Capitalizing on New Opportunities for Systemic Improvement in Mathematics Education

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FYI

Electronic copies of slides are available

by request

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and posted at

nctm.org/briars

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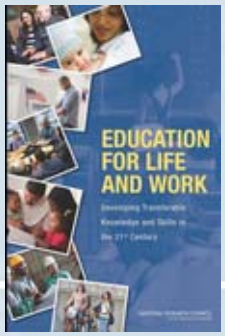
Agenda

- New developments/reports about mathematics curriculum, teaching and learning.
- Implications for K-12 mathematics education
- Actions for moving forward

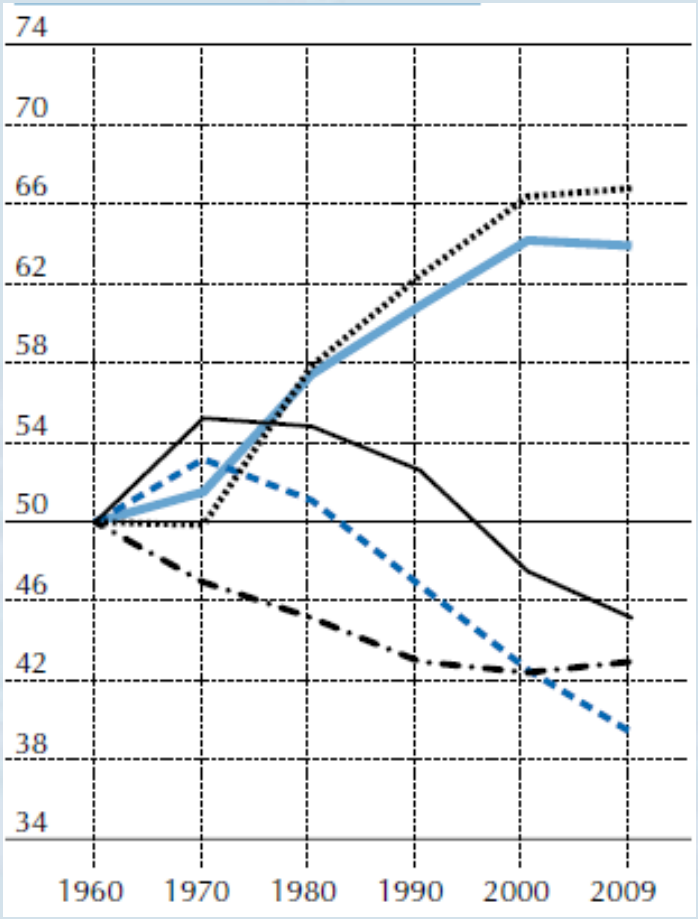
College and Career Readiness

Business and political leaders are asking schools to ensure that students leave high school "college and career ready", possessing 21st century competencies that will prepare them for adult roles as citizens, employees, managers, parents, volunteers and entrepreneurs.

How do you define 21st century competencies?



Trends in the Demand for Skills in the US



Non-routine analytic
Non-routine interactive
Non-routine manual
Routine cognitive
Routine manual

