</p> <p align="center">$12 \div 3 = \square$</p>

Understanding the Structure of a Word Problem

- A goal of **Cognitively Guided Instruction** is that young children become independent problem solvers who are able to approach and solve word problems without having to rely on having a teacher tell them how to do it.
- A number of factors influence whether a problem is appropriate for a child to solve independently and understanding these factors helps the teacher decide which word problems to use during instruction.

If the Problem Involves a Situation That the Child Can Act Out

- **SRU (Action Direct):**
 - Grandfather had 8 strawberries. He gave 3 of them to Grandmother. How many strawberries does Grandfather have now?
- **SRU (Action Indirect):**
 - Grandfather gave 3 strawberries to Grandmother. He had 8 strawberries. How many strawberries does Grandfather have now?

If the Child is Able to Model the Problem with Counters or Drawing

- **CDU direct modeling situation:**
 - Grandfather has 8 strawberries. Grandmother has 5 strawberries. How many more strawberries does Grandfather have?
- **CRU situation that requires ability to analyze:**
 - Grandmother has 5 strawberries. She has 3 fewer strawberries than Grandfather. How many strawberries does Grandfather have?

Modeling Multiplication and Division Problems

- **Multiplication Problem:**
 - Grandmother has 4 piles of strawberries. There are 3 strawberries in each pile. How many strawberries does Grandmother have?
- **Measurement Division Problem:**
 - Grandmother gave 12 strawberries to some children. She gave each child 3 strawberries. How many children got strawberries?
- **Partitive Division Problem:**
 - If Grandfather shares 12 strawberries with 3 friends, how many strawberries will each friend get?

Modeling or Acting Out a Problem in Order

- **Joining problem that can be solved in the word order given:**
 - Grandmother had 5 strawberries. Grandfather gave her 8 more. How many strawberries does Grandmother have now?
- **Joining problem that cannot be solved in the order given:**
 - Grandmother had some strawberries. Grandfather gave her 8 more. Then she had 13 strawberries. How many strawberries did Grandmother have before Grandfather gave her any?

The Location of the Unknown Influences the Problem Difficulty

- **SRU location of unknown at end of problem: ($8 - 3 = \underline{\quad}$)**
 - Grandmother had 8 strawberries. She gave 3 to Grandfather. How many strawberries does Grandmother have now?
- **SCU location of unknown in middle of problem: ($8 - \underline{\quad} = 5$)**
 - Grandfather had 8 strawberries. He gave some to Grandmother. Now he has 5 strawberries. How many strawberries did Grandfather give to Grandmother?
- **SSU location of unknown at start of problem: ($\underline{\quad} - 3 = 5$)**
 - Grandfather had some strawberries. He gave 3 strawberries to Grandmother. Then he had 5 strawberries left. How many strawberries did Grandfather have before sharing with Grandmother?

Children's Intuitive Solution Strategies

- Children go through developmental thinking processes when learning to solve word problems, and these processes are often intuitive; students naturally seek a solution through self-discovery and inquiry.
- Teachers must promote the development of mathematical reasoning by clearly understanding the relationships among the different types of word problems.

Components of Cognitively Guided Math Instruction

- **Direct Modeling (Concrete-Representational-Abstract)**
 - Act out the problem.
 - Follow the sequence or steps for completing the problem.
 - Use manipulatives.
 - Use composition or decomposition of numbers.
- **Counting Strategies**
 - Use the following strategies: counting-all, counting-on & counting-back.
- **Flexible Base-Ten Strategies**
 - Use invented or alternative algorithms.
- **Derived Facts/Number Facts**
 - Recall basic facts for addition, subtraction, multiplication and division.
 - Use derived facts, doubles or near doubles

Direct Modeling Strategies

- **Direct Modeling** strategy represents each number in the problem with concrete objects.

Child's Solution to JRU

$$6 + 5 = 11$$

“Grandfather had six strawberries.

One, two, three, four, five, six.”

(The child sets out six counters.)

“Grandmother gave him five more.

One, two, three, four, five.” (Child sets out five counters and then pushes both sets together and

counts all of the counters.)

“Now, he has eleven strawberries.”

Child's Solution to SRU

$$11 - 5 = 6$$

“Grandmother had eleven

strawberries. One, two three, four, five, six, seven, eight, nine, ten,

eleven, twelve.” *(Child sets out eleven counters.)*

“She gave five to Grandfather. One, two, three, four, five. *(Child counts out and*

removes five counters from the group of eleven and counts the remaining counters.)

