

## **It's All About That Base: Creatively Connecting Fractions and Decimals**

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### **NOTES**

## Base-X Positional Representations

1. Find the missing representations. **Do the movements related to each number represented!**

Base-10 number representation	Base-5 number representation	Base-5 movement representation
$78_{\text{TEN}}$		3 snaps, 0 claps, 3 steps
		4 snaps
$124_{\text{TEN}}$		
$126_{\text{TEN}}$		
		1 spin, 1 snap, 1 step
	$2012_{\text{FIVE}}$	
	$1000_{\text{FIVE}}$	
		"The largest number you can create with the moves you have"

2. Complete the following for different bases (10, 3, 5, and 7).

Base-10 number representation	Base-3 number representation	Base-5 number representation	Base-7 number representation
	$102_{\text{THREE}}$		
		$102_{\text{FIVE}}$	
			$102_{\text{SEVEN}}$

**Extension:** Which is larger,  $42_{\text{FIVE}}$  or  $112_{\text{THREE}}$ ? Can you *determine* and *justify* the answer without using base-10 numbers?

**“X-imals”**

<b>Symbols</b>	<b>Words</b>	<b>Picture</b>
23.5 <sub>SIX</sub>		
23.5 <sub>EIGHT</sub>		
222.22 <sub>FOUR</sub>		
	2 groups of 9, 0 groups of 3, 2 ones, 2-thirds, and 3-ninths	

**Connecting Fractions and X-imals**Find  $\frac{1}{2}$  as an X-imal in each base

Base 6

Base 8

Base 10

Base 5

## Picturing Different Fraction and Decimal Notational Forms

3 loaves of bread shared by 4 people

## Connecting Symbols and Pictures

For each situation, draw a picture that shows the amount of bread for each person's fair share in a way that matches the different symbol systems.

### 1 loaf shared by 4 people

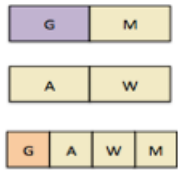
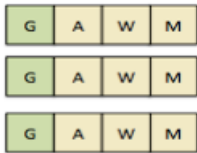
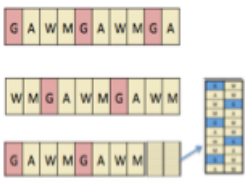

The standard fraction way	The decimal way
The base 5-imal way	Another way (your choice)

### 2 loaves shared by 6 people

The standard fraction way	The decimal way
The base 5-imal way	Another way (your choice)

### 3 loaves shared by 7 people

The standard fraction way	The decimal way
The base 5-imal way	Another way (your choice)

<b>Egyptian Fraction</b>		$\frac{1}{2} + \frac{1}{4}$	<ul style="list-style-type: none"> <li>• All numerators are 1</li> <li>• No denominator repeated</li> </ul>
<b>Standard Fraction</b>		$\frac{3}{4}$	Each person gets one piece from each loaf
<b>Decimal</b>		$0.25$	Only allowed to make pieces that are powers of 10 "You must always cut into 10 pieces"
<b>X-imal</b>		$0.33..._{\text{FIVE}}$	Only allowed to make pieces that are powers of X "You must always cut into X pieces"

**More Explorations with X-imals**

<b>Exploration 1</b> Find: <ul style="list-style-type: none"><li>• <math>1/3</math> in base 4</li><li>• <math>1/6</math> in base 7</li><li>• <math>1/5</math> in base 6</li></ul> What do you notice? Why?	<b>Exploration 2</b> Find $1/2$ in base 2, 3, 4, 6, 7, etc. What do you notice? Can you determine what $1/2$ will be in base-N?
<b>Exploration 3</b> Find: <ul style="list-style-type: none"><li>• <math>1/3</math> in base-7</li><li>• <math>2/9</math> in base-10</li></ul> What do you notice? Why?	<b>Exploration 4</b> In base-6, which fractions terminate? Which repeat? Can you find a way to predict if a fraction will terminate or repeat in base-6 without doing the division?