Math and Music:

An Interdisciplinary Approach to Transformations of Functions

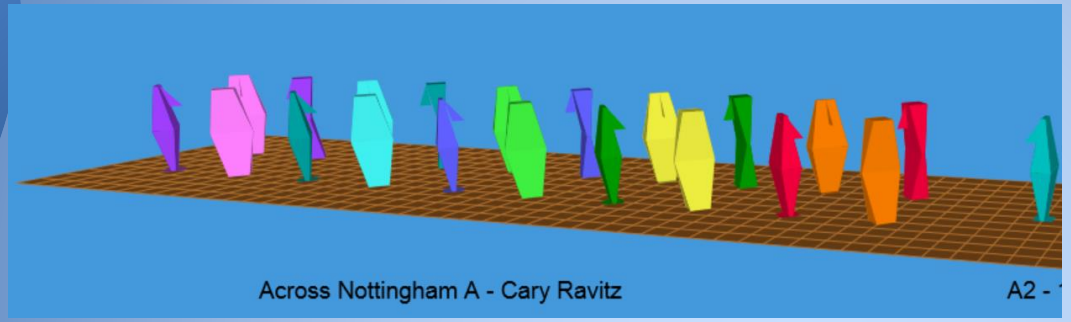
NCTM Annual Conference, San Francisco, CA April 2016

Maria Hernandez, NCSSM Mathematics Instructor
Phillip Riggs, NCSSM Music Instructor

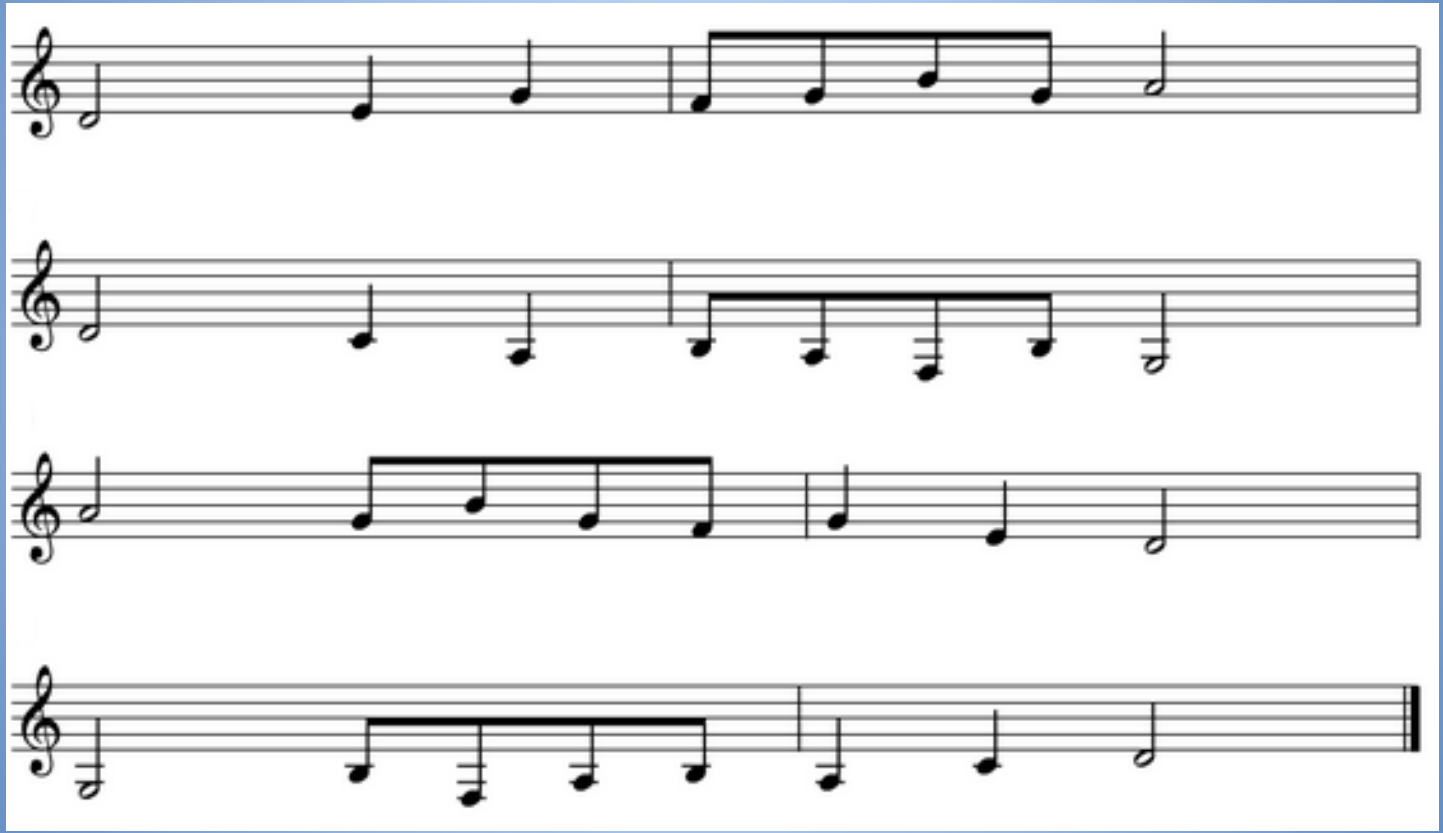
Math and Music in the Summer

- Study western and eastern scales
- Create musical instruments - Boomwhackers!
- Perform with created instruments
- Create melodies using Note Flight
- Create mathematical models using GeoGebra
- Explore connections between contradance and Group Theory
- Learn contradance steps and leading a community dance
- Create their own contra dance.

