Lunchroom Ladies, Truck Drivers, and Native American Beadworkers: Mathematical Modelers

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2007 (Moscone)

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Session Goals

- Participant Introduction / Interaction
- Analyzing modeling tasks
- Content connections
- Pedagogical connections
- Discussion

Mathematical Modeling

- An application process where K-12 students engage with varying levels of sophistication. (National Council of Teacher of Mathematics, 2000)

- The understanding that models can be developed, revised, and reapplied within a context.

- Mathematical conclusions are interpreted and reported within the context of the original inquiry.

Representative mathematical models are, "fundamental to how people understand and use those ideas" and aid individuals to "acquire tools that significantly expand their capacity to model and interpret physical, social and mathematical phenomena" (NCTM, 2000, p.4)

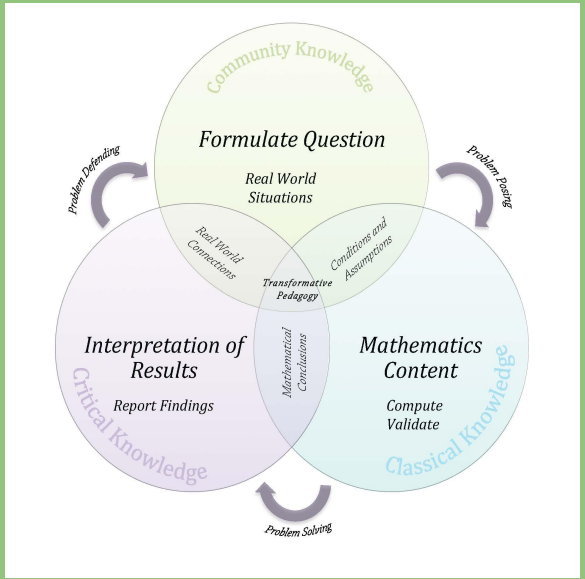
Socio-Critical Mathematical Modeling

Kaiser and Sriraman (2006)

Socio-critical modeling approach provides an “emancipatory perspective” that leads to a critical understanding of the surrounding world”. (p. 304)

Critical Mathematics Education (CME) using socio-critical modeling empowers learners to use:

- Communal knowledge to frame problems that are central to their lived experience.
- Classical knowledge to develop mathematical competencies.
- Critical knowledge to gain a comprehensive understanding of the Socio-political context for the problem (Gutstein, 2007)



Connecting Research to Practice

The work described in this presentation is part of a research study, within a senior level course for prospective middle school teachers.

- Design mathematical modeling tasks for middle school.
- Support middle school students' creation of a mathematical model.
- Modify perceptions of who is a credible source of mathematical knowledge.
- Apply a critical mathematics education perspective.

Designing a Modeling Task

Given the following contexts, explore possible math modeling activities relevant to a middle school classroom!

Social Context 1: School Cafeteria (focus on lunchroom ladies)

Social Context 2: Interstate commercial driving (focus on truck drivers)

Social Context 3: Native American history (focus on beadworkers, basket weavers)

Social Context 4: (You suggest a context....)

Lunchroom Ladies

Ms. Mary is a head cook in a middle school cafeteria. She must follow a set of guidelines to ensure that students receive the right amount of each food group per day. There is also a regulation that lunches must be within a certain calorie range.

- Create a weekly lunch plan that Ms. Mary would approve.
- On a given day, how much of each food type would Ms. Mary have to prepare? Describe how you arrived at the solution.

Make sure that your meal plans conform to the serving size regulations and the nutritional requirement.

Lunchroom Ladies

Nutritional and serving size regulations.

Meal Pattern	Amount Served Per Week (Minimum Per Day)
Fruit (Cups)	2 ½ (½)
Vegetables (Cups)	3 ¾ (¾)
Grains (serving)	8-10 (1)
Meats/ Meat Alternatives (Ounces)	8-10 (1)
Fluid Milk (Cups)	5 (1)

