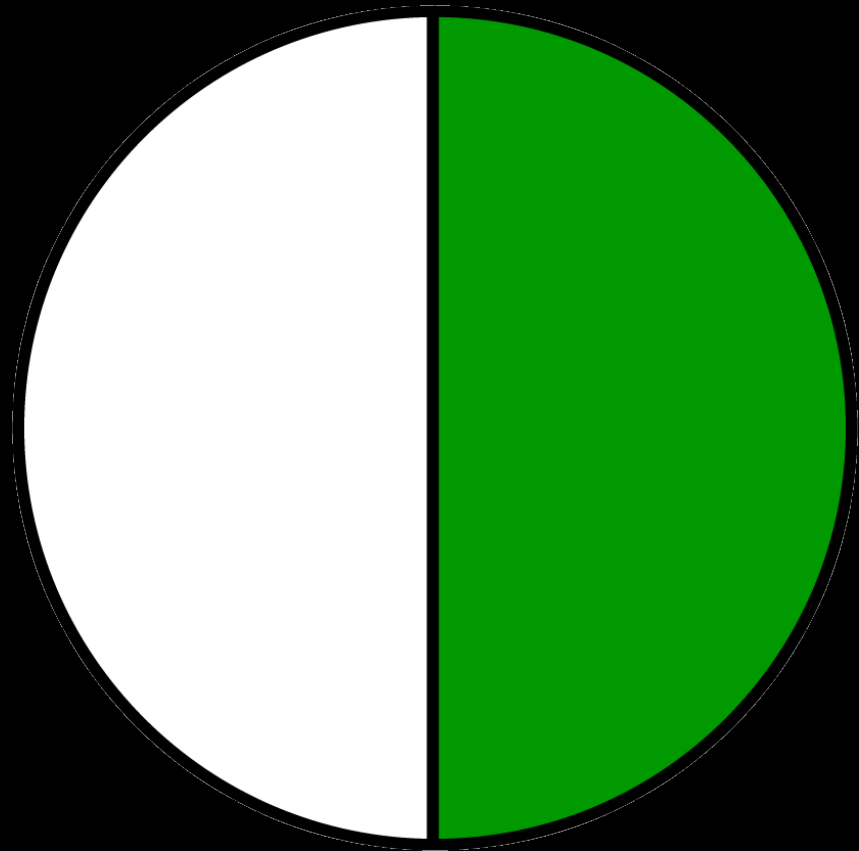




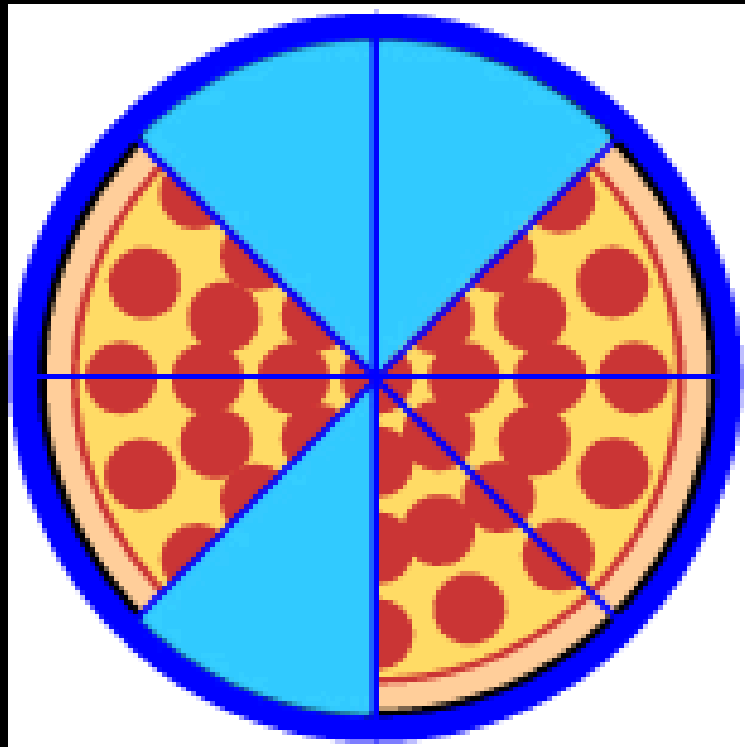
# THE FIVE FUNCTIONS OF FRACTIONS

Claudia Bertolone-Smith, University of Nevada, Reno.

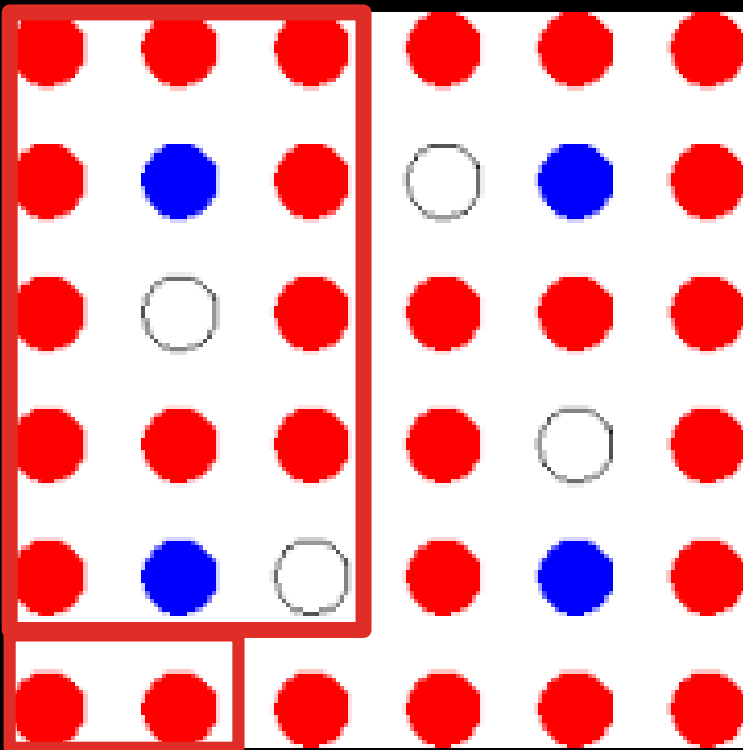
PICTURE  $\frac{1}{2}$  IN YOUR HEAD? WHAT  
DO YOU SEE?