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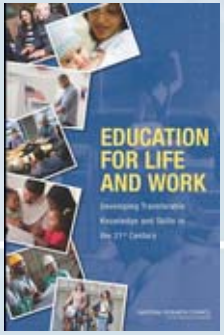
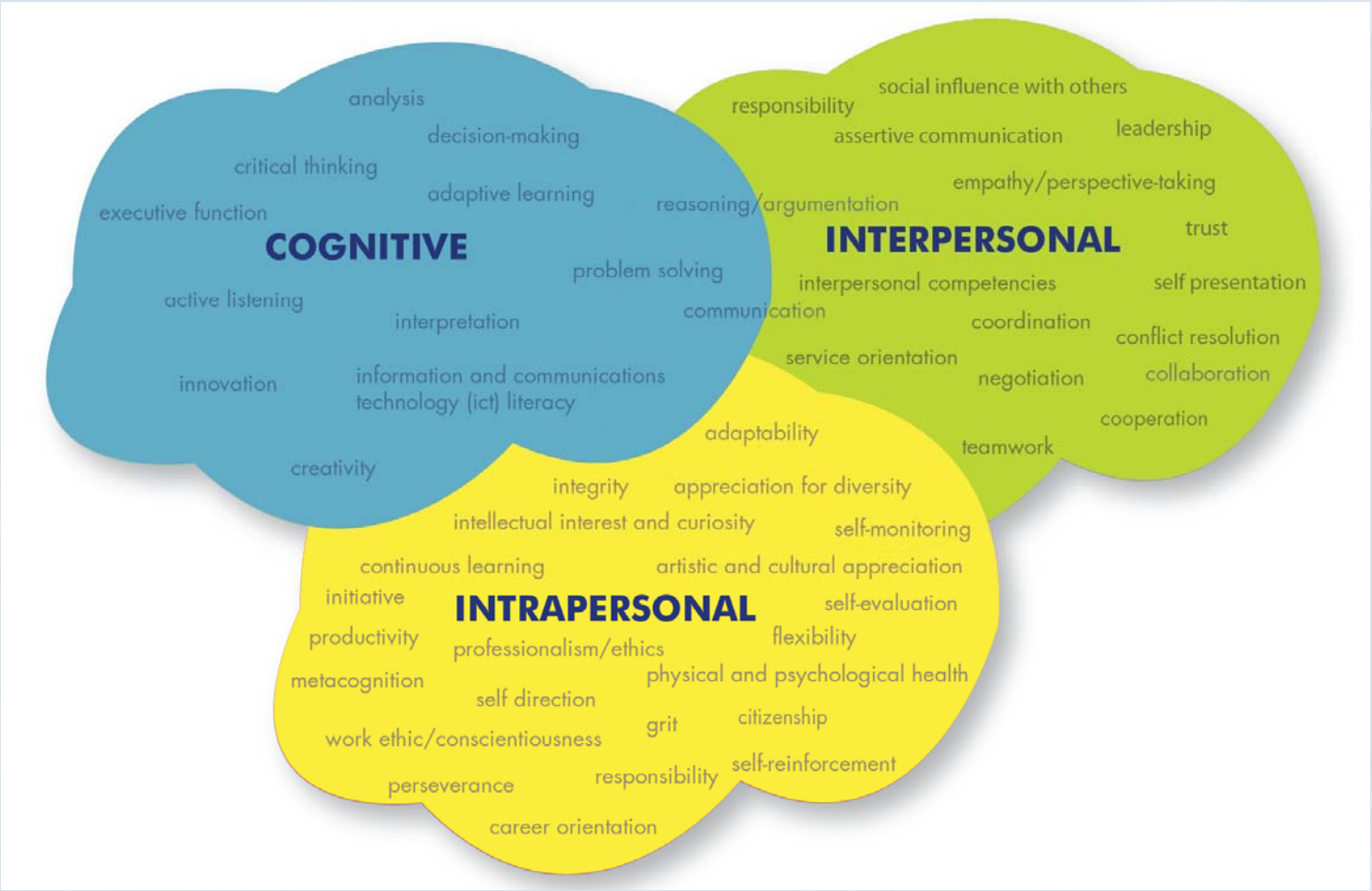
Average change in task inputs across education-industry cells, in percentiles of the 1960 task distribution



21st Century Competencies



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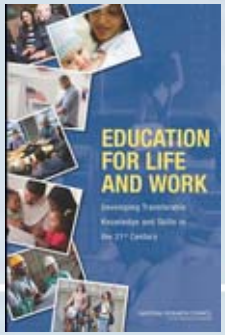
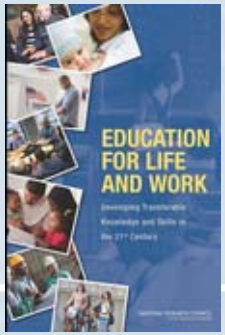
Education for Life & Work, NRC 2012



21st Century Competencies

- Cognitive domain
  - Thinking, reasoning, and related skills
- Intrapersonal domain
  - Self management, including regulating one's behavior and emotions to reach goals;
- Interpersonal domain
  - Expressing information to others, as well as interpreting others' messages and responding appropriately.

Education for Life & Work; Guide for Practitioners, p. 4



21st Century Competencies

"Deeper learning" is the process through which an individual becomes capable of taking what was learned in one situation and applying it to new situations (i.e., transfer). . . . The product of deeper learning is **transferable knowledge, including content knowledge in a domain and knowledge of how, why, and when to apply this knowledge to answer questions and solve problems.** We refer to this blend of both knowledge and skills as "21st century competencies."

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Education for Life & Work, p. 5

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

McDonald's Claim

Wikipedia reports that 8% of all Americans eat at McDonald's every day.

320 million Americans and 14,350 McDonald's…

Could the Wikipedia report be true?

Create a mathematical argument to justify your position.

If you believe the claim is not true, determine a reasonable percentage.

McDonald's Claim

Calculations:

- 8% of 320 million people/day = 25.6 million people/day
- 25.6 million people ÷ 14,350 McDonald's
  = 1,784 people/McDonald's/day
  ≈ 2,000 people/McDonald's/day

Discussion: Assumptions/Definitions

- What does it mean to "eat at"?
- What does "8% .. eat at everyday"?
- # of hours a typical McDonald's is open each day
- # of registers, # of drive-thru lanes
- Americans eating in McDonald's outside of the US?

McDonald's Claim Problem

- What mathematics *content* is needed to complete the task?
- Which mathematical *practices* are needed to complete the task?