“Now she has ‘one, two, three, four, five, six.

She has six.

Counting Strategies

- A child using a **Counting On/Back** strategy is able to hold a number in her/his mind and count on or back from that number while keeping track of the quantity that is added or subtracted using fingers, tally marks, or counters.

Child's Solution to JRU

$$6 + 5 = 11$$

“I don't have to count the six again. I just have to add five to it. I say, ‘Seven, eight, nine, ten, eleven.’ (*Child holds up a finger with each count.*) I have eleven.”

Child's Solution to SRU

$$11 - 5 = 6$$

“I know Grandmother had eleven strawberries. I know she gave five away. So, I count five down. ‘Eleven, ten, nine, eight, seven.’ I have six left.”
(*Child folds a finger down with each count.*)

Flexible Base-Ten Strategies

- **Alternative or Invented Strategies** refers to any strategy other than the traditional algorithm or does not involve the use of physical materials.

Child's Solution to JRU

$$19 + 3 = 22$$

“Nine ones and three ones equal twelve ones. One ten joined with twelve ones equals two tens and two ones.” The sum is 22.
(Partial Sums)

Child's Solution to SRU

$$22 - 19 = 3$$

“Two ones take away nine ones equal (negative) seven. Two tens take away one ten equal one ten. One ten take away (negative) seven equals three ones. The sum is 3.”
(Partial Differences)

Recalled or Derived Fact Strategies

- A child possessing good number sense is able to solve problems in flexible ways, often breaking numbers down and recombining them by using known facts, which is referred to as **deriving**.

Child's Solution to JRU

$$5 + 5 + 1 = 11$$

“I know that five and five is ten. I took one from the six to make five. But I must add one back on. It is eleven.”

Child's Solution to SRU

$$10 - 5 = 5 \text{ or } 11 - 5 = 6$$

“I know that ten take away five is five, but I started with eleven. The answer must be one more. It is six.”

Sample Problems

- Sample Problem #1 : Grade 1

- Michelle has 8 marbles. How many more does Michelle need to have 25 marbles? (JCU Type)

$$8 + \underline{\quad} = 25$$

- Sample Problem #2: Grade 2

- Julie has 69 stickers. She has 14 more than Sarah. How many stickers does Sarah have? (CRU Type)

$$69 - 14 = \underline{\quad}$$

- Sample Problem #3: Grade 3

- 7 families went on a camping trip. Each family brought 24 drinks. How many total drinks were brought by the families? (Multiplication Type)

$$24 \times 7 = \underline{\quad}$$

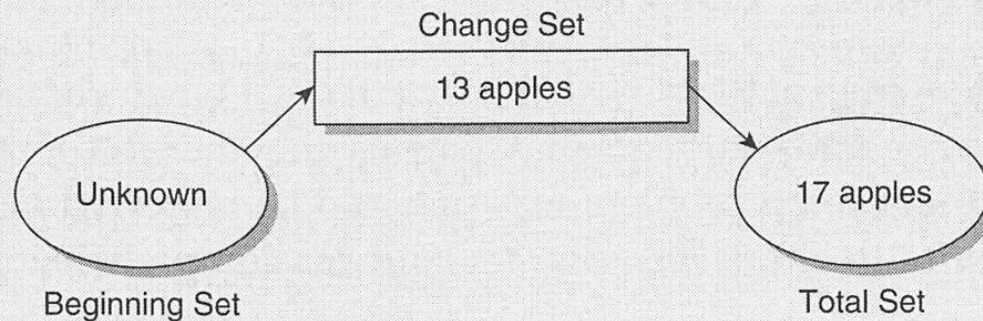
Schematic Diagrams for Word Problems

The Change Schema

One type of problem requires a change schema. These problems include a set of information that indicates change in other information in the problem.

John had some apples. Paul gave him 13 more apples. Now John has 17 apples. How many did John have in the beginning?

The change information (13 apples) must be subtracted from the total resultant set of information (17) in order to determine the start set. This change problem may be represented as follows.