The Group Theory and Contradancing



What do you notice about Alyssa’s melody?



Musical Transformations Compositional Techniques

Who has musical background?

Noteflight Handout

Composition Tools

Involve your music teachers

Link to Mathematics

Create a Data Set Representation of Notes in the Melody:

- Mapping the rhythm to the x-axis
- Mapping the pitch (frequency) of the note to the y-axis

Map Rhythm to *x*-coordinates of data points

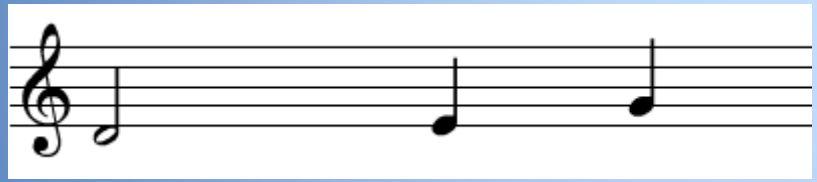
1. Will we create all of our melodies using 4/4 time. Each quarter beat is associated with an integer on the x-axis. So if you have 4 quarter notes in the first measure of your melody, the corresponding x-coordinates of the points will be 0, 1, 2, 3.

We will use rational number to represent notes that are shorter than quarter notes. So if your first measure has 4 eighth notes and 2 quarter notes, then the x-coordinates of the points for the first measure will be 0, 0.5, 1, 1.5, 2, 3.

Map Pitches to *y*-coordinates of data points

The pitch of the first note in your first measure will correspond to a data point having a y-coordinate of 0. The y-coordinates of all other points will be relative to that note.

For example, if we start on a D, then the y-coordinate of the E note, will be 1, and the y-coord of F is 2, etc...

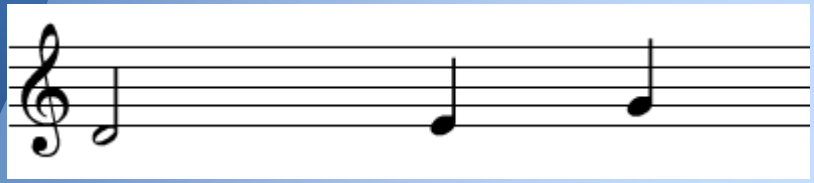


= (0,0), (1,0), (2,1), (3,3)

Note: The half note is represented using two points with a repeated y-value. This scheme doesn't distinguish between a half note and 2 repeated quarter notes.

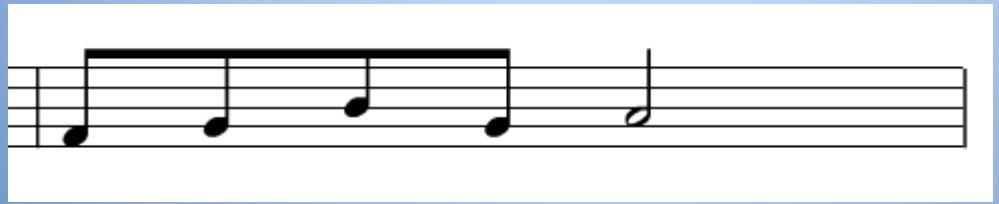
Example continued:

(0,0), (1,0), (2,1), (3,3)