Standards for Mathematical Practice

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Standards for Mathematical Practice

SMP 4: Model with Mathematics

- Make assumptions and approximations to simplify a complicated situation
- Identify important quantities in a practical situation
- Analyze relationships mathematically to draw conclusions
- Interpret their mathematical results in the context of the situation
- Reflect on whether the results make sense.

SMP 3. Construct viable arguments and critique the reasoning of others

- Make conjectures
- Justify their conclusions and communicate them to others
- Recognize and use counterexamples
- Compare the effectiveness of two plausible arguments
- Listen and respond to the arguments of others for sense making and clarity

Guidelines for Assessment and Instruction in Mathematical Modeling Education (GAIMME)

- What is mathematical modeling?
- What does it look like in pK-8, high school, and college?
- Appendices with resources and examples

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What is Mathematical Modeling?

Mathematical modeling is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena.

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What is Mathematical Modeling?

Mathematical modeling is a process made up of the following components:

- Identify the problem
- Make assumptions and identify variables
- Do the math
- Analyze and assess the solution
- Iterate
- Implement the model

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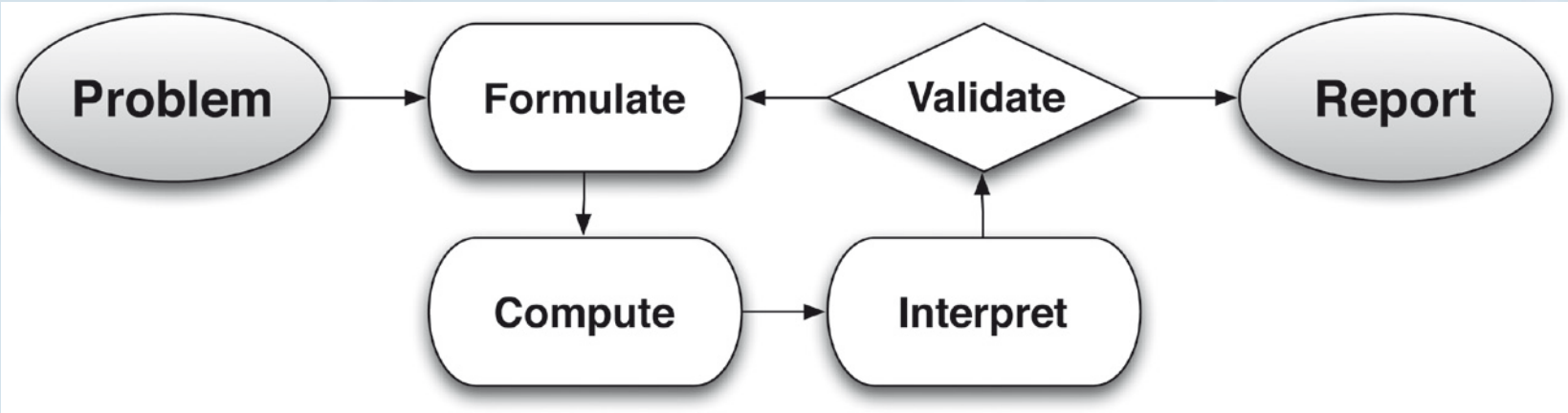
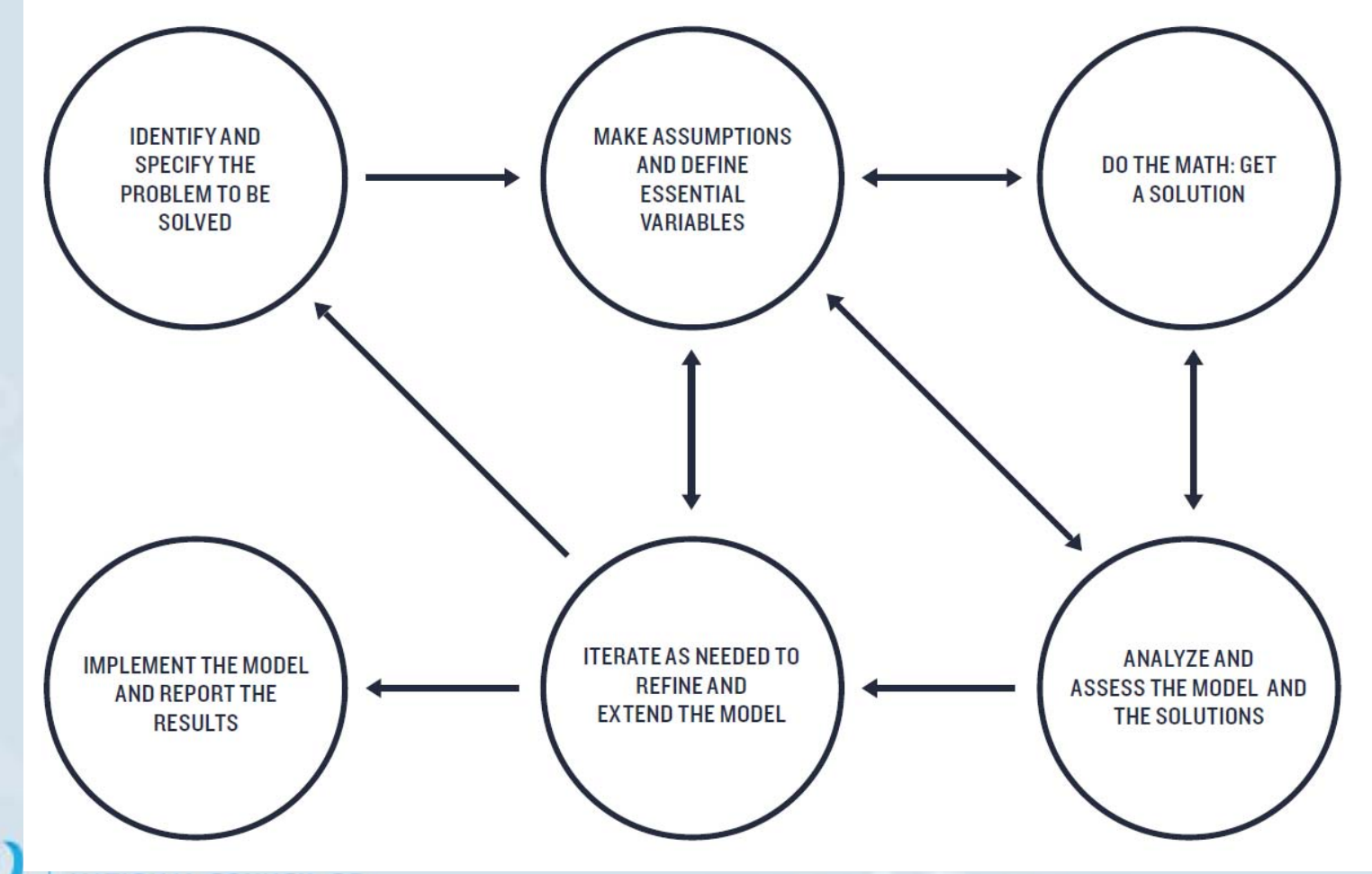
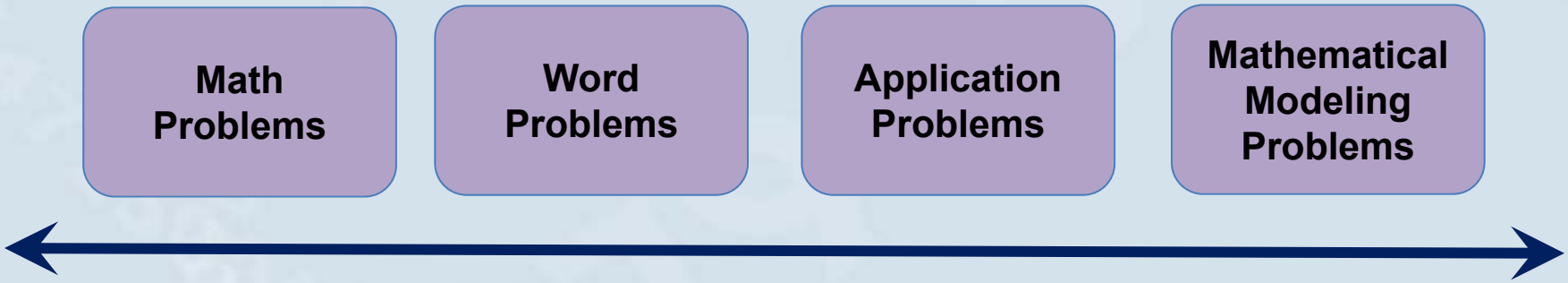
CCSS Modeling Cycle



