Problems for Eager achievers

Factors and Areas of Rectangles

There are several different rectangles with sides that are integer units and a perimeter of 24. Do any of them have area of 24 square units? Sketch the figures. Which has the largest area and which has the smallest?

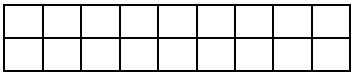
Are there any rectangles with sides that are integer units, where the length of the perimeter is the same as the number of square units in the area?

Are there other rectangles where the length of the perimeter is the same as the number of square units in the area? If so what are their dimensions?

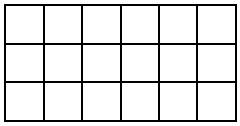
Rectangular "Personalities"

The rectangular personalities of a number N are the number of rectangles that can be made by lining up N squares:

For example 18 has two rectangular personalities:



Which number less than 1,000 has the most rectangular personalities?



Decimal Patterns

1/7 has an interesting decimal expansion that repeats the block of six digits 142857. Investigate the decimal patterns for the sevenths, 2/7, 3/7, …etc. and for some other fractions with odd number denominators. What generalizations can you make and justify?

What other decimals repeat? What are their patterns? What about 13^{ths} or 17ths or ???

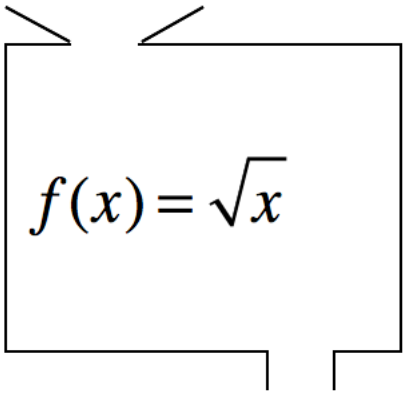
Trapezoids

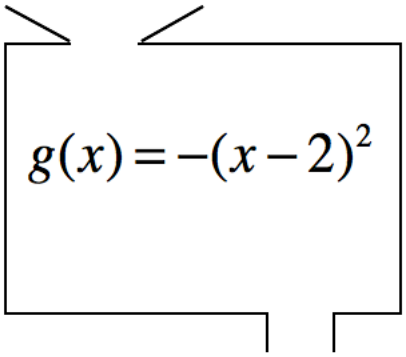
A trapezoid has four sides and at least one pair of opposite sides that are parallel. Will cutting and rearranging pieces to form a parallelogram or rectangle work to find the area of a trapezoid?

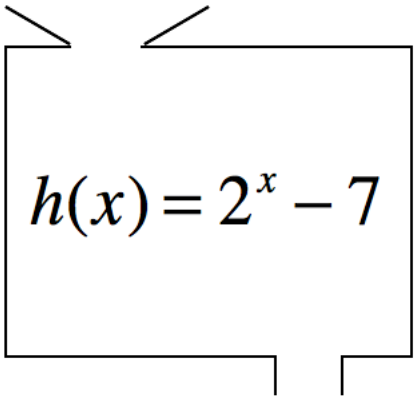
How many ways can you come up with to determine the area of a trapezoid?

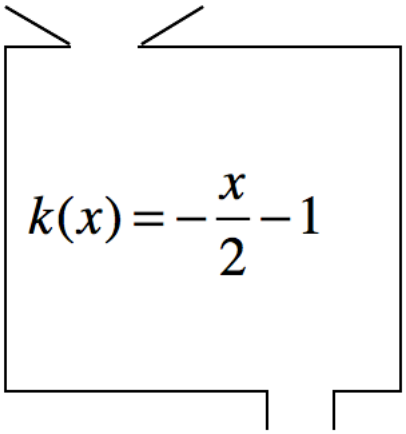
Represent each method using a and b as the bases and h as the height and show algebraically that all the different ways of dissecting are equivalent.

Function Machines









Start with an input of 6 and arrange all four machines in order to come out with 11.
For groups that finish faster try other numbers 4 in -5 out, or 0 in and -5 out. Can you make up your own pairs of "reasonable input/out put numbers?
Is there another number you can put in and get 11 out?
Once you find an order for the machines that gives you an such as -5 or 11 will there always be two possible inputs for that output? Why or why not?

Exponent Efficiency

Rewrite this expression in simpler form. Show as many ways as you can to do this.

$$\left(\frac{2x^{-2}y^4}{x^5y}\right)^{-3}$$

Then decide which is the best way and defend your choice.

[Next 3 from: Steve Bryant, George Graham & Ken Wiley (1965). *Non Routine Problems,* Mcgraw Hill.

See Through the Structure of a Problem

Start with this system to solve: $x + y = 7$
$3x - 2y = 1$

Then give those who finish easily this one:

$$\frac{1}{x} + \frac{1}{y} = 7$$
$$\frac{3}{x} - \frac{2}{y} = 1$$

OR

Start with: $x^2 + y^2 = 7$
$x^2 + y^2 = 25$

And then this:

$$\frac{1}{x} + \frac{1}{y} = 7$$
$$\frac{1}{x^2} + \frac{1}{y^2} = 25$$

Ponder this!

$$x^{x^{x^{x^{\cdot^{\cdot^{\cdot}}}}}} = 2$$

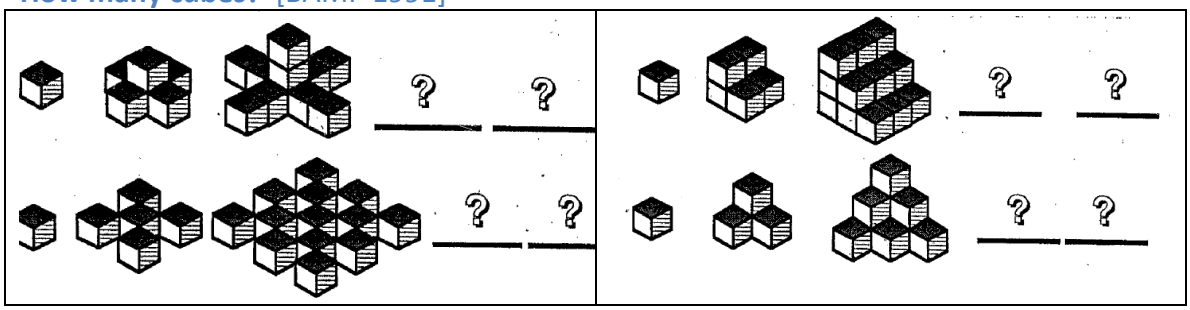
Explore beyond the parabola investigation of *a, h, k*

This group has finished and answered all questions. So they are clear on *a, h, k* when the equation is its graphing form.
But how are *a, h,* and *k* related to *a, b,* and *c* when the equation is in standard form?
How are and *a, b,* and *c* related to the graph. What can you find out using Desmos?

Do Parabolas have slope?

Start with what you know about linear functions, their slopes and their tables.
Investigate the tables for some quadratic functions?
What about a cubic or a 4^{th} degree polynomial?
Can you justify your conjecture for quadratic polynomials?

How many cubes? [BAMP 1991]



Adding Integers Placing the +'s and –'s

Alice is playing the following game: she writes in one row all numbers:

1, 2, 3, ... ,2001

She then starts placing the signs + or – between any two adjacent numbers. In the end, she calculates the value of the resulting expression. She wins if this value is 0.

Can Alice can ever win? Explain and justify your reasoning