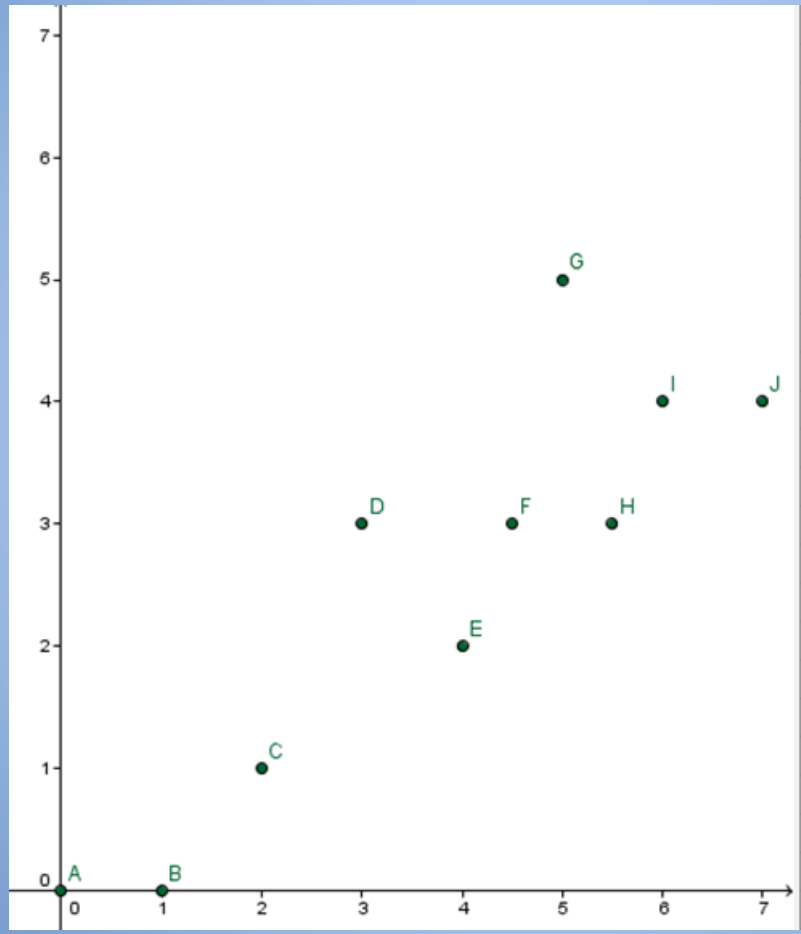


The measure below is associated with the data points

(4,2), (4.5,3), (5,5), (5.5,3), (6,4), (7,4)



Plotting the Data in GeoGebra



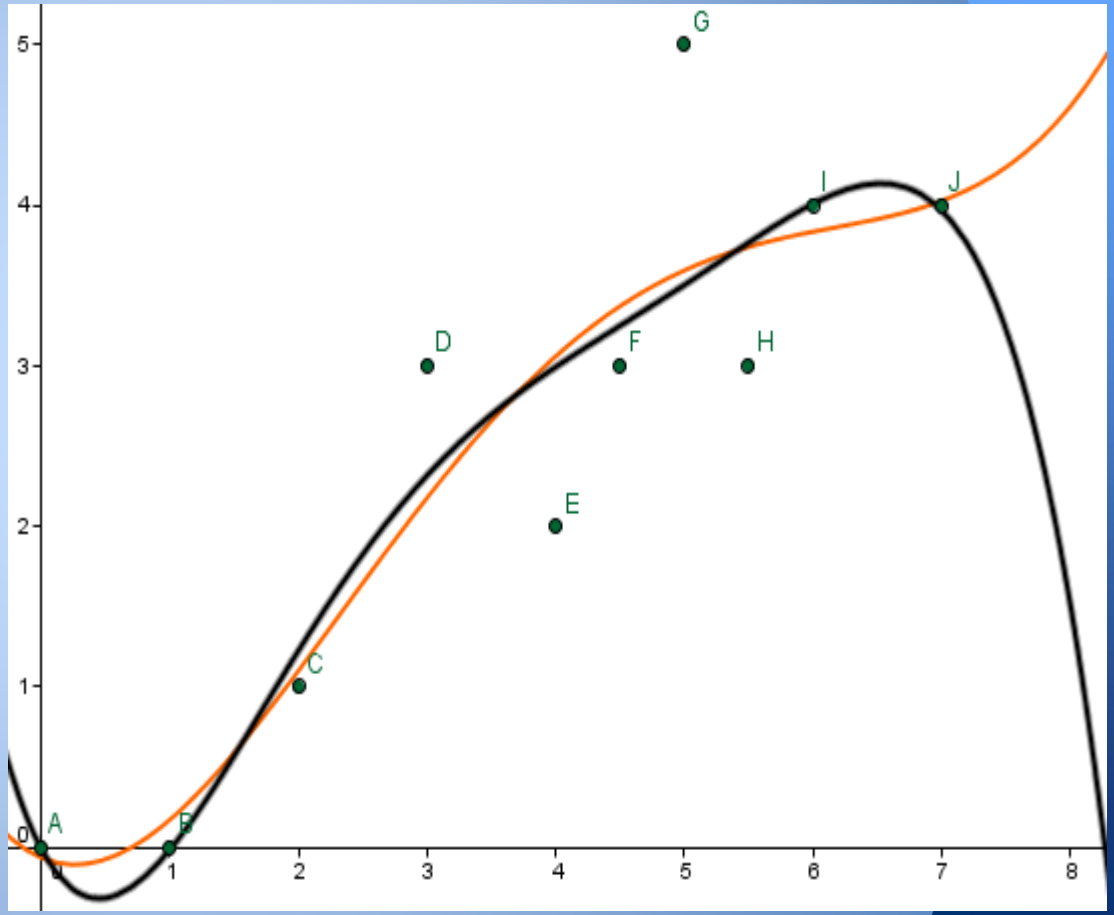
	A	B
1	0	0
2	1	0
3	2	1
4	3	3
5	4	2
6	4.5	3
7	5	5
8	5.5	3
9	6	4
10	7	4
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
21		
22		