GAIMME Modeling Cycle



Guidelines for Assessment and Instruction in Mathematical Modeling Education (GAIMME)



Source: Garfunkel, Sol, et al. (Ed.). *Guidelines for Assessment and Instruction in Mathematical Modeling Education*. Boston/Philadelphia: Consortium for Mathematics and its Applications/ Society for Industrial and Applied Mathematics, 2016.

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Example: Driving for Gas

Word Problem

You drive 12.3 miles to get to work each day and gas costs $3.00 a gallon. How much do you spend on gas each day? If you shared the cost with someone else by carpooling, how many days would you have to carpool to reduce your cost by 1/3?

Source: NCTM/SIAM Joint Committee on Mathematical Modeling Across the Curriculum

Example: Driving for Gas

Application Problem

Suppose there is a station on your normal route that sells gas for $3.00 a gallon. A station 5 miles off your route sells gas for $2.85 a gallon. Should you drive the extra distance to buy gas at that station?

Source: NCTM/SIAM Joint Committee on Mathematical Modeling Across the Curriculum

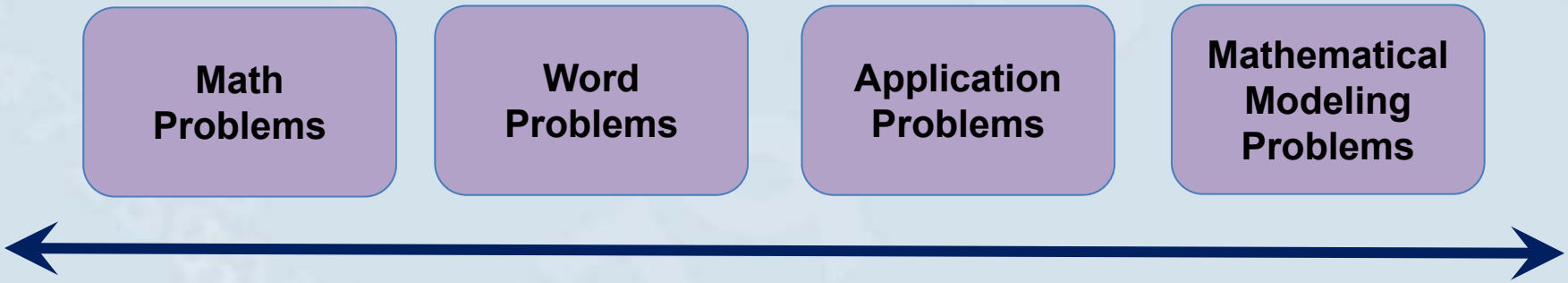
Example: Driving for Gas

Mathematical Modeling Problem

You drive to school every day. On the route you take from home to school, there are several gas stations. Unfortunately, the prices on your route are always high. A friend tells you she buys her gas at a station several miles off your normal route where the prices are cheaper. Would it be more economical for you to drive the extra distance for the less expensive gas than to purchase gas along your route?

Source: NCTM/SIAM Joint Committee on Mathematical Modeling Across the Curriculum

Where Does the McDonald's Claim Problem Fit in this Continuum?



Source: Garfunkel, Sol, et al. (Ed.). *Guidelines for Assessment and Instruction in Mathematical Modeling Education*. Boston/Philadelphia: Consortium for Mathematics and its Applications/ Society for Industrial and Applied Mathematics, 2016.

SMP 4: Model with Mathematics

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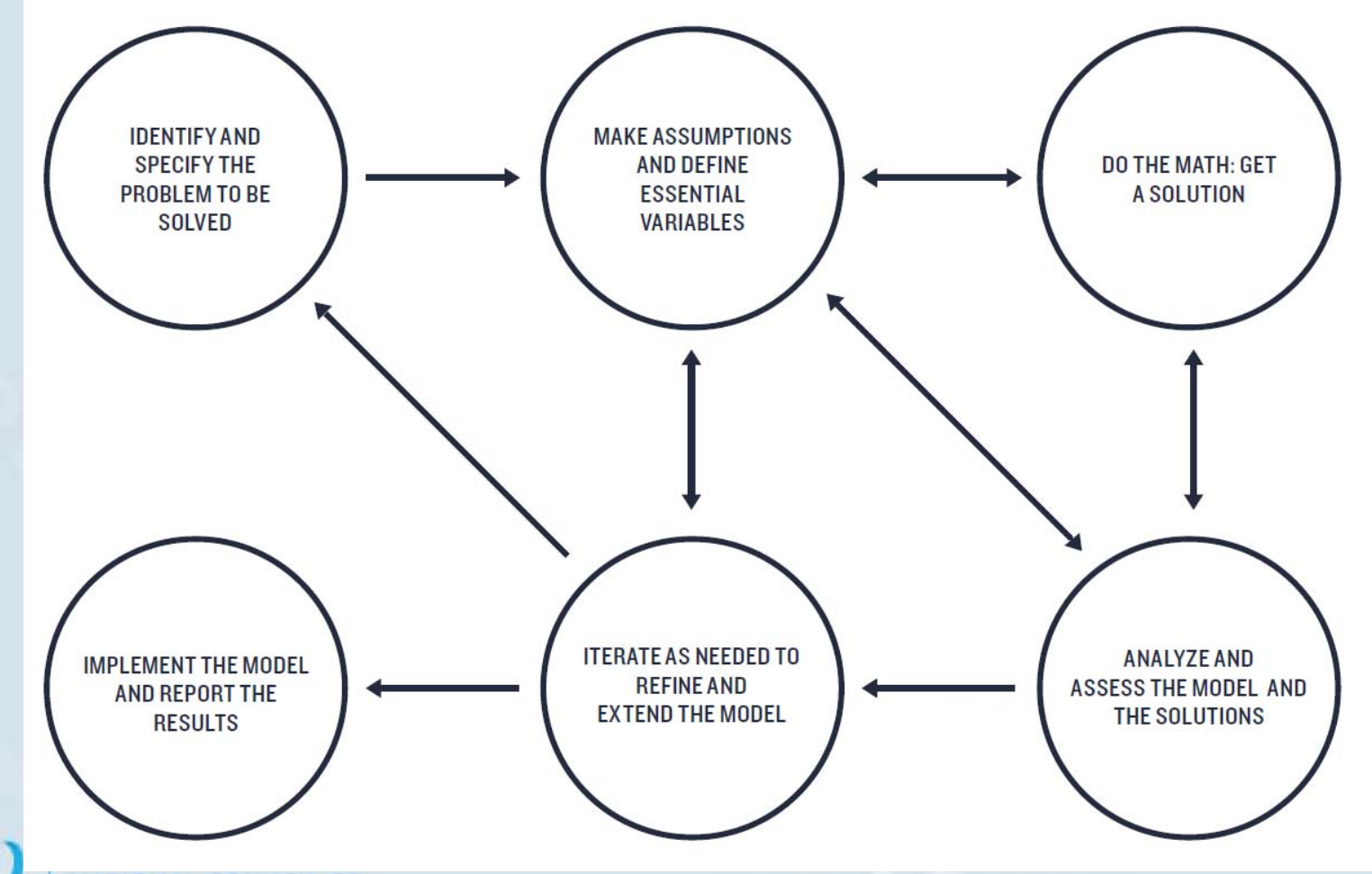
GAIMME Modeling Cycle



