Putting Probability into Practice and the Practices into Probability

Rick A. Hudson
University of Southern Indiana

April 15, 2016

National Council of Teachers of Mathematics Annual Meeting

San Francisco, CA

Why Probability into Practice?

Integrated Math

Common Core

- Algebra I
- Geometry
- Algebra II
- Precalculus
- Calculus

Quantitative Reasoning

Finite Mathematics

(AP) Statistics

Why Probability into Practice?

Each of the 6 faces of a certain cube is labeled either R or S. When the cube is tossed, the probability of the cube landing with an R face up is 1/3.

How many faces are labeled R ?

A. Five B. Four C. Three D. Two E. One

Only 40% of 8th-grade students chose the correct answer.

Perez & Daiga (2016)

Why Practices into Probability?

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Goals for the Session

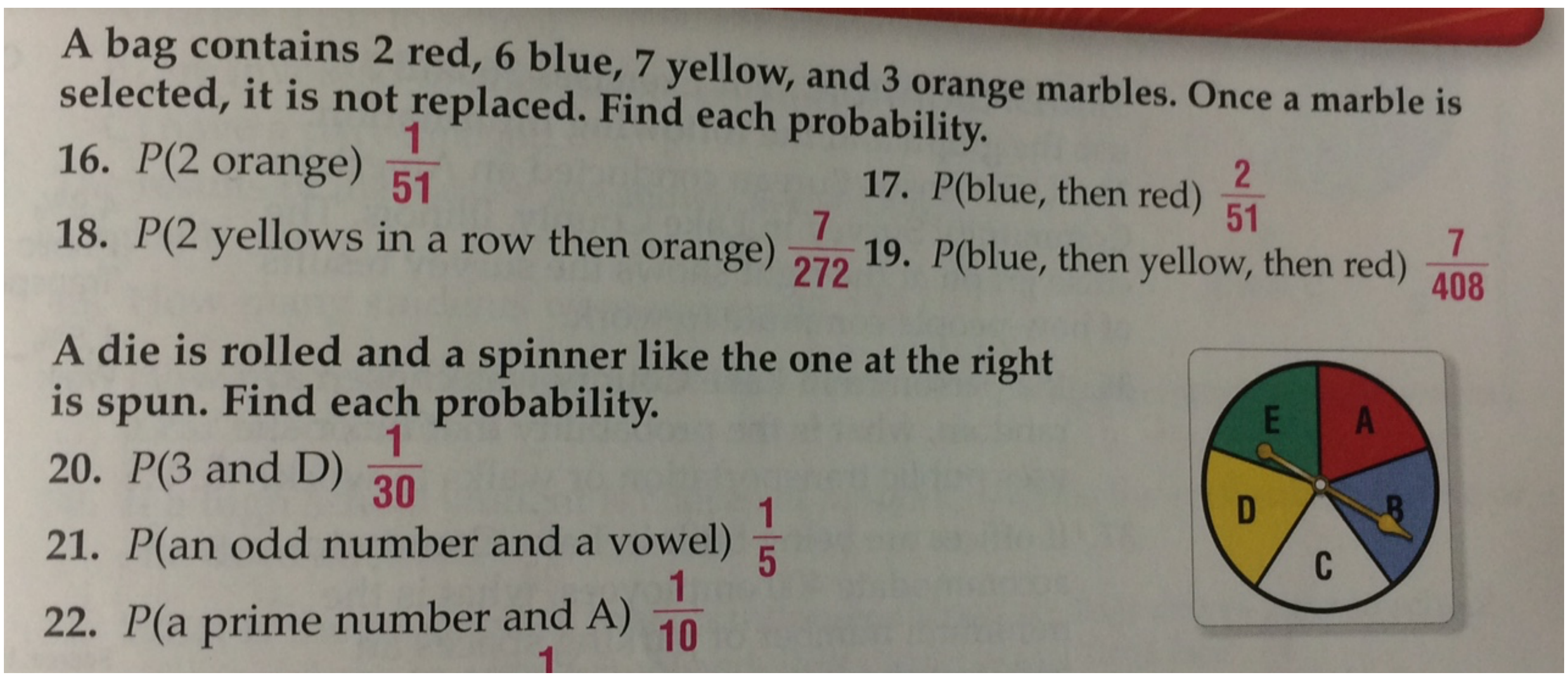
- Engage in Probability Tasks at the high school/middle school level that have potential to embed the Standards for Mathematical Practice (SMPs) in rich ways.