Creating a Function to “Fit” the Data

Using FitPoly in GeoGebra

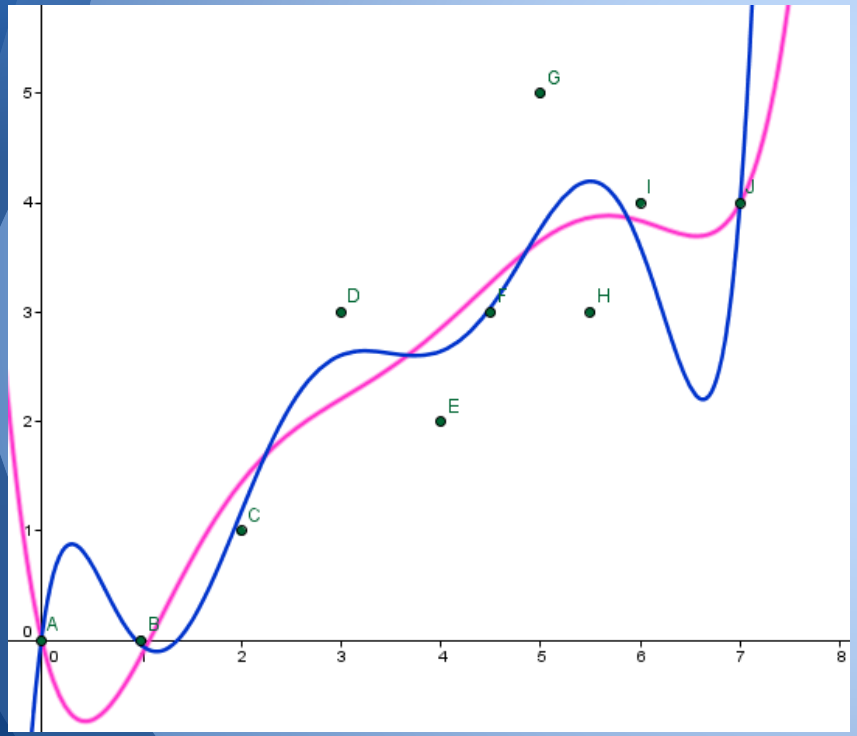
Orange = 4th degree

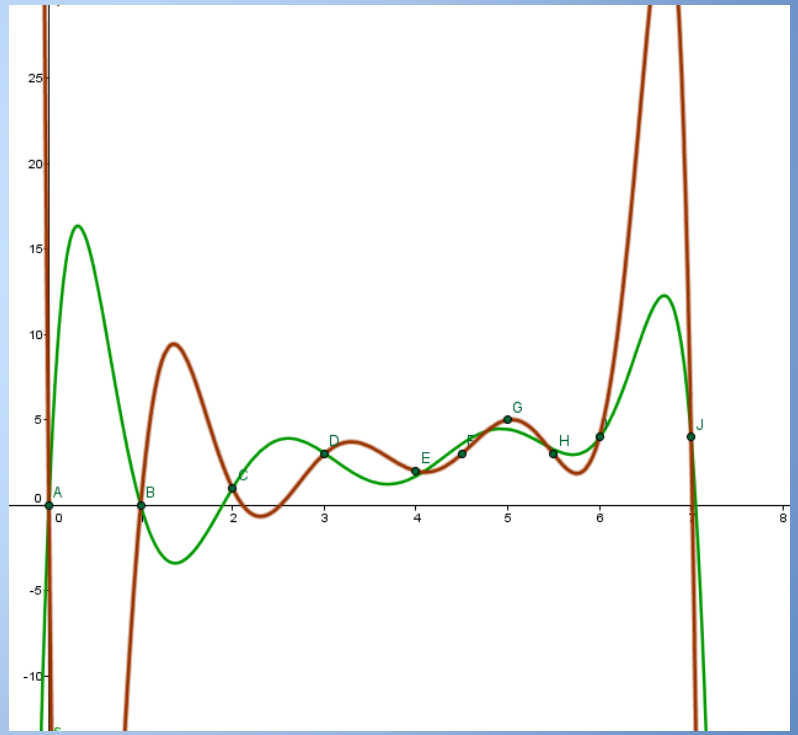
Black = 5th degree



degree

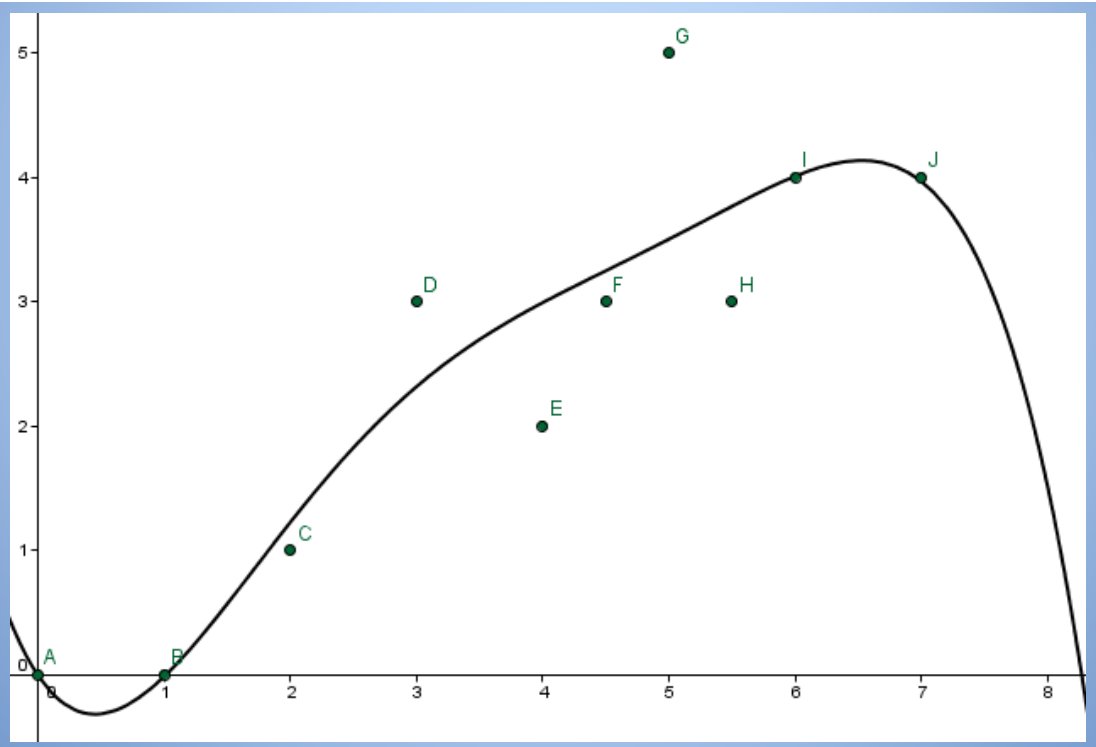
Blue = 7th degree **Brown** = 9th degree