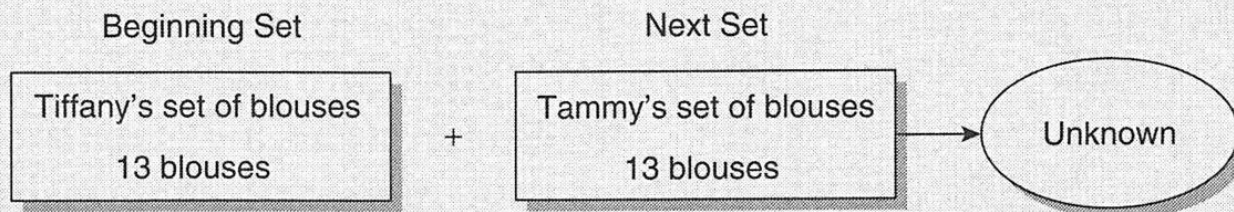
Schematic Diagrams for Word Problems

The Group Schema

In a group schema, items are grouped together from various sets. Consider the following problem:

Tiffany owns 13 blouses that she wears to school. Her twin sister Tammy owns 13 blouses. When these girls swap clothes for school, how many blouses can they choose from?

A group schema would be represented as follows.



Schematic Diagrams for Word Problems

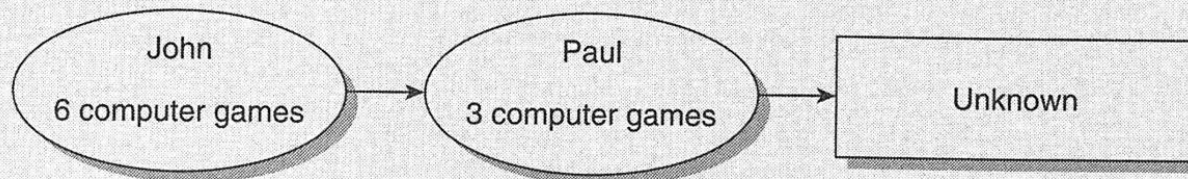
The Comparison Schema

Some word problems present “comparison” problems, which require the student to determine and subsequently compare values.

John has 6 computer games. He has 3 more than Paul. How many games does Paul have?

In order to solve this problem, the child must have a comparison schema or mental concept that includes three pieces of information: two reference quantities (the number of computer games that John has, and the difference) and a derived piece of information involving the comparison answer. Note also that the cue word *more* usually means that a child should add, but in this example it indicates subtraction.

A comparison schema would be represented as follows:



Teaching a Learning Strategy

1. Explain the strategy and what it can be used for with some initial guided practice.
 - Make a poster of the strategy steps and keep it in front of the class.
2. Explain the differences between spoken language and thought.
 - Be sure that students understand that strategies are ways of thinking that can assist them in solving mathematical problems for the rest of their lives.
3. Introduce the strategy steps one at a time.
 - Model each step separately and in combination with other steps.
 - Reinforce each step and practice each step over a period of several days.
 - Explicitly teach students to memorize the steps, to recognize the steps, and to identify the activities done in each step.
4. Use direct instruction to teach the steps.
 - Once the students say and write the steps independently, they can begin to use them to solve problems.

STAR Learning Tactic

- **S**earch the word problem.
 1. Read the problem carefully.
 2. Ask, “What do I know, and what do I need to find?”
 3. Write down the facts.
- **T**ranslate the words into an equation in picture form.
 1. Choose a variable to solve for.
 2. Identify the operations necessary (look for cue words).
 3. If possible, represent the problem with manipulatives.
 4. Draw a picture of the equation, including know facts and operations.
- **A**nsWER the problem.
 1. Perform the necessary operations, solving for the unknown.
- **R**eview the solution.
 1. Reread the problem.
 2. Ask, “Does the answer make sense? Why or why not?”
 3. Check the answer.

Teaching Perspectives of Problem Solving

1. Teach problem solving as a goal
 - a. How to problem solve (Open minded)
 - b. Set goals with students (Calculation, key word recognition, problem type, multi-step, etc.)
2. Teach problem solving as a process
 - a. Develop math concepts through selected problems
 - b. More than one way to arrive at an answer
3. Teach problem solving as a tool (applications and modeling)
 - a. Real world problems
 - b. Word Problems

Sample Problem

Grade 2: Solve the problem and identify the problem type.

Sheila took all the dollar bills out of her two pockets. The total was \$9. How much did she have in each pocket? Show how you know the answer is correct.

Extension: Rewrite the problem to create a different problem type.

Problem Solving Organizer

Robinwood Elementary

Target:
What I will learn is...

Know it:
Here is my solution...

Show it:
What I am thinking is... (picture)

See it:
What I know about the task is ...

Say it:
What I want to find is...

I can _____
What I will do ... _____

The form is shaped like a rocket with a large black flame trail at the bottom. A thought bubble is located to the left of the rocket's nose cone.