- Use Technology to Simulate Probabilistic Situations

Meet someone new!

- Form a group of 3-5 individuals who you can work with throughout this workshop.
- Discuss: What are your favorite probability activities? Why do you like them?

Probability in Curricula…



A bag contains 2 red, 6 blue, 7 yellow, and 3 orange marbles. Once a marble is selected, it is not replaced. Find each probability.

16. P(2 orange) $\frac{1}{51}$
17. P(blue, then red) $\frac{2}{51}$
18. P(2 yellows in a row then orange) $\frac{7}{272}$
19. P(blue, then yellow, then red) $\frac{7}{408}$

A die is rolled and a spinner like the one at the right is spun. Find each probability.

20. P(3 and D) $\frac{1}{30}$
21. P(an odd number and a vowel) $\frac{1}{5}$
22. P(a prime number and A) $\frac{1}{10}$

Pick a State

Reactions

[redacted] It was right on mine. Brilliant

Like · Reply · March 20 at 8:03am

[redacted] Wow, u got my color right crazy

Like · Reply · March 19 at 6:46pm

[redacted] This is amazing! Any man that can guess what's going on in a woman's head really is magic! 😳😁 Am I right Rick Lax

Like · Reply · 👍 30 · March 20 at 3:33pm · Edited

TURN & TALK:
Explain how it works!

Alabama	Hawaii	Massachusetts	New Mexico	South Dakota
Alaska	Idaho	Michigan	New York	Tennessee
Arizona	Illinois	Minnesota	North Carolina	Texas
Arkansas	Indiana	Mississippi	North Dakota	Utah
California	Iowa	Missouri	Ohio	Vermont
Colorado	Kansas	Montana	Oklahoma	Virginia
Connecticut	Kentucky	Nebraska	Oregon	Washington
Delaware	Louisiana	Nevada	Pennsylvania	West Virginia
Florida	Maine	New Hampshire	Rhode Island	Wisconsin
Georgia	Maryland	New Jersey	South Carolina	Wyoming

Alabama	Hawaii	Massachusetts	New Mexico	South Dakota
Alaska	Idaho	Michigan	New York	Tennessee
Arizona	Illinois	Minnesota	North Carolina	Texas
Arkansas	Indiana	Mississippi	North Dakota	Utah
California	Iowa	Missouri	Ohio	Vermont
Colorado	Kansas	Montana	Oklahoma	Virginia
Connecticut	Kentucky	Nebraska	Oregon	Washington
Delaware	Louisiana	Nevada	Pennsylvania	West Virginia
Florida	Maine	New Hampshire	Rhode Island	Wisconsin
Georgia	Maryland	New Jersey	South Carolina	Wyoming

What was Rick's probability of choosing the right color?

47/50 or 94%

Discuss State Letters: Did we engage in any SMPs?

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Let 'Em Roll

adapted from
Carlton & Mortlock (2005)

- Game Description: The contestant rolls five dice a maximum of three times in an attempt to win a new car. The five dice are exactly the same: Each die has a car pictured on three sides and dollar amounts ($500, $1000, and $1500) on the other three sides. A contestant who rolls cars on all five dice wins the car. If a rolled die does not show a car, the contestant can either take the money shown on the die and leave the game or "freeze" the die or dice showing the car and roll the remaining dice, should the contestant have rolls remaining. If the contestant obtains five dice with cars by the end of three rolls, she or he wins the car; if not, she or he wins the total dollar amount shown on the dice on the last turn.
  Carlton & Mortlock (2005)

Video: Let ‘Em Roll

- Find the theoretical probability by considering what the probability of a single outcome resulting in 4 cars (C$CCC) and then determine how many different outcomes there are.

$$P(C\$CCC) = 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 = .03125$$

$$P(\$CCCC) = 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 = .03125$$
$$P(CC\$CC) = 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 = .03125$$
$$P(CCC\$C) = 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 = .03125$$
$$P(CCCC\$) = 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 = .03125$$

$$P(4\ cars) = 5 \cdot .03125 = .15625\ or\ 15.625\%$$

- Based on your previous finding, you may complete the table representing a probability distribution at the right. Now find the probability that each of the other possible events will occur, and complete the table.