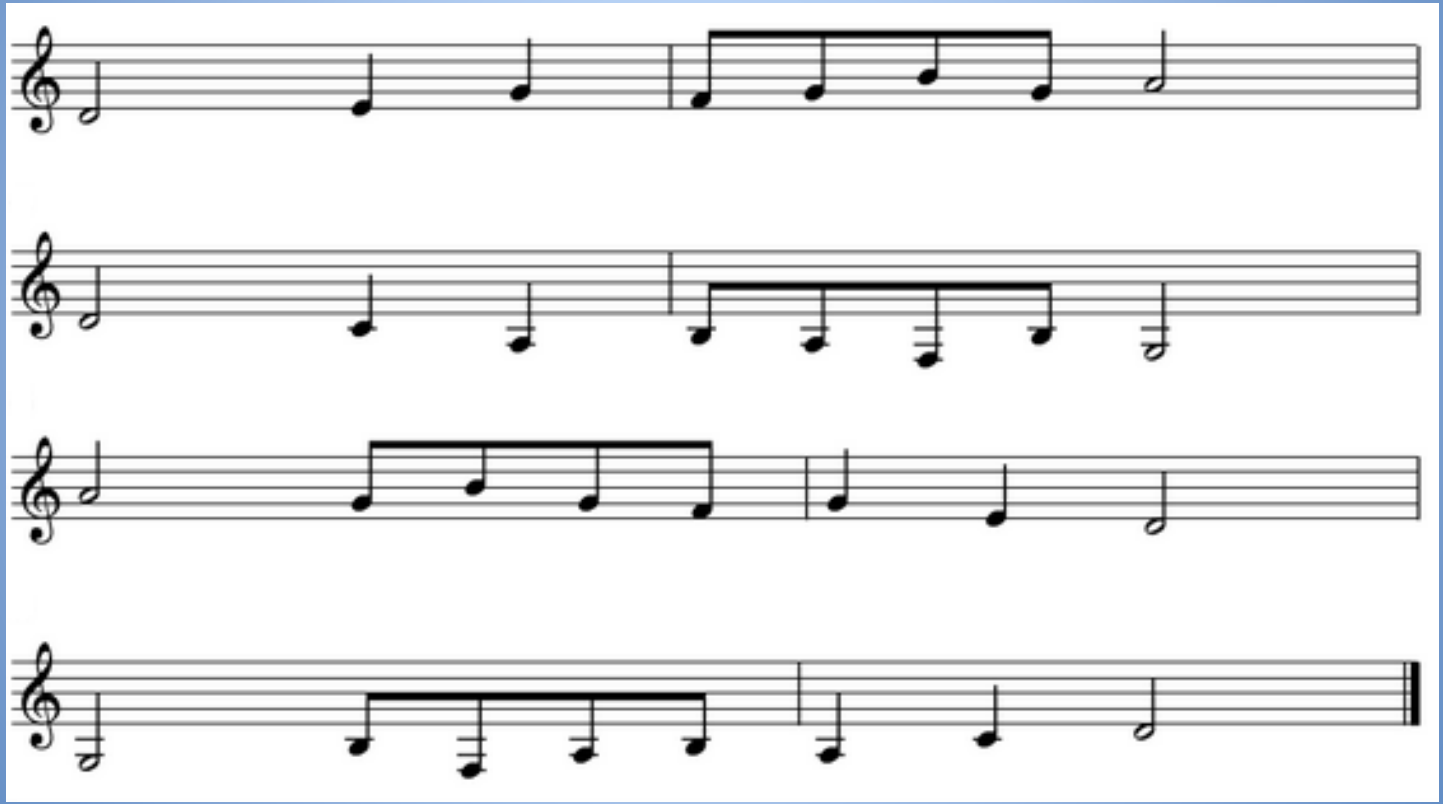


We will use the 5th degree polynomial.