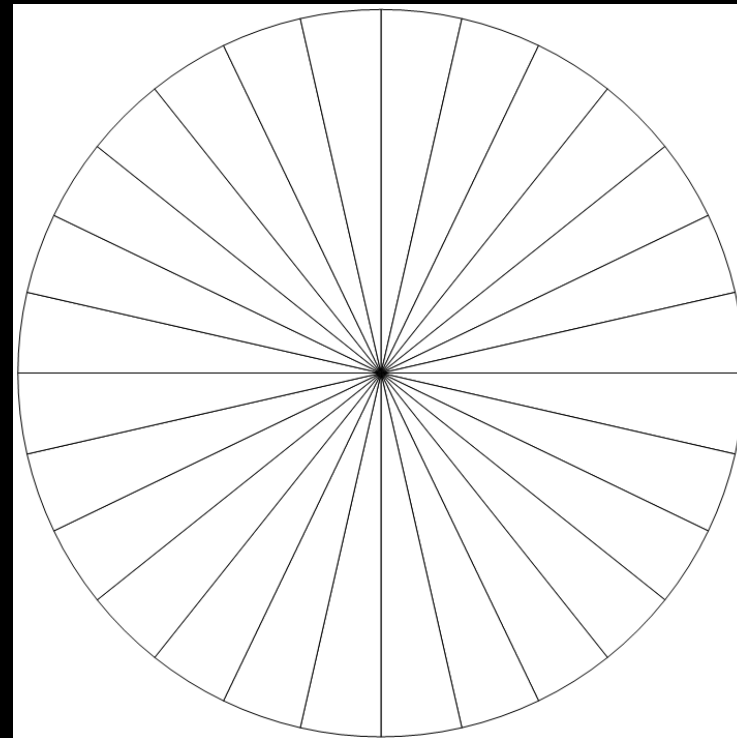
PICTURE 5/8 IN YOUR HEAD? WHAT  
DO YOU SEE?



# PICTURE 17/36 IN YOUR HEAD? WHAT DO YOU SEE?



How many out of 36??



# OUR JOURNEY THROUGH THE FIVE FUNCTIONS OF FRACTIONS

**Part-whole**

**Measurement**

**Unfinished  
Division**

**Operator**

**Rate/Ratio**

# TOOLS REQUIRED!

A ruler.



A race.



A WILLINGNESS TO LET GO OF PART-WHOLE  
THINKING!

# BUILD A RULER FOR THE RATIONAL RULER COMPANY!

## Rational Ruler Regulations

1. Use fractions, improper fractions, and whole numbers for the rulers (no mixed numbers).
2. The spaces made on the ruler must be made up of equal sections.
3. The lines are labeled with the correct fractions.
4. Rational Rulers are neat and tidy.

# BUILD A RULER FOR THE RATIONAL RULER COMPANY!

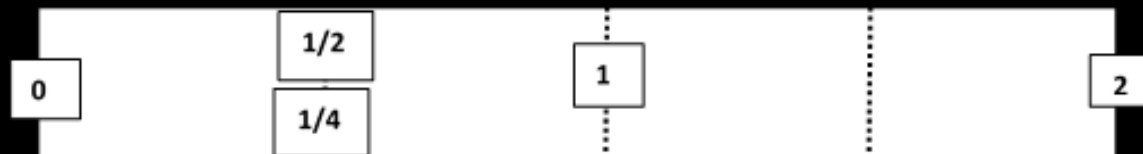
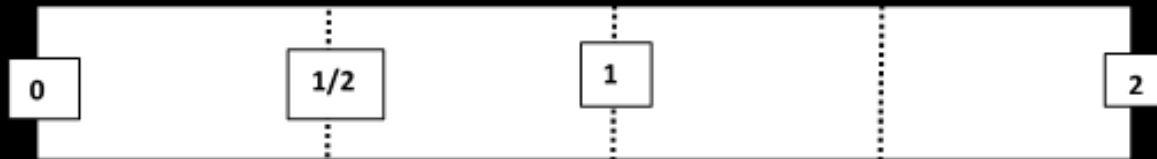
Fold into 4 equal parts.

It starts at 0 and ends at 2.

What should we label the lines?

