Create Debate

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Goals of Today's Session:

- Share strategies that create debate in the mathematics classroom
- Understand why debate is so critical to learning mathematics

Key Ideas of Why Debates are Important

- Debates increase engagement and investment in the topic being studied
- Debates prime students to learn new methods
- Debates develop student reasoning and critical thinking

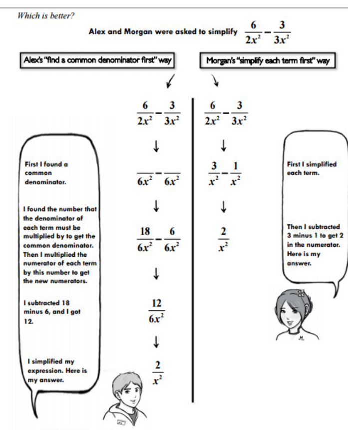
Research has shown that teaching becomes more effective when common mistakes and misconceptions are systematically exposed, challenged and discussed….Cognitive conflicts occur when the learner recognizes inconsistencies between existing beliefs and observed events. This happens, for example, when a learner completes a task using more than one method and arrives at conflicting answers. ….Research has shown that such conflicts, when resolved through reflective discussion, lead to more permanent learning than conventional, incremental teaching methods, which seek to avoid learners making 'mistakes'.

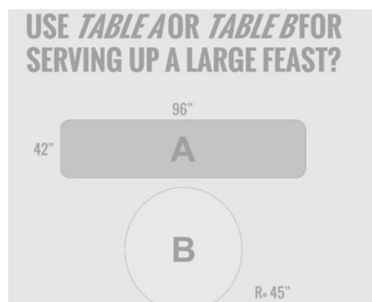
-Malcom Swann, University of Nottingham

"The researchers found that when students were given problems to solve, and they did not know methods to solve them, but they were given opportunity to explore the problems, they became curious, and their brains were primed to learn new methods, so that when teachers taught the methods, students paid greater attention to them and were more motivated to learn them. The researchers published their results with the title "A Time for Telling," and they argued that the question is not "Should we tell or explain methods?" but "When is the best time do this?"

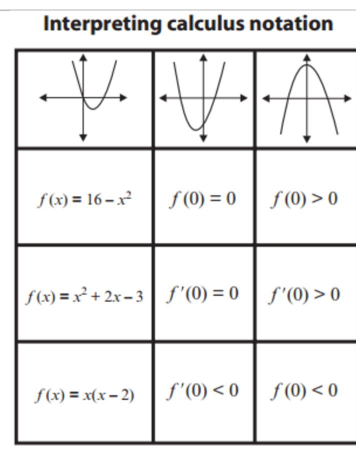
-Jo Boaler, Mathematical Mindsets

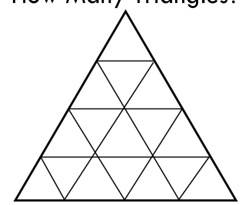
Structures for Creating Debate

Structure	Description	Examples	Resources
1. Challenges -Testing Theorems	Create a challenge for students to overcome – often framed (without explicitly so to students) as the search for a counterexample to a theorem. The search for a counterexample helps illuminate the prerequisites, or requirements, for a theorem to be put in place.	Can you make a triangle from these side lengths – (4 – 4 – 10)? *Triangle Inequality Theorem* Can you draw a line going through points a and b such that no tangent is parallel to the secant line connecting a and b? *Mean Value Theorem*	
2. Analyzing Student Work	Students work a problem to get a sense of what's being asked and what might be a strategy to solve the question. Students are then introduced to 4 to 9 examples of student work to discuss and analyze. They are asked to debate and discuss which methods are correct, which misconceptions some student work has, and which methods are the best/most elegant. Also provides an opportunity to highlight excellent student mistakes and debate which mistakes are the most common, most helpful, or most interesting.	Draw a distance/time graph in the interval [0,6] such that the net distance traveled is between 4 and 8 and the net displacement is 1. *4 different examples of student work are shown to analyze.*	
3. Contrasting Cases	These scripted discussions between two peers (often like a comic book discussion) show the students engaging in a back and forth as they discuss different answers they got for a question or two different strategies they used to solve the question. Students discuss which method or solution is better, and dig into the thinking of each student.		Harvard GSE Contrasting Cases Curriculum http://tinyurl.com/ContrastingCases Scripted Debates