$$p(x) = -0.01\,x^5 + 0.1\,x^4 - 0.69\,x^3 + 2.1\,x^2 - 1.51\,x - 0.01$$



Use transformations of functions to create a math representation of the entire melody…*Play the Melody*



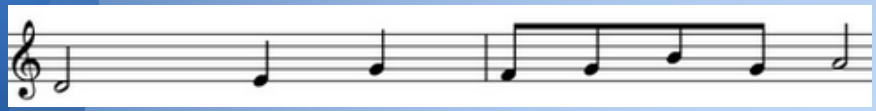
Back to Data Set and Function

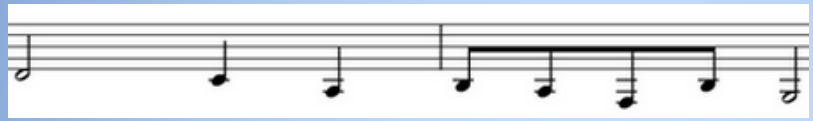
1. Limit the domain of the original function [0,7]
2. Reflect original about the *x*-axis on [8,15]
3. Reflect original about the *y*-axis on [16,23]
4. Reflect the original about the *x*- and *y*-axes on [24, 31]

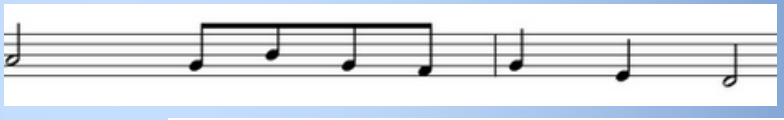
f(x) = fitpoly[list1, 5]

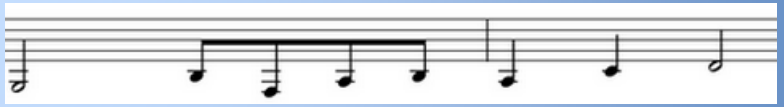
1. g(x) = if [0 <= x <= 7,f(x)]
2. h(x) = if [8 <= x <= 15, -g(x-8)]
3. k(x)= if [16 <= x <= 23, g(-(x-23))]
4. m(x)= if [24 <= x <= 31, -g(-(x-31))]