Fruits		**Vegetables**	
Apple sauce	50	Broccoli	80
Apple slices	50	Carrots	75
Banana	75	Peas	60
Grapes	60	Green Beans	65
Pineapple chunks	60	Lettuce (Salad)	10
Orange Juice	45	Tomatoes	40
Orange Slices	45		
Apple Juice	45		
Grains		**Meats**	
Roll	65	Breaded Chicken Patty	160
Hamburger Bun	150	Grilled Chicken Patty	120
Hot Dog Bun	125	Hamburger	185
Bread Stick	100	Sausage Links	110
Sliced Bread	70		
Fluid Milk		**Extras**	
Chocolate Milk	90	Marinara Dipping Sauce	40
Regular Milk	90	Ranch Dressing	95
Strawberry Milk	90	Ketchup	35
		Mustard	35
		Mayonnaise	60
		Chips	100

Truck Driving

Mr. Joe is a truck driver. As part of this job requirement, he has to deliver shipments to different warehouses across the country. He starts from [your location]. He starts from this city and delivers shipment to three other cities in the United States [outside of your state].

- What is the shortest route that he can take?
- How long will it take him to complete it?
- The release date for the iPhone 6 is January 1st at midnight (12:00 AM). When should Mr. Joe leave to make sure all of the stores got the iPhone 6s before the grand release?

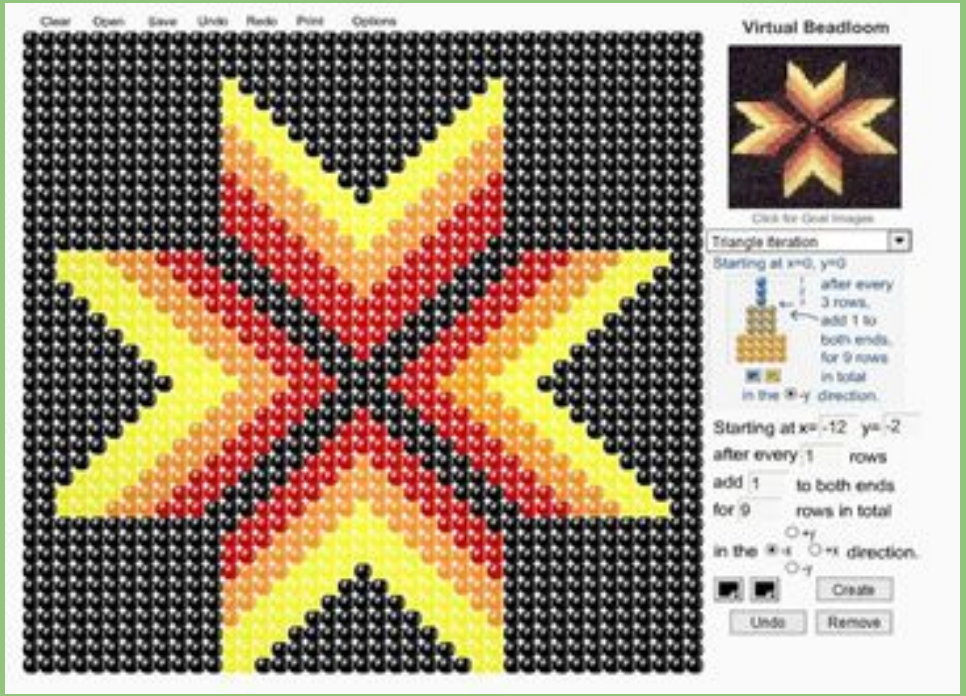
Routes to Boston, Cincinnati, Charleston and Houston

Route 1	Route 2	Route 3
		

Native American Geometry

Visit http://homepages.rpi.edu/~eglash/eglash.htm.

1. Virtual Bead Loom
2. Pacific Northwest Basket Weaver
3. Navajo Rug Weaver



Assignment

Mathematical Modeling Project

For this project you will:

- Select an everyday activity that is personally meaningful to you. This topic (negotiated with the instructor) will become central to your mathematical investigations.
- Identify, explore, understand, and describe a significant mathematical phenomenon from the chosen topic.
- Work with a group of middle school students to share your work.
- Write a project report and deliver a class presentation.