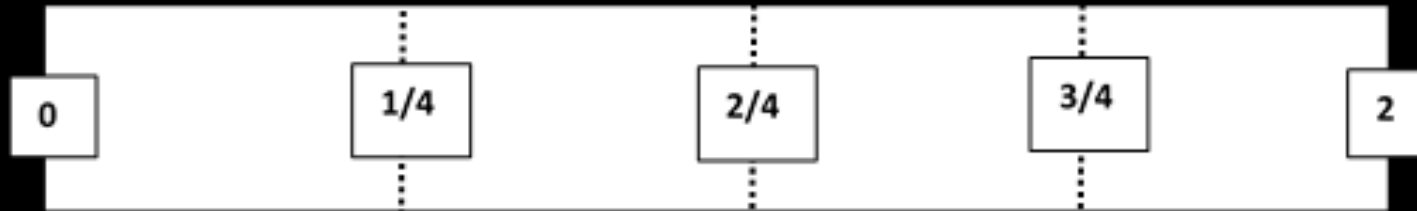


# STUDENT CONCEPTIONS



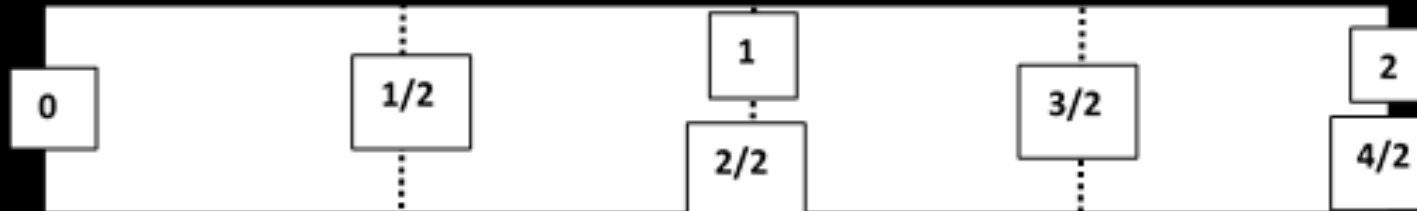
# WHEN IS IT $\frac{1}{2}$ ? WHEN IS IT $\frac{1}{4}$ ?

## Part-whole Thinking



Part-whole thinking sees the entire ruler as the whole regardless of indicators otherwise, and uses the fraction to label the number of partitioned pieces in relationship to the whole.

## Measurement Thinking



Measurement thinking considers the units with the whole and sees the ruler as a piece of a continuous number line. The unit is defined and fractions are applied as labels that iterate across the whole.

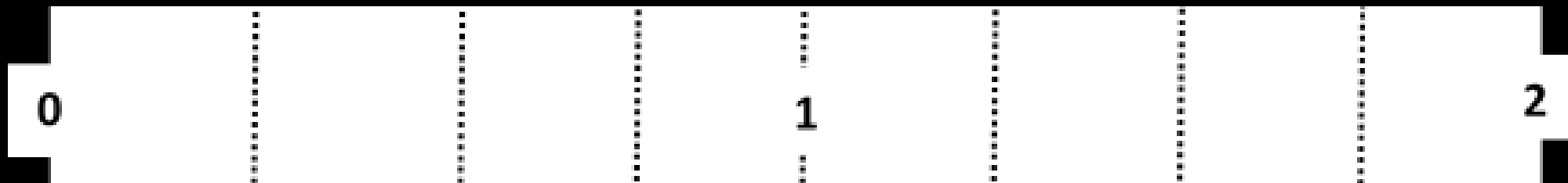
# BUILD A RULER FOR THE RATIONAL RULER COMPANY!

Fold into 8 equal parts.

Label the middle fold 1.

What should we label the lines?

# TWO SCENARIOS: WHEN IS IT $\frac{1}{8}$ ? WHEN IS IT $\frac{1}{4}$ ?



*Scenario 1:* Jose is running a race. The race is two miles. He runs from the start to the first fold. He asks, "How much of the race have I run?????"

*Student 1:* What did you get (Student 2)?

*Student 2:* I got  $\frac{1}{4}$ .

*Student 3:* Well, we think it is  $\frac{1}{8}$ .

*Student 2:* Cause there are 4 spaces and one whole.

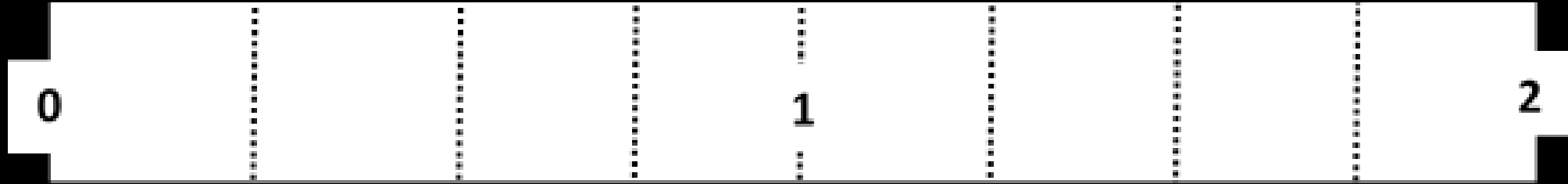
*Student 1:* Yeah, but that's only one mile.

*Student 1:* This is the whole race.

*Student 3:* If it was split into miles then it would be  $\frac{1}{4}$

*Student 3:* It says how much of the race.