4. Estimation w/Reasoning	These debates get at students' number sense and reasoning. Students are asked to estimate a quantity, or evaluate whether a quantity is an over- or under- estimate. Their reasoning is supported by both mathematical justifications as well as personal experience/references that inform their number sense. Using the Over/Under game is another way to encourage debate. Over/Under involves a number being set as a borderline prediction, and then the person has to decide if the actual number is over or under the set number.	Estimate the number of 5 dollar bills in the school right now. Over/Under on the number of snow days for the coming year: 5	http://www.estimation180.com/
5. Odd One Out	In these situations, students are asked to analyze a series of pictures, graphs, or numerical expressions and give a justification as to why each of the four does not belong with the other three. A similar title for these types of problems is "Which one doesn't belong"	Which one is the odd one out? (a) 20,14,8,2… (b) 3,7,11,15,… (c) 4,8,16,32,… 36 | 99 9 | 123	Odd One Out (And Other Strategies from main author of MARS Tasks) http://bit.ly/1qPowWI Which One Doesn't Belong http://wodb.ca/
6. Would you rather?	A question format that asks students to choose between two different choices and justify their answers with mathematical reasoning. A take-off of the popular game, these situations are all geared towards questions that lend themselves to mathematical justifications, but don't necessarily have a correct answer.	Would you rather have two slices of a medium pizza or one slice of a large pizza? 	Would you rather math http://www.wouldyourathermath.com/

7. What if…?	Students are asked how changing one parameter or aspect of a question would change the solution. Often, variables are used to allow students to explore the extremes of a situation and play with the different values that can be substituted. These often lend themselves to explorations on Geogebra or Desmos.	Take the formula for gravitational pull – what if the G changed, what would happen? What if the m changed, what would happen?	PBS Learning Series Video – Chris Luzniak http://bit.ly/1S8Yh6b
8. Opinionated	Students are asked to have an opinion that is not traditionally math related, but the justification of their response is grounded in math. Question stems are "Which solution is the most elegant?" "Which solution is the best" "Which strategy is the most efficient?" Since there is no "right" answer, debates rely upon students combining their mathematical thinking and that of the solutions presented to justify their answers.	Consider the two solutions shown on the board – which method is the most elegant? Look at the three sets of mistakes made by these three students. Which mistake is the best? Why?	Ignite Talk – Chris Luzniak "Debate That" http://bit.ly/1Xunmdm
9. Always/ Sometimes/ Never	The most common type of debatable problems on tests and in textbooks. Given a True or False question, students have to decide if the statement is Always True, Sometimes True, or Never true. In justifying their response, students volunteer justifications, counterexamples, or full on proofs to establish their claim.	Two squares are similar – always, sometimes, or never. Every function has a y intercept. $(x+4)^2 = x^2 + 4^2$	

10. Why do we do that?	Building a deeper understanding of fundamental concepts can come from stepping back and putting students in the position of the creators of mathematical devices and norms and ask, from a historical perspective, why certain decisions were made. In debating this reasoning, students consider and challenge assumptions they use and are primed to make sense of the interesting history of the mathematical tools we use. This also proves to be an elegant entryway into exposing students to the history of mathematics through the lens of other cultures and a way to explore methods developed and more commonly used in other countries.	Why do we rationalize the denominator? Why are there 360 degrees in a circle, and not 370? Or 400? When did humans start using negative numbers? Why do we call it a logarithm?	James Tanton Website, Curriculum, and Videos http://www.jamestanton.com/
11. Extreme Cases	There are many challenging extreme cases both to understand as a student, but also to explain to a student. Further, students cannot appreciate the challenges of these extremes until they are put in a position to try to explain them themselves. Asking these questions and having the class debate several plausible solutions puts students in a situation to make sense of the answer once explained.	Why is 2^0 1? What is 0 divided by 0? Is .999=1?	
12. Card Sort	Students are given a set of cards that have either expressions, images, graphs, tables, or some combination of these, and they are asked to sort these cards into pairs, groups, or a two way table, all depending on the prompt. Debate and discussion emerge as students wrestle over how to categorize the cards.		Odd One Out (And Other Strategies from main author of MARS Tasks) http://bit.ly/1qPowWI MARS Tasks http://map.mathshell.org/index.php

13. Debatable Problems – Introducing New Content	These problems are accessible to all students and create an opportunity for an initial prediction or understanding to be challenged. These problems are low floor/high ceiling and create an investment for students in figuring out an easier, more elegant method. These problems prime students to make sense of new content.	Which one would you rather have - $10,000 a day for 30 days or .01, .02, .04, … for 30 days as a mean to introduce exponential growth The Monty Hall Problem for an introduction to probability trees.	NRich Math http://nrich.maths.org/ MARS Tasks http://map.mathshell.org/index.php Illustrative Mathematics https://www.illustrativemathematics.org/
14. Debatable Problems – Introducing Problem Solving Techniques	These problems are immediately engaging and interesting but challenging to make sense of and often require the use of a specific problem solving habit. It may be creating a small case, trying something, organizing one's thoughts. Another extension of these questions is to ask "How many ways…" This takes a traditionally closed question and can open it up to challenge students to think systematically about their approach.	How Many Triangles?  x and y are prime numbers and $x + y = 12$. xy 38 $0 < a < b$ $(a + b)(a + b)$ $(b - a)(b - a)$	Park School Habits of Mind http://bit.ly/1qPnZ7a 501 Quantitative Reasoning Questions http://bit.ly/1V1mgbZ
15. Other…			