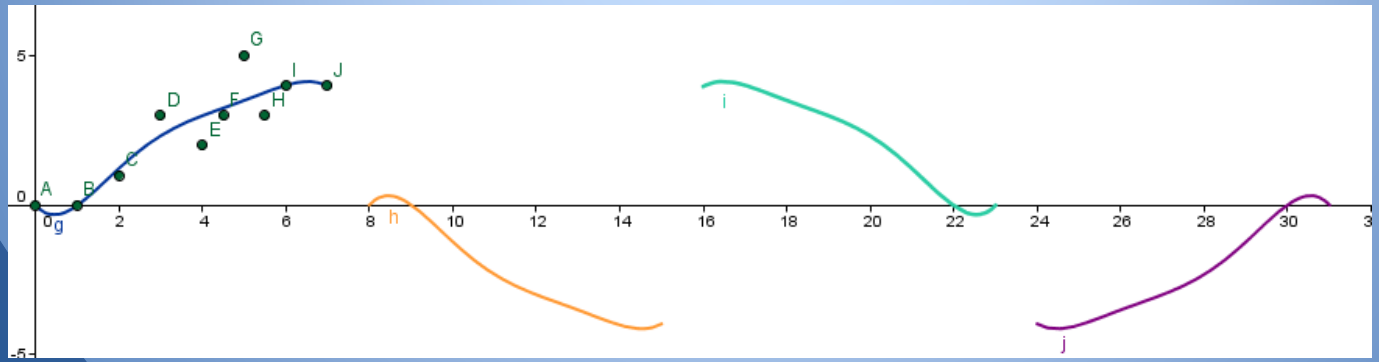
Putting the Music and the Math Together











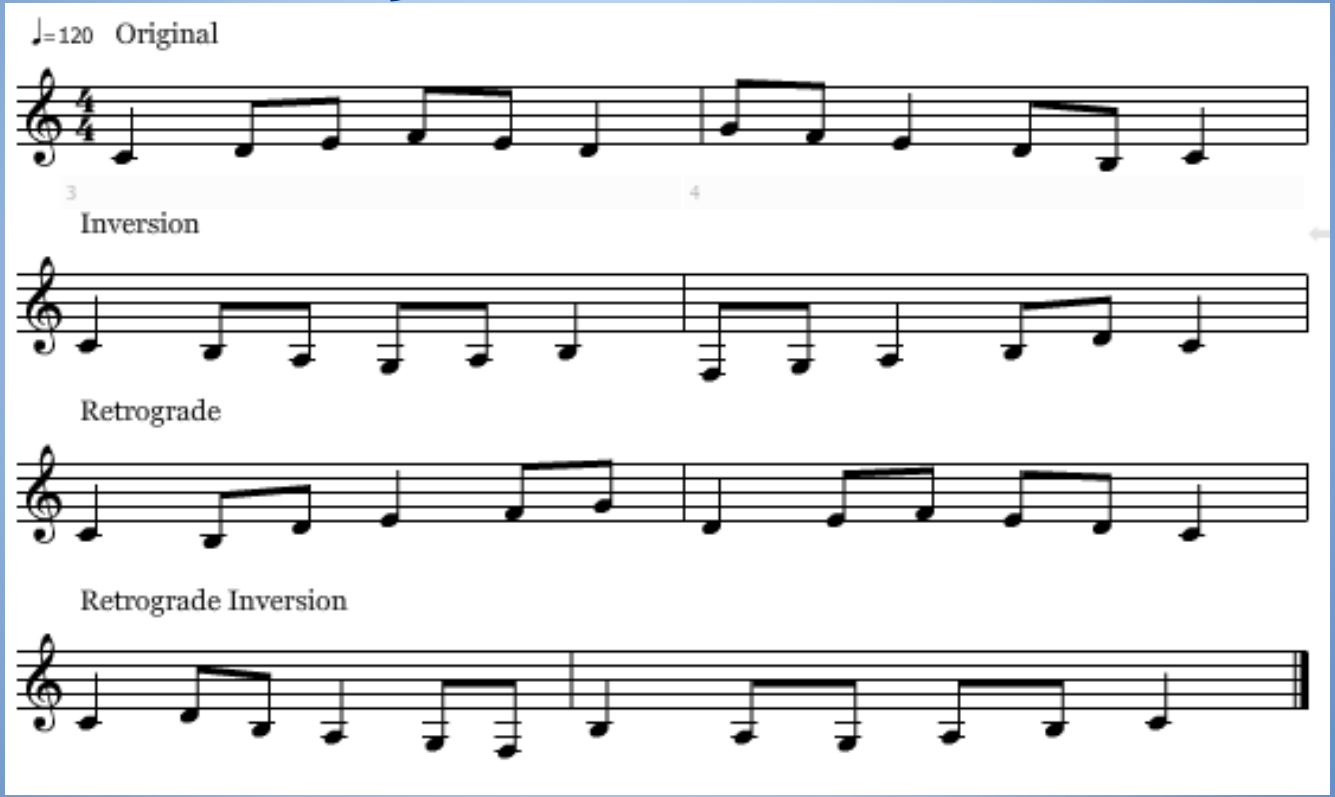
Mathematical / Musical Equivalents

Musical	Mathematical
Inversion	Reflect across the *x*-axis
Retrograde	Reflect across the *y*-axis
Retrograde Inversion	Reflect across both axes
Diminution	Compress horizontally
Augmentation	Stretch horizontally
Change the key	Vertical shift
Offset the timing (Round)	Horizontal shift

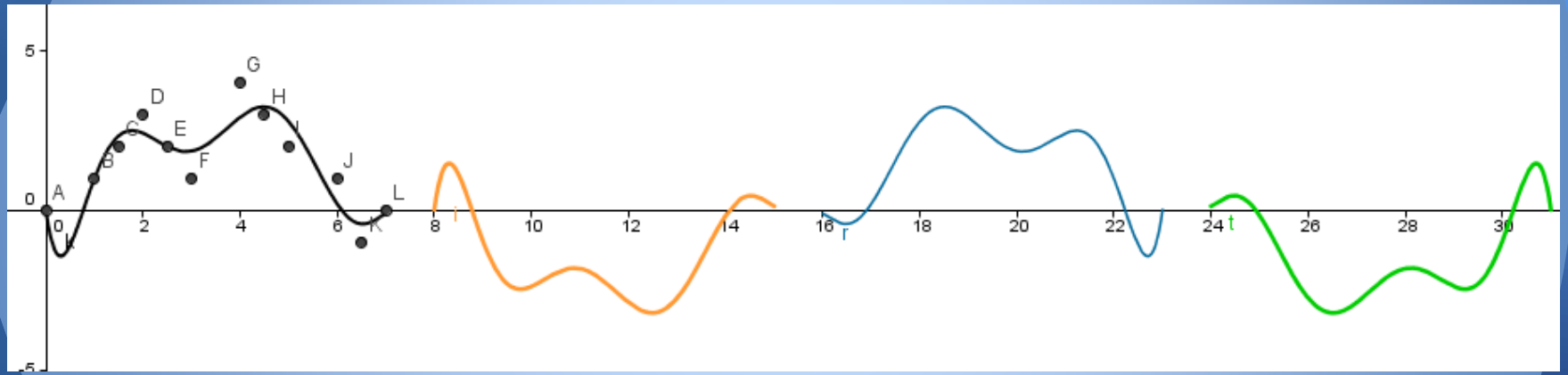
Student Examples

Noteflight	GeoGebra
Non-Music Student	Non-Music Student
Music Student	Music Student

Jules' Melody



Jules' Mathematical Representation of His Melody



Student Reflections

"In this lab, we created music and then applied it to math. I used to take music in middle school, but that was a loooong time ago. I also learned that the measures of a music sheet with notes are like a graph with points. ***Apparently the horizontal space/line in which the first note is on is the equivalent to a math x-axis.*** *I thought it was pretty cool to see how taking the inverse of notes was the same for points. My favorite part of this lab was definitely playing the 4 different music lines and comparing them. It looked like I had done something really complicated when I actually just represented my music in different transformation on a graph."*