X (Number of Dice with Cars)	P(X) (Probability of X occurring)
0	
1	
2	
3	???
4	.15625 = 15.625%
5	

How Many Ways to get 3?

CAR CAR CAR $ $ CAR $ $ CAR CAR

$$ {}_5C_3 = 10 $$

CAR $ CAR $ CAR $ $ CAR CAR CAR

For 3 dice to result in cars on one roll, there would be

$_5C_3 = 10$ possible outcomes using the dice, each having a probability of $(0.5)^5$.

$10(0.5)^5 = .3125$

X (Number of Dice with Cars)	P(X) (Probability of X occurring)
0	.03125 = 3.125%
1	.15625=15.625%
2	.3125 = 31.25%
3	.3125 = 31.25%
4	.15625=15.625%
5	.03125 = 3.125%

Binomial Probability Distributions

A binomial experiment consists of a series of repeated trials satisfying these conditions:

- There must be a fixed number of trials. (n)
- Each trial must have 2 possible outcomes – one success, one failure.
- The probability of success must be the same for every trial, call in p. The probability of failure is q =1-p.
- The trials are independent.

If X represents the number of successes in n trials, $P(X) = {}_nC_x \cdot p^x \cdot q^{(n-x)}$

A new task

- What is the probability that a Let 'Em Roll player wins by rolling 1 car on the first roll, 3 cars on the second roll, and 1 car on the third roll?

$$
\begin{aligned}
&{}_5C_1(.5)^5 \cdot {}_4C_3(.5)^4 \cdot {}_1C_1(.5)^1 \\
&= 5\,(.5)^5 \cdot 4\,(.5)^4 \cdot 1\,(.5)^1 \\
&\approx .02 = 2\%
\end{aligned}
$$

Winning!

- What is the probability that a player wins the car?

$P(5) = {}_5C_5\,(.5)^5$ $=3.125\%$

$P(4,1) = {}_5C_4\,(.5)^5 \cdot {}_1C_1\,(.5)$ $=7.8125\%$

$P(4,0,1) = {}_5C_4\,(.5)^5 \cdot {}_1C_0\,(.5) \cdot {}_1C_1\,(.5)$ $=3.9062\%$

$P(3,2) = {}_5C_3\,(.5)^5 \cdot {}_2C_2\,(.5)^2$ $=7.8125\%$

$P(3,1,1) = {}_5C_3\,(.5)^5 \cdot {}_2C_1\,(.5)^2 \cdot {}_1C_1\,(.5)$ $=7.8125\%$

$P(3,0,2) = {}_5C_3\,(.5)^5 \cdot {}_2C_0\,(.5)^2 \cdot {}_2C_2\,(.5)^2$ $=1.9531\%$

$P(2,3) = {}_5C_2\,(.5)^5 \cdot {}_3C_3\,(.5)^3$ $=3.9062\%$

$P(2,2,1) = {}_5C_2\,(.5)^5 \cdot {}_3C_2\,(.5)^3 \cdot {}_1C_1\,(.5)$ $=5.8594\%$

$P(2,1,2) = {}_5C_2\,(.5)^5 \cdot {}_3C_1\,(.5)^3 \cdot {}_2C_2\,(.5)^2$ $=2.9297\%$

$P(2,0,3) = {}_5C_2\,(.5)^5 \cdot {}_3C_0\,(.5)^3 \cdot {}_3C_3\,(.5)^3$ $=0.4883\%$

$P(1,4) = {}_5C_1\,(.5)^5 \cdot {}_4C_4\,(.5)^4$ $=0.9766\%$

$P(1,3,1) = {}_5C_1\,(.5)^5 \cdot {}_4C_3\,(.5)^4 \cdot {}_1C_1\,(.5)$ $=1.9531\%$