# TWO SCENARIOS: WHEN IS IT $1/8$ ? WHEN IS IT $1/4$ ?

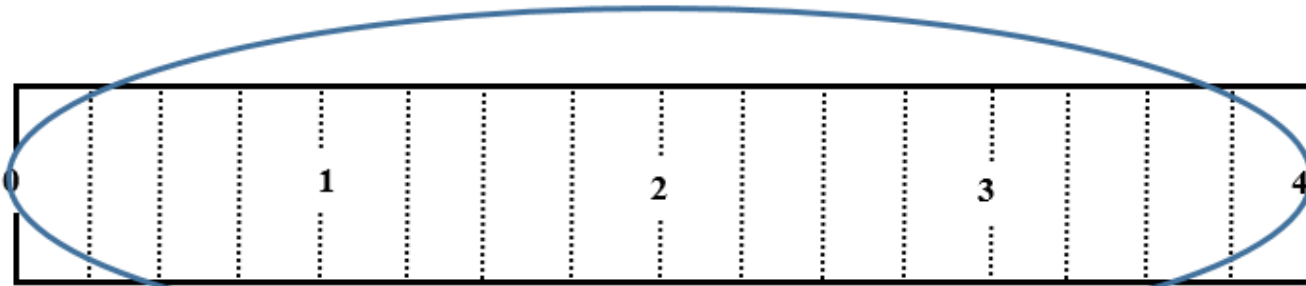


*Scenario 2:* Talia is running the same race. She stops at the first fold and asks, “How much of a MILE have I run?????”

# EXPLICIT SHIFT: PART WHOLE TO MEASUREMENT

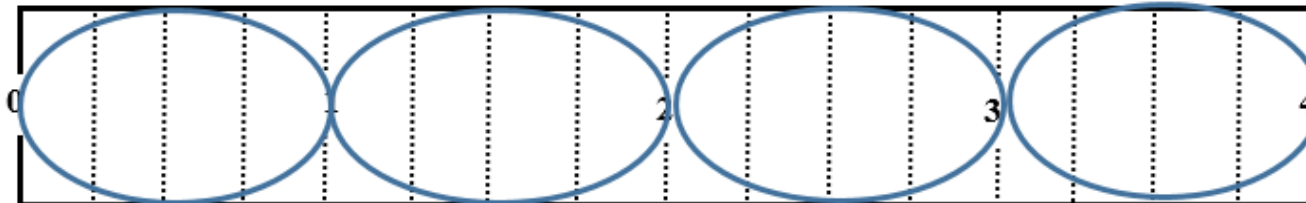
## PART/WHOLE THINKING

Label this ruler with part/whole thinking. What is each line out of the WHOLE unit?



## MEASUREMENT THINKING

Label this ruler with MEASUREMENT THINKING. What part is each line out of 1 km?



Rational Rulers  
Require  
MEASUREMENT  
THINKING!



# BUILD A RULER FOR THE RATIONAL RULER COMPANY!

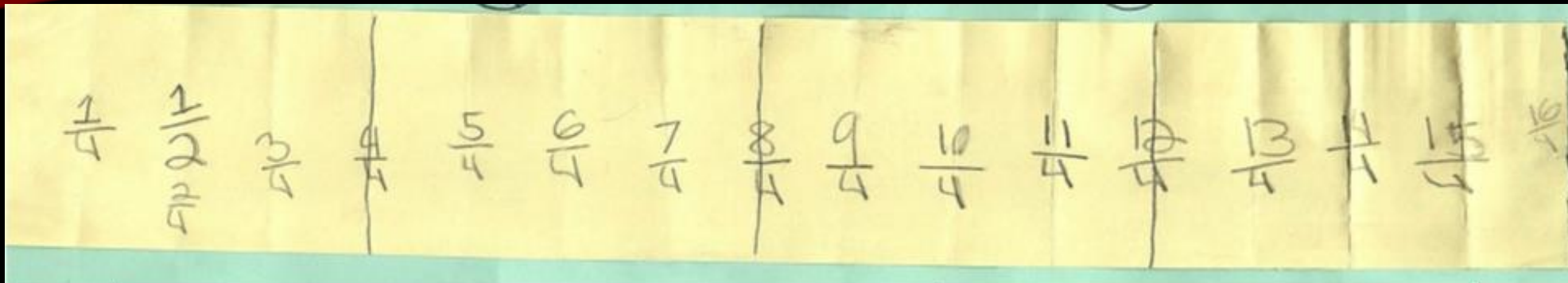
Fold into 16 equal parts.

Label the fourth fold 1.

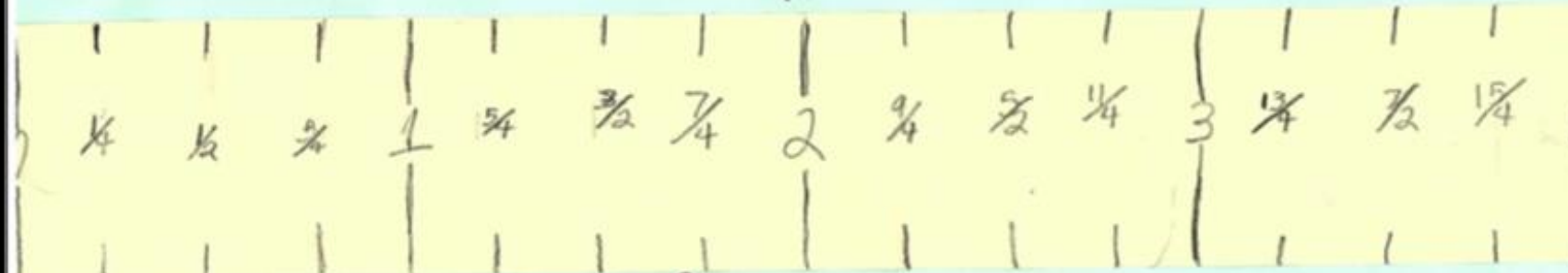
What should we label the lines?