Suggested Outline for the Project Report

- **Introduction.** Describe the everyday activity; Identify outside collaborators and document your interactions with these professionals. Include pictures/diagrams/other artifacts as applicable to your work.
- **Math Explorations (Focus on mathematical modeling)**
  1. Develop a mathematical activity/task – Remember to situate your mathematical task in the chosen context. Keep your task open-ended so that there is ample scope to apply the modeling framework.
     Making sense of a situation; determining given and needed information; making assumptions; problem posing; computing a solution; Interpreting findings in context; validating findings; revising and repeating the process; reporting the solution
  2. What connections to the CCSS-M content standards are explicit/implicit? Discuss.
- **Reflection**. Share your struggles and insights. Discuss opportunities and challenges both from a learner's perspective and a teacher's perspective. You will be in a better position to write this after your session with the middle school students. Use student work and your understanding of their work to guide your reflections.

Student Work - Lunchroom Ladies

I chose the same menu for all five days of the week.

Apple Sauce	½	50
Carrots	¾	75
Bread Stick	1	100
Sausage Links	1	110
Regular Milk	1	90

Total calorie count: 525

There are about 700 students in our school – not all of them eat lunch at school. So here is my answer for 600 students.

- Fruit: 600 * 0.5 = 300.0 cups
- Vegetables: 600 * 0.75 = 450.0 cups
- Grains: 600 x 1 = 600 servings
- Meat: 600 x 1= 600 ounces
- Milk: 600 x 1 = 600 cups

First Attempt: Student realized that this simplistic plan neither fulfilled the nutritional requirements nor the recommended serving size regulations.

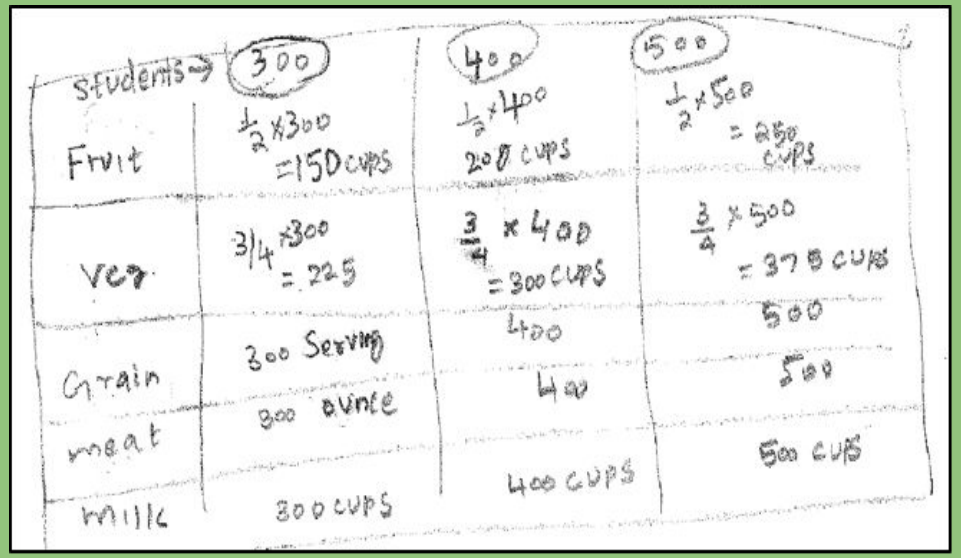
Food type	Fruit			Veg			Grains			Meat			Fluid			Total Calorie count
	Name	*SS*	*CC*	*Name*	*SS*	*CC*	*Name*	*SS*	*CC*	*Name*	*SS*	*CC*	*Name*	*SS*	*CC*	
Mon	Apple sauce	1/2	50	Carrot	3/4	75	Bread	1	100	Sausage	1	110	Reg. Milk	1	90	**425**
Tue	Banana	½	75	Peas	3/4	60	Roll	1	65	Ham burger	1	185	Choc. Milk	1	90	**570**
Wed	Grapes	½	60	Broccoli	3/4	80	Bun	1	150	Grilled chicken	1	120	Str. milk	1	90	**500**
Thu	Orange	½	45	Green beans	3/4	65	Hotdog bun	1	125	Breaded chicken	1	160	Reg. milk	1	90	**485**
Fri	Apple juice	½	45	Green beans	3/4	65	Bread	1	70	Ham burger	1	185	Choc. milk	1	90	**455**
Weekly serving		2 1/2		Weekly serving		3 3/4	Weekly serving		5	Weekly serving		5	Weekly serving		5	

Second Attempt: Student realized that they still had not attended to the nutritional requirements and the serving size regulations.