Having Kittens

Here is a poster published by an organization that looks after stray cats.

Cats can't add but they do multiply!

In just 18 months, this female cat can have 2000 descendants.

Figure out whether this number of descendants is realistic.

Here are some facts that you will need:

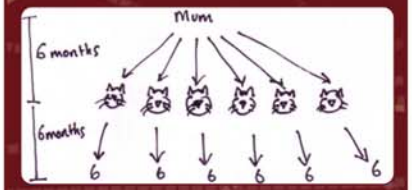
Length of pregnancy About 2 months	Age at which a female cat can first get pregnant About 4 months	Number of kittens in a litter Usually 4 to 6
Average number of litters a female cat can have in one year 3		Age at which a female cat no longer has kittens About 10 years

http://map.mathshell.org/

Mathematics Assessment Project
CLASSROOM CHALLENGES
Formative Assessment Lessons for High School

MARS Mathematics Assessment Resource Service

Home | About | News | Lessons | Tasks | Tests | PD Modules | TRU Math Suite | Standards



Search...

Grade: 6 | 7 | 8 | High School

High School

- Number and Quantity
  - Modeling Population Growth: Having Kittens
  - Classifying Rational and Irrational Numbers
  - Evaluating Statements about Rational and Irrational

Modeling Population Growth: Having Kittens

Mathematical goals

This lesson unit is intended to help you assess how well students are able to:

- Interpret a situation and represent the constraints and variables mathematically.
- Select appropriate mathematical methods to use.
- Make sensible estimates and assumptions.
- Investigate an exponentially increasing sequence.
- Communicate their reasoning clearly.

Introduction

This lesson is designed to help students develop strategies for modeling. Note that a video of this lesson is available in the professional development materials.

- Before the lesson, students attempt the problem individually. You then review their work and write questions to help students improve their solutions.
- At the start of the lesson, students work individually answering your questions. Then, in small groups, students work collaboratively on the task before evaluating some sample solutions. In a whole-class discussion, students explain and compare the alternative solution strategies they have seen and used.
- In a follow-up lesson, students review what they have learned.

Materials required

Lesson (complete)
(5440.8K PDF)

Projector Resources
(2494K PPT)

Using the Classroom Challenges
To make the most of these materials:

- Read more about the purpose of the *Classroom Challenges*...
- Download *A Brief Guide for teachers and administrators* (PDF)

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NCTM GAIMME Sessions

106- What Is the Current State of Mathematical Modeling Education?
Thursday, April 14, 2016: 9:45 AM-11:00 a.m., 306 (Moscone)

251- GAIMME: Mathematical Modeling for Elementary School
Thursday, April 14, 2016: 3:30 PM-4:30 p.m., 305 (Moscone)

274- GAIMME: Mathematical Modeling for Middle School
Friday, April 15, 2016: 8:00 AM-9:00 a.m., 305 (Moscone)

403- GAIMME: Mathematical Modeling for High School
Friday, April 15, 2016: 12:30 PM-1:30 p.m., 305 (Moscone)