# STUDENT WORK



Rational Ruler  
Company





# EQUIVALENT FRACTIONS EMERGE

The image shows a handwritten table on a piece of paper with a decorative header. The header consists of a black background with a wavy, multi-colored border in shades of red, orange, yellow, and green. The table itself is written on a light-colored, slightly aged paper and lists equivalent fractions for integers from 1 to 15. Each integer is written at the top of a column, and its equivalent fractions with denominators 4, 8, 12, 16, 20, 24, and 32 are listed below it. The fractions are written in a simple, hand-drawn style.

Integer	Equivalent Fractions
1	$\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}, \frac{6}{24}, \frac{7}{28}$
2	$\frac{2}{4}, \frac{4}{8}, \frac{6}{12}, \frac{8}{16}, \frac{10}{20}, \frac{12}{24}, \frac{14}{28}$
3	$\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \frac{18}{24}, \frac{21}{28}$
4	$\frac{4}{4}, \frac{8}{8}, \frac{12}{12}, \frac{16}{16}, \frac{20}{20}, \frac{24}{24}, \frac{28}{28}$
5	$\frac{5}{4}, \frac{10}{8}, \frac{15}{12}, \frac{20}{16}, \frac{25}{20}, \frac{30}{24}, \frac{35}{28}$
6	$\frac{6}{4}, \frac{12}{8}, \frac{18}{12}, \frac{24}{16}, \frac{30}{20}, \frac{36}{24}, \frac{42}{28}$
7	$\frac{7}{4}, \frac{14}{8}, \frac{21}{12}, \frac{28}{16}, \frac{35}{20}, \frac{42}{24}, \frac{49}{28}$
8	$\frac{8}{4}, \frac{16}{8}, \frac{24}{12}, \frac{32}{16}, \frac{40}{20}, \frac{48}{24}, \frac{56}{28}$
9	$\frac{9}{4}, \frac{18}{8}, \frac{27}{12}, \frac{36}{16}, \frac{45}{20}, \frac{54}{24}, \frac{63}{28}$
10	$\frac{10}{4}, \frac{20}{8}, \frac{30}{12}, \frac{40}{16}, \frac{50}{20}, \frac{60}{24}, \frac{70}{28}$
11	$\frac{11}{4}, \frac{22}{8}, \frac{33}{12}, \frac{44}{16}, \frac{55}{20}, \frac{66}{24}, \frac{77}{28}$
12	$\frac{12}{4}, \frac{24}{8}, \frac{36}{12}, \frac{48}{16}, \frac{60}{20}, \frac{72}{24}, \frac{84}{28}$
13	$\frac{13}{4}, \frac{26}{8}, \frac{39}{12}, \frac{52}{16}, \frac{65}{20}, \frac{78}{24}, \frac{91}{28}$
14	$\frac{14}{4}, \frac{28}{8}, \frac{42}{12}, \frac{56}{16}, \frac{70}{20}, \frac{84}{24}, \frac{98}{28}$
15	$\frac{15}{4}, \frac{30}{8}, \frac{45}{12}, \frac{60}{16}, \frac{75}{20}, \frac{90}{24}, \frac{105}{28}$

# BUILD A RULER FOR THE RATIONAL RULER COMPANY!

Create the ruler for task 1.

Work with your team to divide this ruler into 9 equal parts. Write the correct fractions on the lines you make.

The ruler is a horizontal rectangle divided into 9 equal segments by vertical lines. The left end is labeled with a box containing the number 0, and the right end is labeled with a box containing the number 3. Above each of the 9 vertical lines, a fraction is handwritten. From left to right, the fractions are:  $\frac{1}{3}$ ,  $\frac{2}{3}$ ,  $\frac{3}{3}$ ,  $\frac{4}{3}$ ,  $\frac{5}{3}$ ,  $\frac{6}{3}$ ,  $\frac{7}{3}$ ,  $\frac{8}{3}$ , and  $\frac{9}{3}$ .

# FRACTION AS AN UNFINISHED DIVISION PROBLEM (EMERGING)

C: If this is three, what I want to know where is 1 and 2? If this is three, where is 1 and 2?

Conner: Here is 1 and here is 2 (points to  $3/3$  and  $6/3$ )

C: If this is one whole, (circles  $3/3$ ) what do we label the lines?

Jordan:  $1/3$

C: What do we label the second line? Everyone say it:

Class:  $2/3$ ,  $3/3$ ...

C: I don't want mixed numbers so what do I label the next line?

Class:  $4/3$ ,  $5/3$ ,  $6/3$

C: What is 6 divided by 3?

Class: 2

C: Oh, that's interesting. It is also two. Numbers are amazing.

Class: Ohh.

C: What's next?

Class:  $7/3$ ,  $8/3$ ,  $9/3$

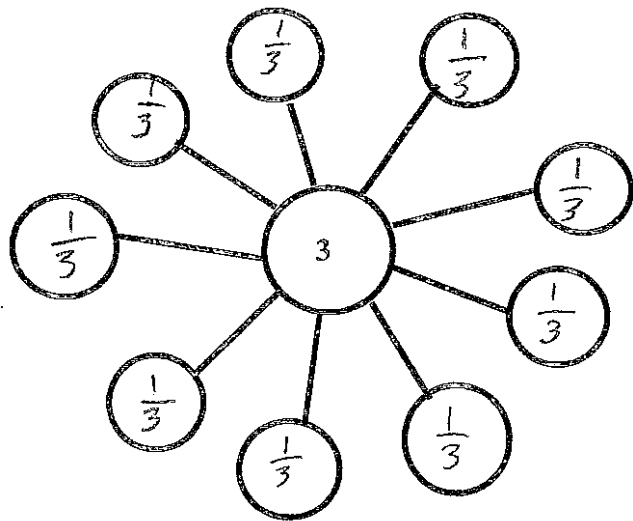
C: What is nine divided by three?