Monitoring and Reflecting

- Assist students in monitoring and reflecting on the problem-solving process.
 - Provide students with a list of prompts to help them monitor and reflect during the problem-solving process.
 - Model how to monitor and reflect on the problem-solving process.
 - Use student thinking about a problem to develop students' ability to monitor and reflect.

List of Prompts for Problem Solving

Sample Question List

- What is the story in this problem about?
- What is the problem asking?
- What do I know about the problem so far ?
What information is given to me? How can this help me?
- Which information in the problem is relevant?
- In what way is this problem similar to problems I have previously solved?
- What are the various ways I might approach the problem?
- Is my approach working? If I am stuck, is there another way I can think about solving this problem?
- Does the solution make sense? How can I verify the solution?
- Why did these steps work or not work?
- What would I do differently next time?

Sample Task List

- Identify the givens and goals of the problem.
- Identify the problem type.
- Recall similar problems to help solve the current problem.
- Use a visual to represent and solve the problem.
- Solve the problem.
- Check the solution.

Model Problem Solving Process

Last year was an extremely dry year in Colorado. Denver usually gets 60 inches of snow per year. Vail, which is high up in the mountains, usually gets 350 inches of snow per year. Both places had 10 inches of snow less than the year before. Mohammed and Marlet live in Colorado and heard the weather report. Mohammed thinks the decline for Denver and Vail are the same. Marlet thinks when you compare the cities decline is different. Explain how both are students are correct.

Solution

Teacher: First, I ask myself, “**What is the story about and what do I need to find out?**” I see that the problem has given me the usual amount of snowfall and the change in each snowfall for each place and that it talks about a decline in both cities. I know that decline means “a change that makes something less.” Now, I wonder how the decline in snowfall for Denver and Vail can be the same for Mohammed and different for Marlet. I know that a decline of 10 inches both cities is the same so I guess that is what makes Mohammed correct. How is Marlet thinking about the problem?

I ask myself, “**Have I ever seen the problem like this before?**” I remember seeing a problem similar to this last week.

Before I go on, I ask myself, “**What steps should I take to solve this problem?**” I looks like I need to divide the change amount by the original amount to find the percent change in snowfall for both Denver and Vail.

- Denver: $10 \div 60 = .166$ or 16.7% or 17 % when rounded to the nearest whole number
- Vail: $10 \div 350 = .029$ or 2.9% or 3% when rounded to the nearest whole number

So the percent decrease in snow for Denver was much greater (17%) compared to Vail (3%).

Finally, I ask myself, “**Does this answer make sense when I reread the problem?**” Mohammed’s answer makes sense because both cities had a decline of 10 inches of snow. Marlet is right because the percent decrease for Vail is much smaller than it is for Denver. Both answers make sense.

References

- Bender, W.N. (2009). *Differentiating math instruction : Strategies that work for K-8 classrooms*. (2nd Ed.) Thousand Oaks, CA: Corwin Press.
- Carpenter, T.P., Fennema, E., & Franke, M.L. (1996). Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal*, 97 (1), 3-20.
- Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2000). *Cognitively Guided Instruction: A Research-Based Teacher Professional Development Program for Elementary School Mathematics*. Research Report.
- Carpenter, T.P., Fennema, E., Peterson, P.L., Chiang, C.P., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching. *American Educational Research Journal*, 26 (4), 499-531.
- Kamii, C., & Rummelsburg, J. (2008). Arithmetic for First Graders Lacking Number Concepts. *Teaching Children Mathematics*, 14(7), 389-394.
- Moscardini, L. (2010). 'I like it instead of maths': How pupils with moderate learning difficulties in Scottish primary special schools intuitively solved mathematical word problems. *British Journal of Special Education*, 37 (3), 130-8.
- Promising Practices Network (2007). *Cognitively guided instruction (CGI)*. Retrieved from <http://www.promisingpractices.net/program.asp?programid=114>.
- Spilde, A. (2013). *Mathematics in a Second Grade Classroom: The Effects of Cognitively Guided Problem Solving* (Doctoral dissertation, Arizona State University).
- Villasenor, A., & Kepner, H.S. (1993). Arithmetic from a problem-solving perspective: An urban implementation. *Journal for Research in Mathematics Education*, 24 (1), 62-9.
- Vygotsky, L. (1978). Interaction between learning and development. *Readings on the development of children*, 23(3), 34-41.