$P(1,2,2) = {}_5C_1\,(.5)^5 \cdot {}_4C_2\,(.5)^4 \cdot {}_2C_2\,(.5)^2$ $=1.4648\%$

$P(1,1,3) = {}_5C_1\,(.5)^5 \cdot {}_4C_1\,(.5)^4 \cdot {}_3C_3\,(.5)^3$ $=0.4883\%$

$P(1,0,4) = {}_5C_1\,(.5)^5 \cdot {}_4C_0\,(.5)^4 \cdot {}_4C_4\,(.5)^4$ $=0.061\%$

$P(0,5) = {}_5C_0\,(.5)^5 \cdot {}_5C_5\,(.5)^5$ $=0.098\%$

$P(0,4,1) = {}_5C_0\,(.5)^5 \cdot {}_5C_4\,(.5)^5 \cdot {}_1C_1\,(.5)$ $=0.2441\%$

$P(0,3,2) = {}_5C_0\,(.5)^5 \cdot {}_5C_3\,(.5)^5 \cdot {}_2C_2\,(.5)^2$ $=0.2441\%$

$P(0,2,3) = {}_5C_0\,(.5)^5 \cdot {}_5C_2\,(.5)^5 \cdot {}_3C_3\,(.5)^3$ $=0.1221\%$

$P(0,1,4) = {}_5C_0\,(.5)^5 \cdot {}_5C_1\,(.5)^5 \cdot {}_4C_4\,(.5)^4$ $=0.0305\%$

$P(0,0,5) = {}_5C_0\,(.5)^5 \cdot {}_5C_0\,(.5)^5 \cdot {}_5C_5\,(.5)^5$ $\underline{=0.00305\%}$

$\approx 51.29\%$

What assumptions have we made about this problem?

Variations

The staff at *The Price Is Right* thinks too many contestants are winning a car in this game, so they are considering changing the games in different ways. There are three proposed changes.

- Proposed Change #1: Using the same dice (3 $, 3 Cars), but increasing the number of dice in the game to 6.
- Proposed Change #2: Continuing to use 5 dice, but changing the dice so they only have 2 sides that are labeled "Car" and 4 sides that are labeled with cash.
- Proposed Change #3: Use 6 dice, where each die has 8 sides total with 3 sides labeled "Car."

For each of the proposed changes, determine (theoretically) the probability that a contestant will roll 4 Cars on the first roll.

Discuss Let 'Em Roll: Did we engage in any SMPs?

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Deal or No Deal Pt.1

How much money should Anteia expect? Should she take the deal?

$400	¼	Offer?
$10,000	¼	$264,000
$400,000	¼	Expected Value?
$750,000	¼	

Expected Value Procedure

$$E(X) = \sum x_i p_i$$

where x_i represents the value of the i^{th} outcome

and p_i represents the probability of the i^{th} outcome.

$$E(X) = \$400 \cdot \frac{1}{4} + \$10000 \cdot \frac{1}{4} + \$400{,}000 \cdot \frac{1}{4} + \$750{,}000 \cdot \frac{1}{4}$$

$$= \$290{,}100$$

Expected Value Concept

If all outcomes are equally likely to occur, the expected value is the same as the mean.

How much money should Anteia expect? Should she take the deal?

$400	¼	Offer?
$10,000	¼	$264,000
$400,000	¼	Expected Value?
$750,000	¼	$290,100

Deal or No Deal Pt. 2

How much money should Anteia expect?
Should she take the deal?

$400	1/3
$400,000	1/3
$750,000	1/3

Offer?
$402,000

Expected Value?
$383,466.67

Deal or No Deal Pt. 3

Variations

$400	50%	Expected Value?
$400,000	40%	$
$750,000	10%	

Discuss Deal or No Deal: Did we engage in any SMPs?