Final Revision

Students made three changes to the final revision.

- Make changes to the grain/meat item so to bring up the serving size.
- Make no changes to the to the milk/vegetable items.
- Increase the calorie count, but not exceed 700.



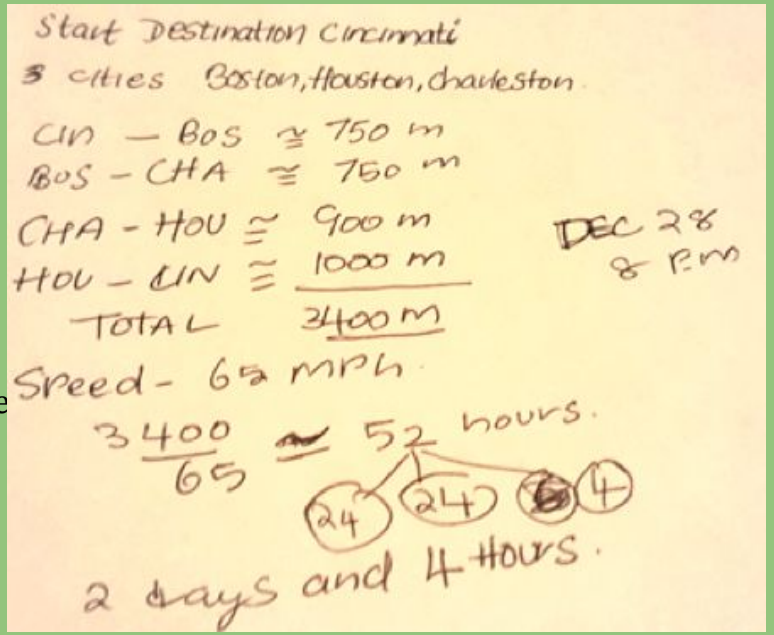
Through my interactions with Ms. Pam, a lunch lady, I noticed that the lunchroom is full of hidden mathematical concepts. She helped me realize the potential for using such contexts to design math activities. I have grown confident in developing [models] and teaching modeling to elementary and middle school students as I no longer believe that these are abstract concepts for only a select group of students. [Sam, prospective middle school teacher]

Student Work - Truck Driver

First, students outlined key steps that were instrumental in the Model development. In parentheses they included an answer or a strategy that will lead to an answer.

- Choose four destinations (Cincinnati, Boston, Charleston and Houston as the four cities, with Cincinnati as the base destination)

- Find distance between two cities (make a table). Students used a ruler to measure the distance between two cities and rounde quarter of an inch. Using the map scale (1 inch = 300 miles), they found the actual distance between the two cities. Since they did not know if this scale is accurate, they also used google maps to check the answer.



Calculating Distances

Start	End	Distance		Google map (miles, *rounded to the nearest 10)*	Google map miles ÷ map miles
		Map inches *(rounded to the nearest quarter of an*	Actual miles *(Map inches * 300)*		
Cincinnati	Boston	2.5	750	880	1.2
Boston	Charleston	2.5	750	970	1.3
Charleston	Houston	3	900	1100	1.1
Cincinnati	Houston	2.75	825	1000	1.2
Houston	Boston	5	1500	1800	1.2
Charleston	Cincinnati	1.5	450	620	1.4

Find the shortest route. Using the table, students determined the shortest route (circled in table) and that it spans 3950 miles. At this time, Student J commented, "*We should exclude the distance to travel from the third city to the base destination Cincinnati, since our goal is to suggest a suitable start time for Mr.Joe so that he can deliver the iPhones by the deadline; the problem does not say that he has to be back home by that deadline*". Thus the total distance was re-calculated to be 3950 - 880 = 3070 miles.