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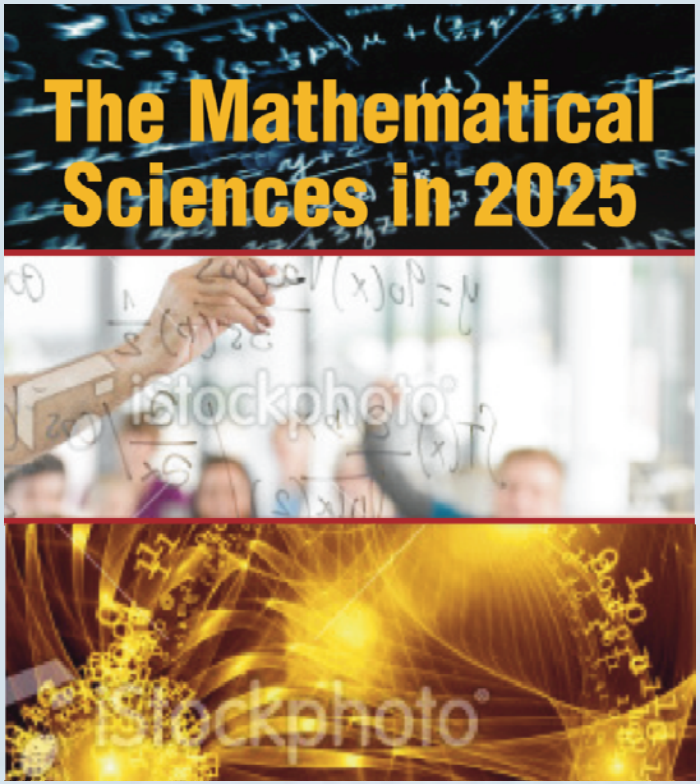
College and Career Readiness



- Examines the mathematical science now and how it needs to evolve to produce the best value for the country by 2025.
- Describes the remarkable success of the mathematical sciences in the opening years of the 21st century.
- Highlights the increasing importance of statistics, modeling and discrete mathematics.

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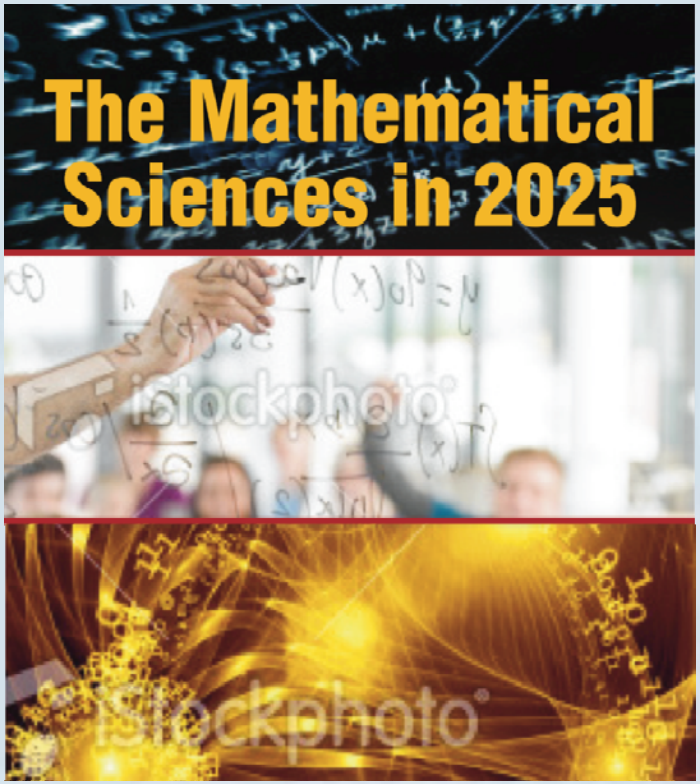
College and Career Readiness



Our high schools focus on getting people prepared for calculus . . .

But we do little to teach statistics, probability, and uncertainty . . .

This is one of the biggest issues facing U.S. mathematical sciences; it is also a big problem in terms of national competitiveness.