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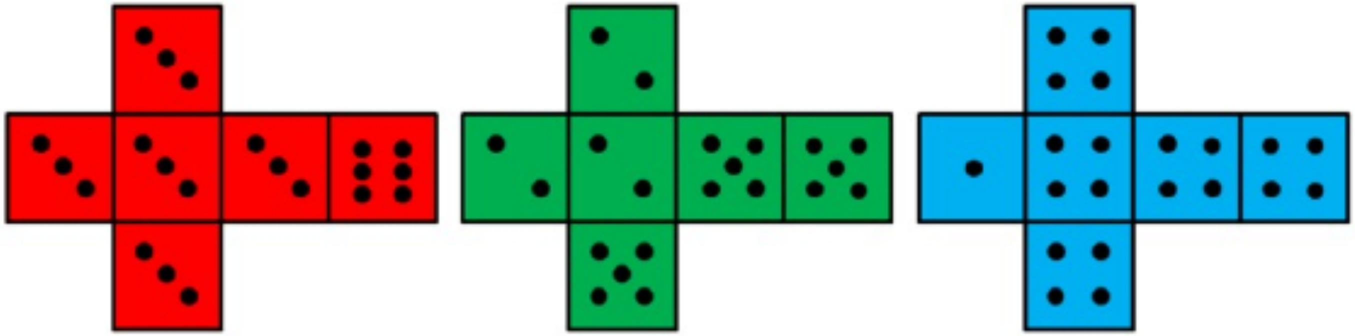
This is a game for two people.

You have three dice; one is red, one is green, and one is blue. These dice are different than regular six-sided dice, which show each of the numbers 1 to 6 exactly once. The red die, for example, has 3 dots on each of five sides, and 6 dots on the other. The number of dots on each side are shown in the table and picture below.

Red	3	3	3	3	3	6
Green	2	2	2	5	5	5
Blue	1	4	4	4	4	4



To play the game, each person picks one of the three dice. However, they have to pick different colors.

- The two players both roll their dice. The highest number wins the round.
- The players roll their dice 30 times, keeping track of who wins each round.
- Whoever has won the greatest number of rounds after 30 rolls wins the game.

Source: Illustrative Mathematics (2016)

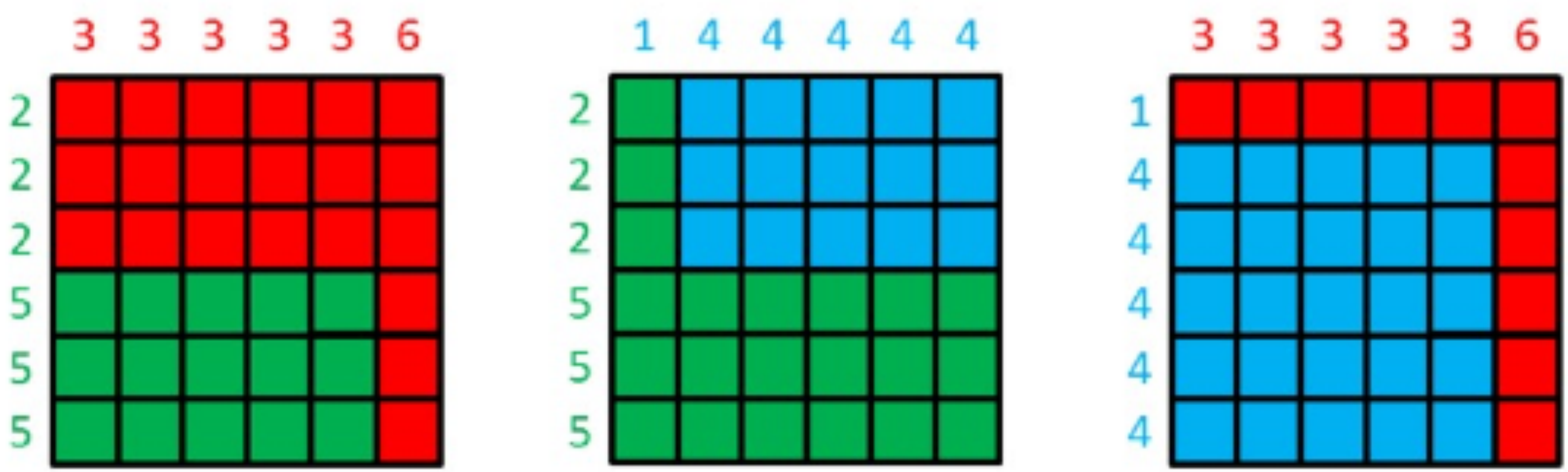
Results

Red vs. Green : RED WINS	58%	42%
Red vs. Blue: BLUE WINS	69%	31%
Green vs. Blue: GREEN WINS	58%	42%

What should be your strategy when picking die?