Class: 3

C: Oh, its also 3. Numbers are amazing.

# DECOMPOSE 3 INTO 9 EQUAL PARTS. WRITE AN ADDITION AND MULTIPLICATION PROBLEM

Decompose 3 into nine equal pieces.



Addition Sentence:

$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{9}{3} = 3$$

Multiplication Sentence:

$$9 \times \left(\frac{1}{3}\right) = 3$$

$$\frac{9}{3} \div 9 = \frac{1}{3}$$

- Number Bonds- AMAZING
- Some students could see the division problem!!

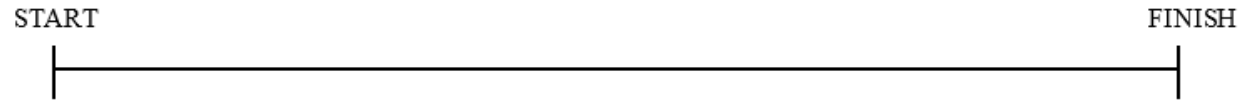
# RACE PLANNING FOR THE RRC

San Francisco wants to plan a fun walk for the students. This is what they would like to have:

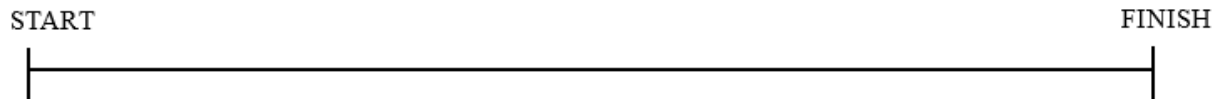
4. At  $\frac{2}{3}$  and  $\frac{1}{4}$  there will be fizzy water.
5. At  $\frac{1}{8}$  and  $\frac{6}{6}$  there will be burritos.
6. At  $\frac{1}{2}$  and  $\frac{6}{8}$  there will be dancers.

USE A STRING FOR ACCURACY!

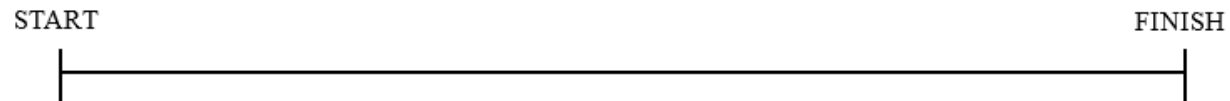
*Race Ruler for thirds and sixths:*



*Race Ruler for halves, fourths and eighths:*



*Final Race Plan:*

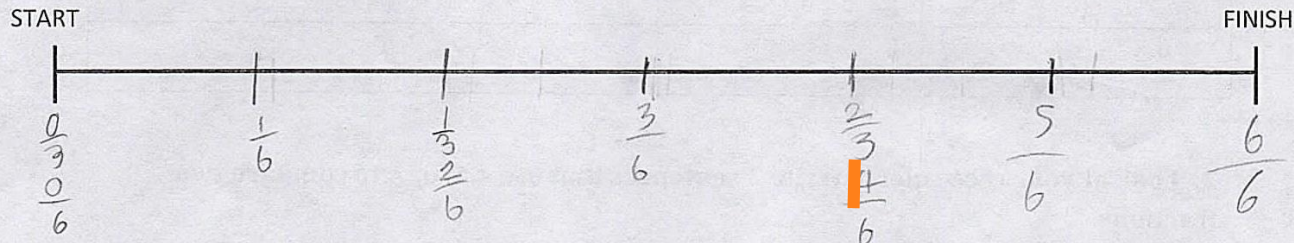


- Thirds

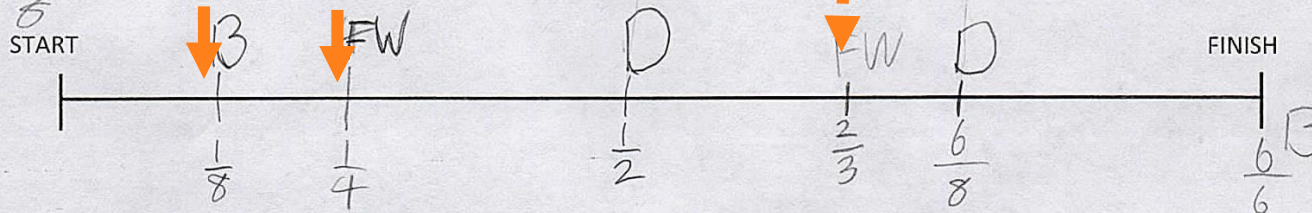
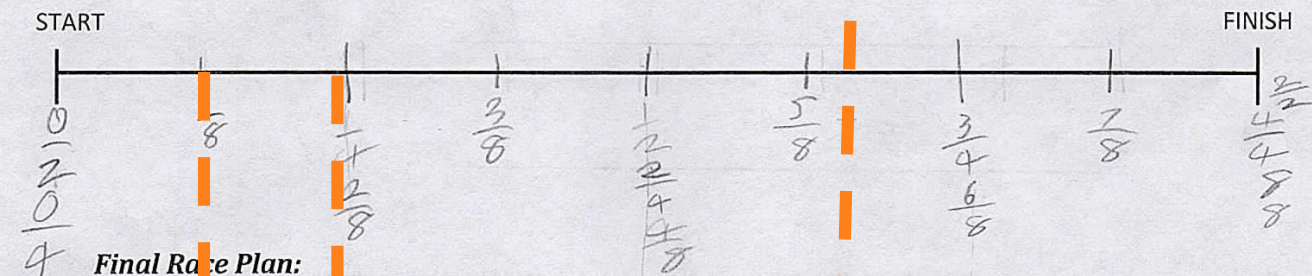