Student Reflections

"It was interesting to see how music and math could relate to each other. I would not consider myself a musically talented person at all, ***but seeing how math could relate to music made it easier for me to understand and allowed me to see how math can relate to things that don't seem remotely close to math itself.*** *It was nice that everybody got to make his or her own music, because you could make it easier or harder based on your knowledge of music."*

Student Reflections

Jules Video

Mason's Video

Student Reflections

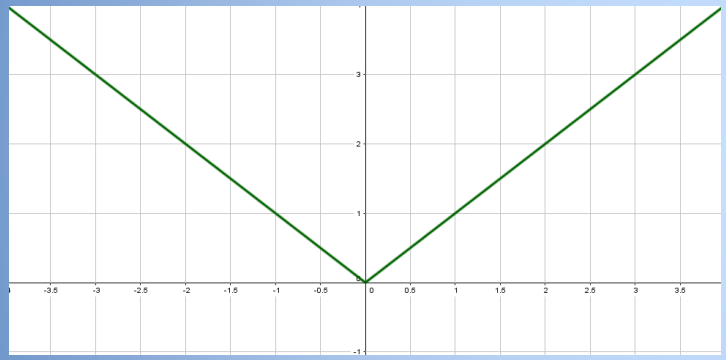
"I learned you can transform the measures of music just like you can transform graphs and that composers actually use that technique to compose music. I liked that we could use note flight and geogebra together to get a better understanding and more practice with transforming graphs. ***I also liked that we applied math to a real life application."***

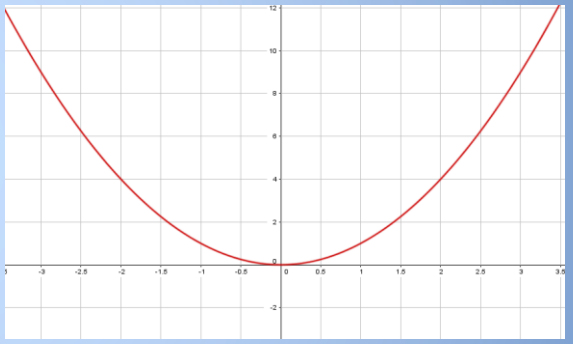
This interdisciplinary activity was a great reminder on how math is applied to our everyday lives. I like how we were able to extrapolate an 8 measure piece of music from two measures. ***I think that it would be nice to go even deeper and make longer pieces or even "reverse engineer" it to find out what a certain graph might sound like if it were music.***

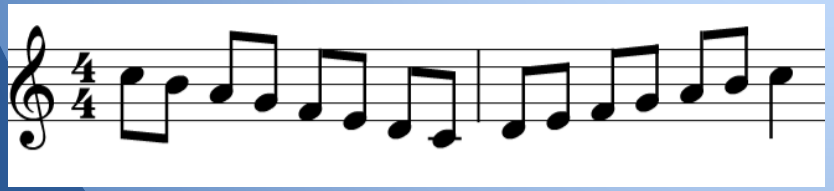
Bach Crab Cannon

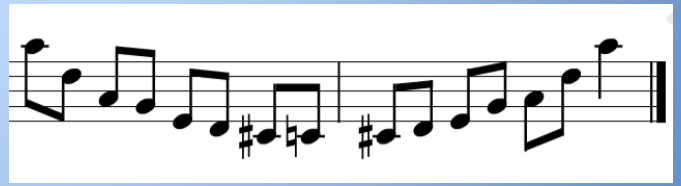
What if we go from Math to Music?

Example: Absolute Value vs Quadratic









Play music

References and Resources

1. The Great Courses: “How Music and Mathematics Relate”, David Kung

2. Mathematics Teacher Article: “Listening to Geometry”, September, 2009

3. Micro Robot Dance Java Applet - Contradance site http://ravitz.us/dance/mrd.php

4. “The Math Behind the Music”, Leon Harkleroad

5. Vi Hart “Doodle Music” & “What’s Up with Noises”

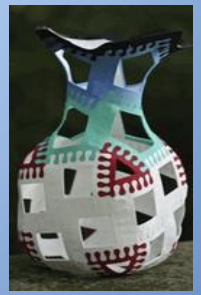
6. J.S. Bach’s Crab Canon https://www.youtube.com/watch?v=xUHQ2ybTejU

Conferences

1. NCSSM Teaching Contemporary Mathematics Conference,
   January 27-28, 2017
   Durham, North Carolina
   http://www.ncssm.edu/tcmconference

2. Anja Greer Math, Science and Technology Conference
   Phillips Exeter Academy,
   June 26 – July 1, 2016
   Exeter, New Hampshire
   https://www.exeter.edu/summer_programs/7325.aspx

3. Bridges Conference – Bridges art, music, math, architecture and culture.
   August 2016, Finland http://bridgesmathart.org/



Knowles Teaching Fellows Program (KSTF)

http://kstf.org/

Duke Masters of Arts in Teaching

https://educationprogram.duke.edu/graduate

Thank you!

Maria Hernandez hernandez@ncssm.edu
Phillip Riggs riggs@ncssm.edu

NCSSM Math and Music Summer Accelerator Students