What if you have to pick first?

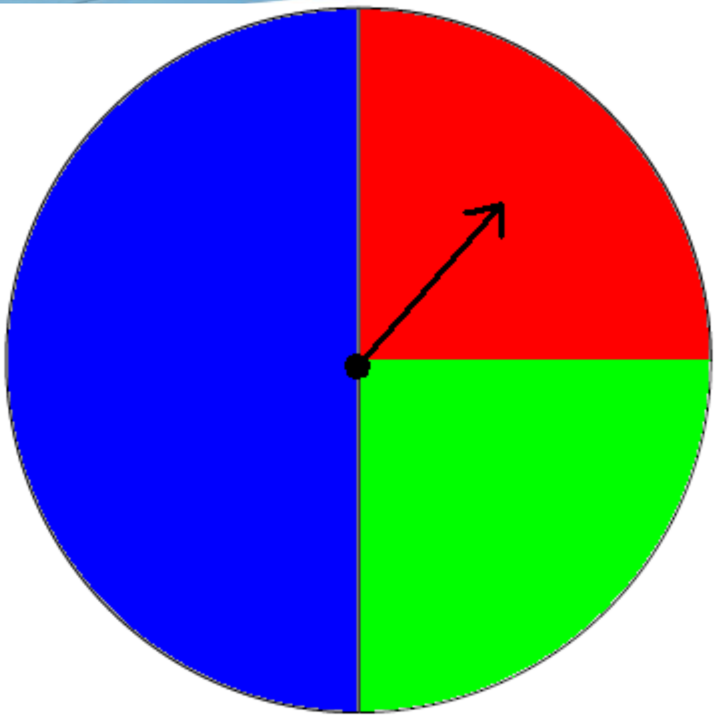
Here are three tables that are color-coded to see who will win which rolls in each of the three possible pairings:



Using Representations to Model

- If I spin the spinner at the right twice, the probability of spinning two different colors is 62.5%.

- Your task: Show how we could "prove" this to a student.



Area Representations

First Spin

Second Spin

Blue Blue 25%	Red Blue 12.5%	Green Blue 12.5%
Blue 50% Blue Red 12.5%	Red 25% Red Red 6.25%	Green 25% Green Red 6.25%
Blue Green 12.5%	Red Green 6.25%	Green Green 6.25%

62.5%

Discuss Red, Green & Blue: Did we engage in any SMPs?

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Double Decks

- You have two decks of cards, each of which is a normal deck of 52 cards. You keep the decks in separate piles, and at the same time you turn over the top card of each deck. Your friend offers to make you a bet. He says that if you continue to flip over the top cards of both decks simultaneously, he will pay you $50.00 if you are able to turn over the same card in each deck before you get to the bottom of the pile. If you are unable to do so, then you will have to pay him $50.00. If you take his bet, who will be more likely to win?

Double Decks

Make a conjecture!