# STUDENT WORK

*Race Ruler for thirds and sixths:*



*Race Ruler for halves, fourths and eighths:*

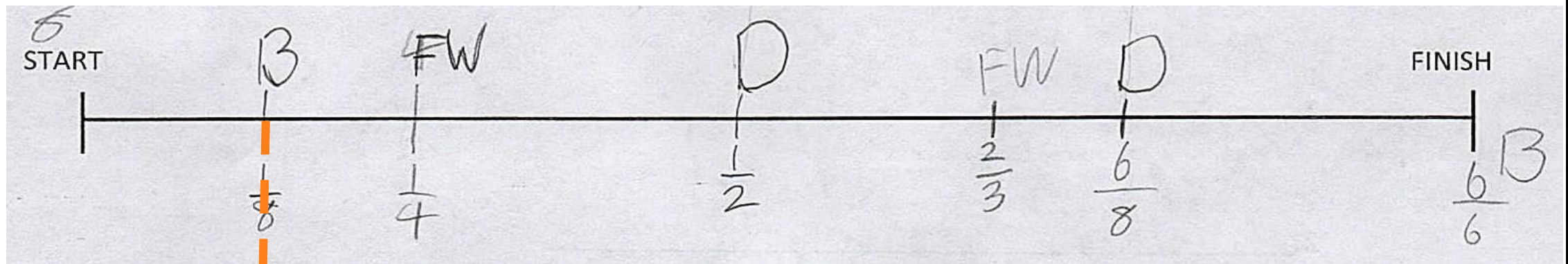


- Thirds much harder to iterate than halves.
- Strings can be used as a tool to test partitions.
- Students could accurately place 5 or more fractions in order and magnitude on an empty number line
- Race rulers essential for accuracy.

# FRACTIONS AS A RATE

What if the race is 16 miles long. Sam runs 1 mile in 10 minutes. How long will it take her to finish  $\frac{1}{8}$  of the race?

**Final Race Plan:**



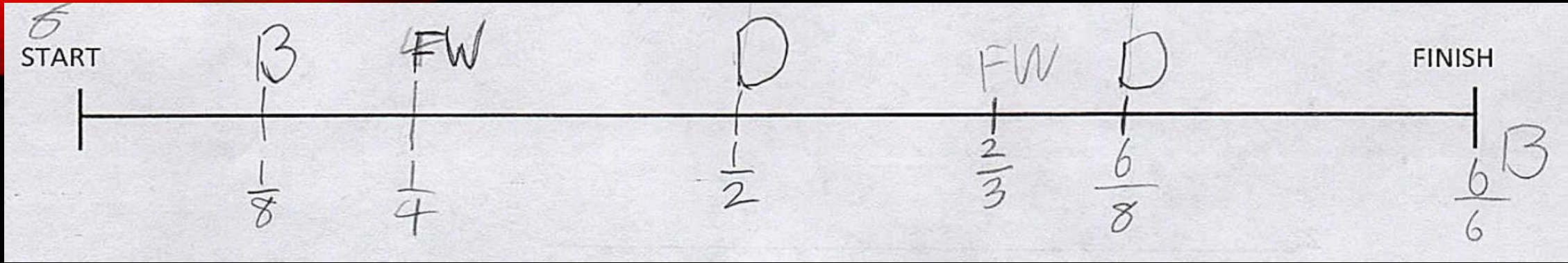
Number of minutes



16 miles

Use the bar as a tape diagram.