Find the duration of the trip (use rate formula). To determine the duration of the trip, students needed to know Mr. Joe's driving speed. Since they could not ask Joe, they looked up the speed limit guidelines on the website http://www.motorists.org/speed-limits/state-chart and found out that, in most states, the speed limit (for trucks) ranged between 55 to 65 mph. Assuming 60 mph as Mr. Joe's average driving speed, they calculated the total time needed as 51 hours (distance / rate = 3070/60 = 51)

Start	City 1	City 2	City 3	End	Total distance (miles)	
					Map	Google maps
Cincinnati	Boston	Charleston	Houston	Cincinnati	3400	3950
Cincinnati	Houston	Charleston	Boston	Cincinnati	3225	3950
Cincinnati	Charleston	Boston	Houston	Cincinnati	3505	4390
Cincinnati	Houston	Boston	Charleston	Cincinnati	3525	4390
Cincinnati	Boston	Houston	Charleston	Cincinnati	3600	4400
Cincinnati	Charleston	Houston	Boston	Cincinnati	3690	4400

Critical Aspects Students Considered

Student J: *If the stores close at say 5:00 p.m. on New Year's Eve, then we have to deliver the phones before that time. Let us aim for 4:00 p.m. on Jan 31. Minus 24 hours, 30th 4:00 p.m.; minus 24 hours, 29th 4:00 p.m. Minus 2 hours leaves us at 1:00 p.m. on December 29th.*

Student K: *But wait, he cannot drive non-stop. He has to take some breaks for eating and going to the bathroom, and sleeping.*

Student J: *He may also stop to fill gas.*

Student I: *There may be accidents on the way too. So we have to factor in time for such things. Let us say, bathroom breaks take 3 hours, eating and filling gas takes 6 hours, and two nights of sleep is 16 hours. So the total is 25 extra hours.*

Student K: *let us add 5 more hours for other things, like delays and accidents. So the total is 30 hours to add to 51 hours, which is 81 hours.*

Student J: *Then we go back 1 day (24 hours) to 1 p.m. on December 28th and go back 6 more hours that day. 12, 11, 10, 9, 8, 7. So he should start at 7:00 a.m.*

Student K: *No it will be 8 a.m. since we are going back from 1:00 p.m, and we have to stop at 8.*

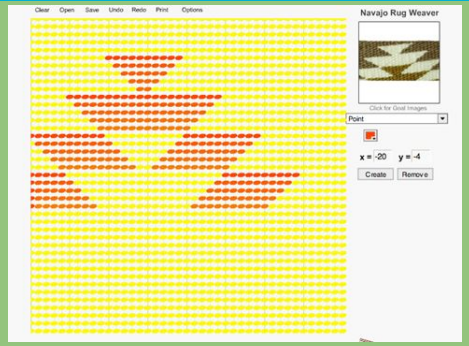
Student Work for Native American Beadwork

Pacific Northwest Basket Weaving

I had some difficulty with recreating this basket. When I looked at the picture, I saw several rectangle joined together. So I began by make rectangles, and translating them to the right and down the same number of units, so that each rectangle of each “layer” was the same size and shape.

Student Work - Use of the Linear Iteration Tool

I used the linear iteration tool to create parallelograms and trapezoids. I used a lot of math to calculate the coordinates of each vertex. For example, the top trapezoid has a base of 10 units. Therefore, when I created the larger trapezoid, I created it so that it would have a base of 20 units. I also wanted to position the starting point, so that it would be centered underneath the trapezoid. This is great math for students to practice. I found it frustrating that you cannot create parallelograms that go off the screen, like the far left parallelogram. I created this by simply using linear iterations and then filling in the last point with the “point” tool.



Examples of Other Modeling Projects

- A Postal worker's view – Figuring out postal codes using modular Arithmetic (extension: Coding and decoding messages)
- Restaurant Mathematics; Financial Mathematics (Statistics / Algebra)
- Walking a day in my parents' shoes – their views and visions of mathematics (problem solving in everyday situations)
- My dad's obsession with logic puzzles – He didn't even like mathematics (Problem solving)
- Mathematics in construction (Pythagorean theorem)
- Dr. Math - Pharmacy / nursing (Measurement, Unit conversions, life-saving lessons)
- Shipping logistics (Data interpretation, Algebra)
- Lunch menus and Mathematics (Data analysis, measurement)
- Seamstressing / Tailoring (Geometry, Number sense)
- Baristas brewing mathematics (Number sense, problem solving, measurement)
- Mathematics from the eyes of immigrant parents (problem solving)

Discussion:

- How does socio-critical mathematical modeling help students build mathematical content knowledge?
- How does socio-critical mathematical modeling help students understand the larger community?
- What other aspects of socio-critical mathematical modeling aid in the development of knowledge?
- What is the relevance of socio-critical mathematical modeling?