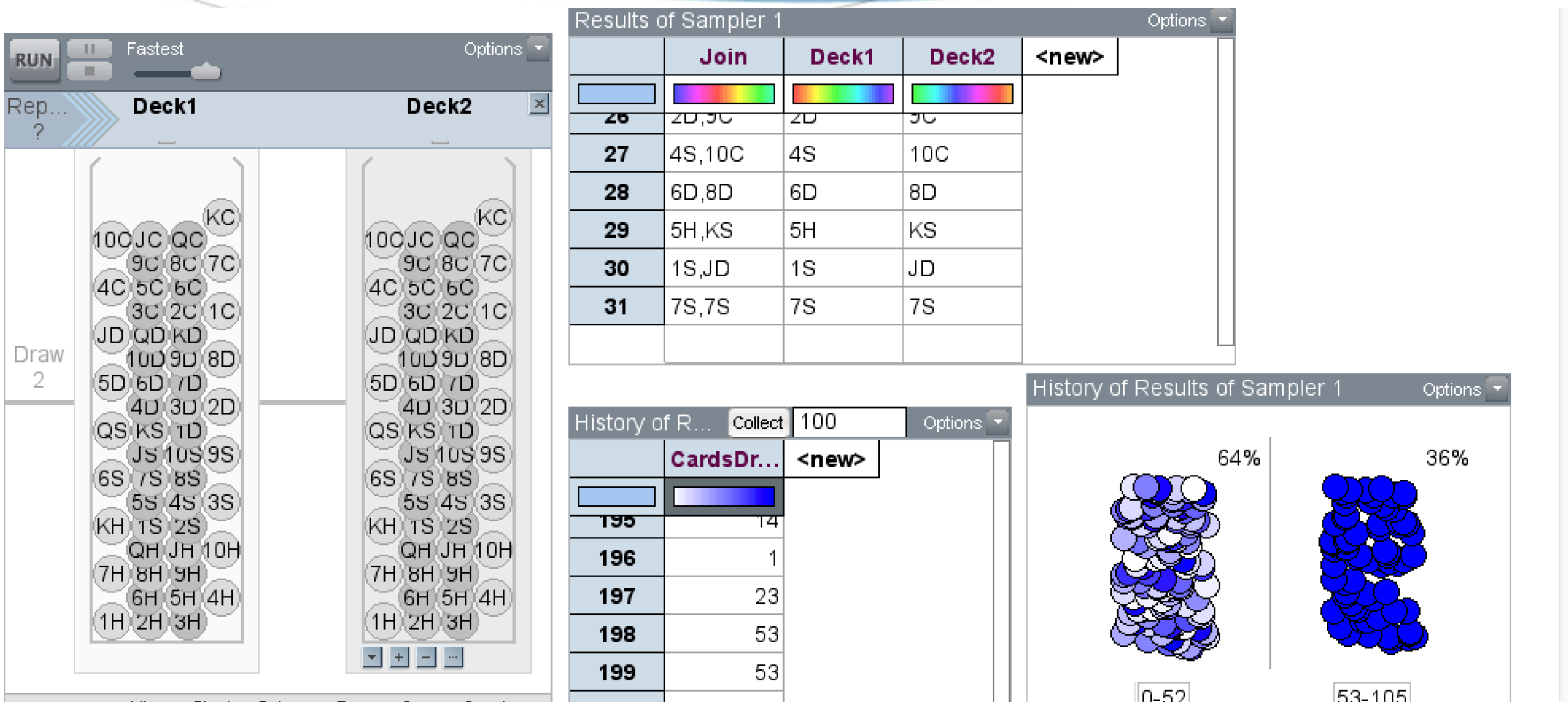
- What percent of the time do you expect to find a match?
- Let's try it with the cards! (Be sure to shuffle!)

Double Deck Results

- Found a Pair:5
- Did Not Find a Pair:5

TinkerPlots

http://www.tinkerplots.com/



Discuss Double Decks:
Did we engage in any SMPs?

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
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Cereal Toys

- Oatie Os Cereal Company decided to promote its cereal by including one of six animal toys in each of its cereal boxes. Assuming that the company will issue equal numbers of toys for each of the six animals and that, when you buy a box of cereal, your chances of getting any of the six toys are the same, about how many boxes of cereal would you expect to need to buy in order to get all six toys? How would your answer change if there were eight toys instead of six?

Discuss Simulation

- How would you develop a simulation for the Cereal Toys Situation?
  - Physically?
  - Using Technology?

Discuss: Take Back

- What have you learned from today's session?
- What can you take back to your classroom?
- What are other ways you can embed the SMPs in your practice of teaching probability?

THANK YOU!

Rick Hudson
rhudson@usi.edu