# FRACTIONS AS AN OPERATOR

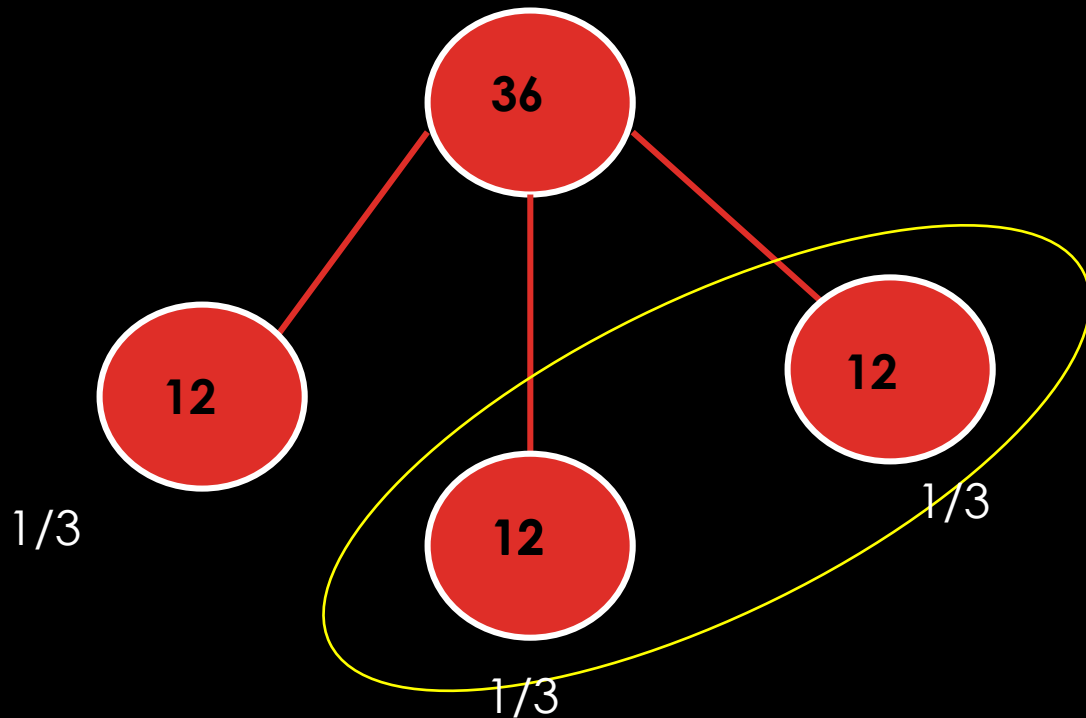


What if this race were 36 miles long. How far would I have to run to get a second fizzy water?????



# NUMBER BONDS: 2/3 OF 36

Break up 36 into 3 EQUAL pieces.



# 2/3

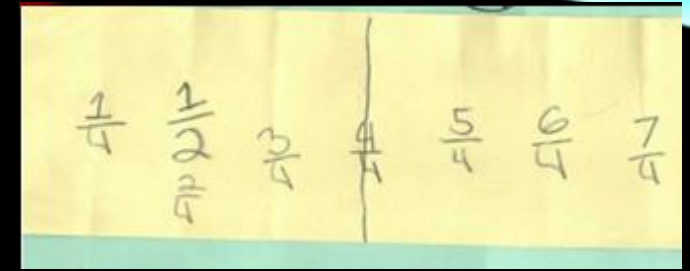
The fraction operates on numbers in this way:

The denominator breaks it into 3 parts.

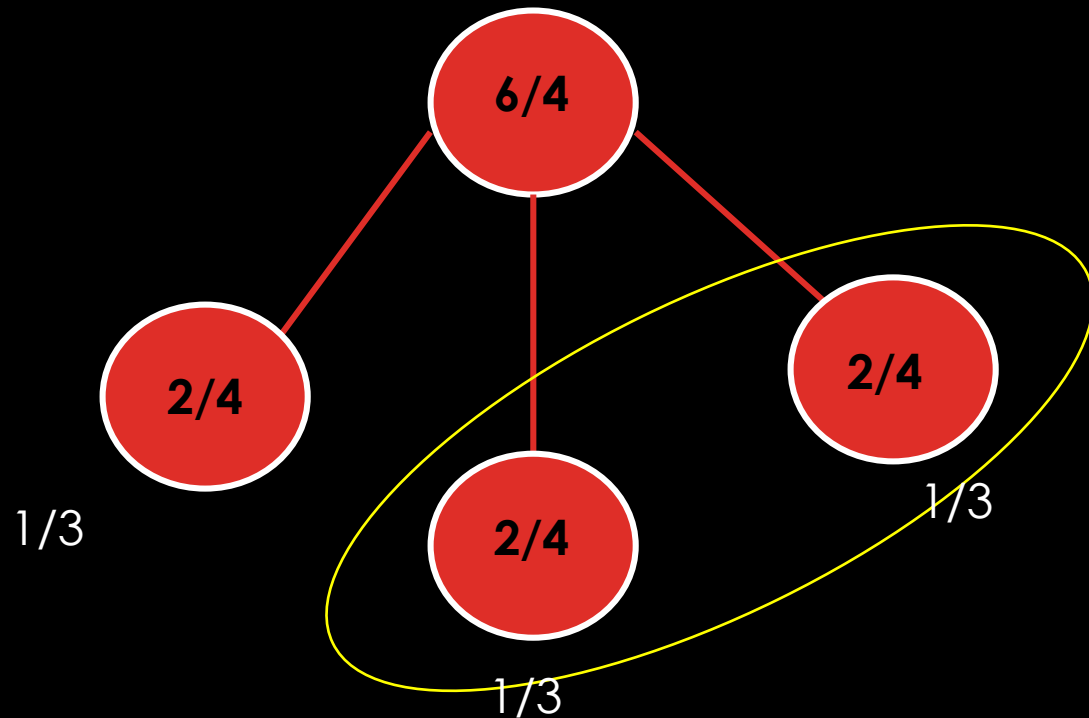
The numerator tells us how many to take

2/3 of 36 is 24

# NUMBER BONDS: $2/3$ OF $6/4$ ??



Break up 36 into 3 EQUAL pieces.



# $2/3$

The fraction operates on numbers in this way:

The denominator breaks  $6/4$  into 3 parts.

The numerator tells us how many to take:

$$\frac{2}{4} + \frac{2}{4} = \frac{4}{4} = 1$$

AND....

$$\frac{2}{3} \times \frac{6}{4} = \frac{12}{12} = 1$$

Can you figure out  $\frac{6}{4}$  divided by  $\frac{2}{3}$  with your ruler????



THANK YOU FOR JOINING ME ON  
THIS EPIC JOURNEY!

If you are interested in testing my unit:  
Rulers, Races and Regions(4<sup>th</sup> grade),  
please email me:

[claudia.bs@charter.net](mailto:claudia.bs@charter.net)