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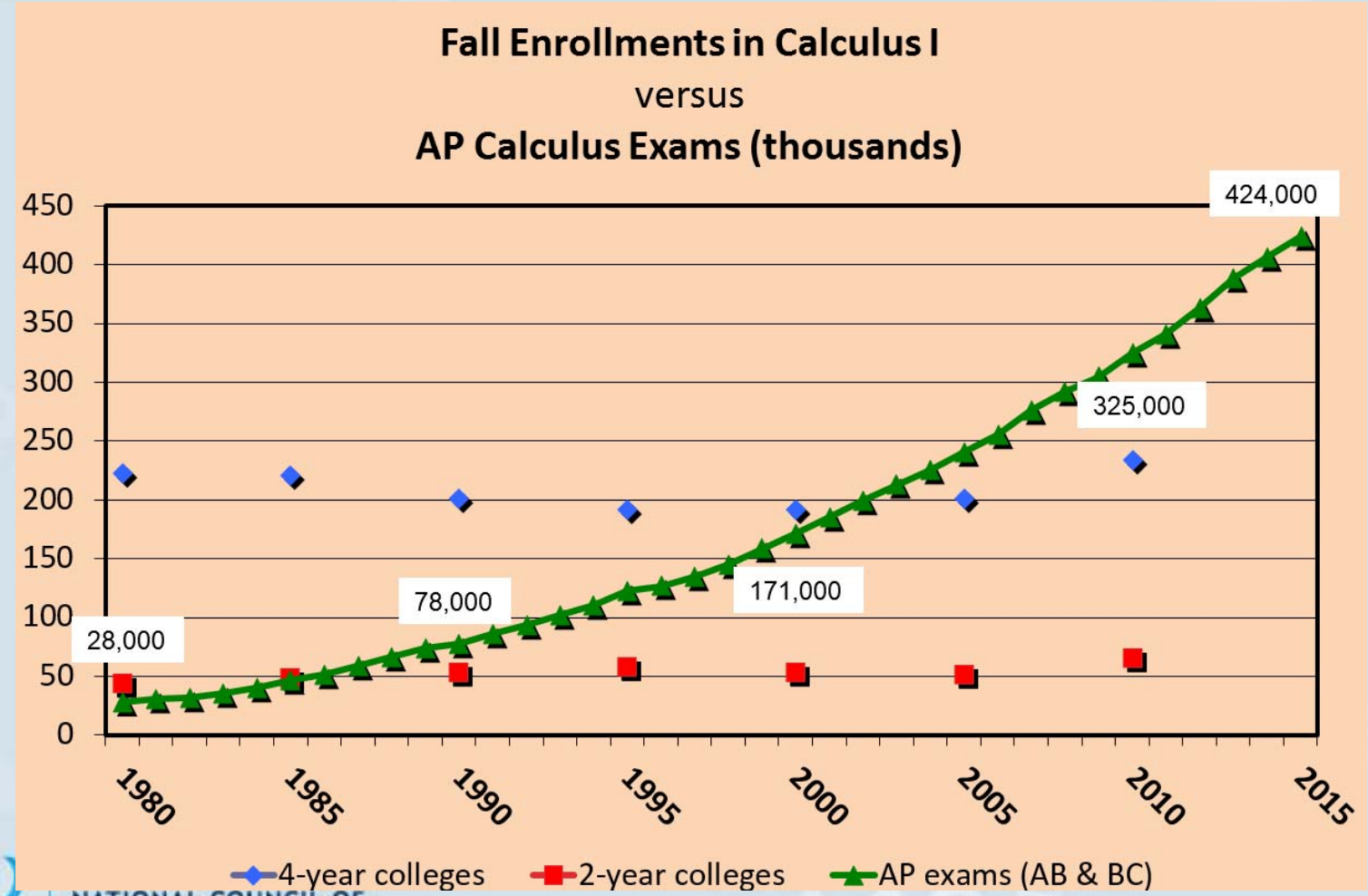
The Rush to Calculus



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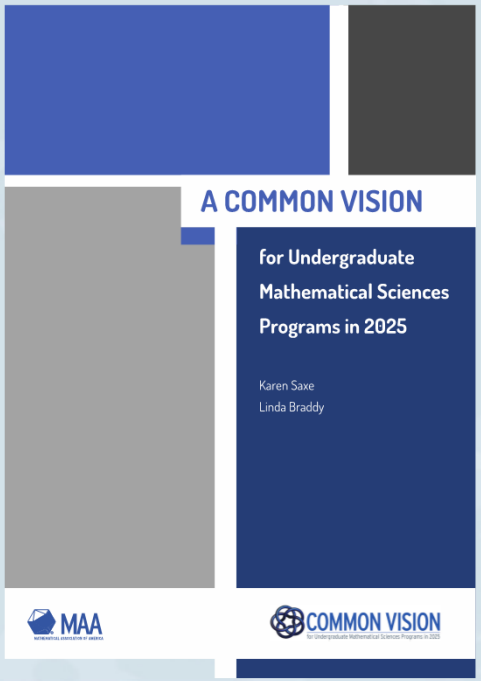
Sources: CBMS and College Board

A Common Vision for Undergraduate Mathematical Sciences in 2025

"The status quo is unacceptable."



Calls on the community to:

1. Update curricula
2. Articulate clear pathways between curricula driven by changes in K-12 and the first courses students take in college
3. Scale up the use of evidence-based pedagogical methods

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Active Learning Increases Student Performance in Science, Engineering, and Mathematics

- Meta-analysis of 225 studies comparing student performance in undergraduate science, technology, engineering, and mathematics (44 studies) (STEM) courses under traditional lecturing versus active learning.
- Active learning interventions included:
  - occasional group problem-solving,
  - worksheets or tutorials completed during class,
  - use of personal response systems
  - studio or workshop course designs.
- Research questions:
  - Does active learning boost examination scores?
  - Does it lower failure rates?

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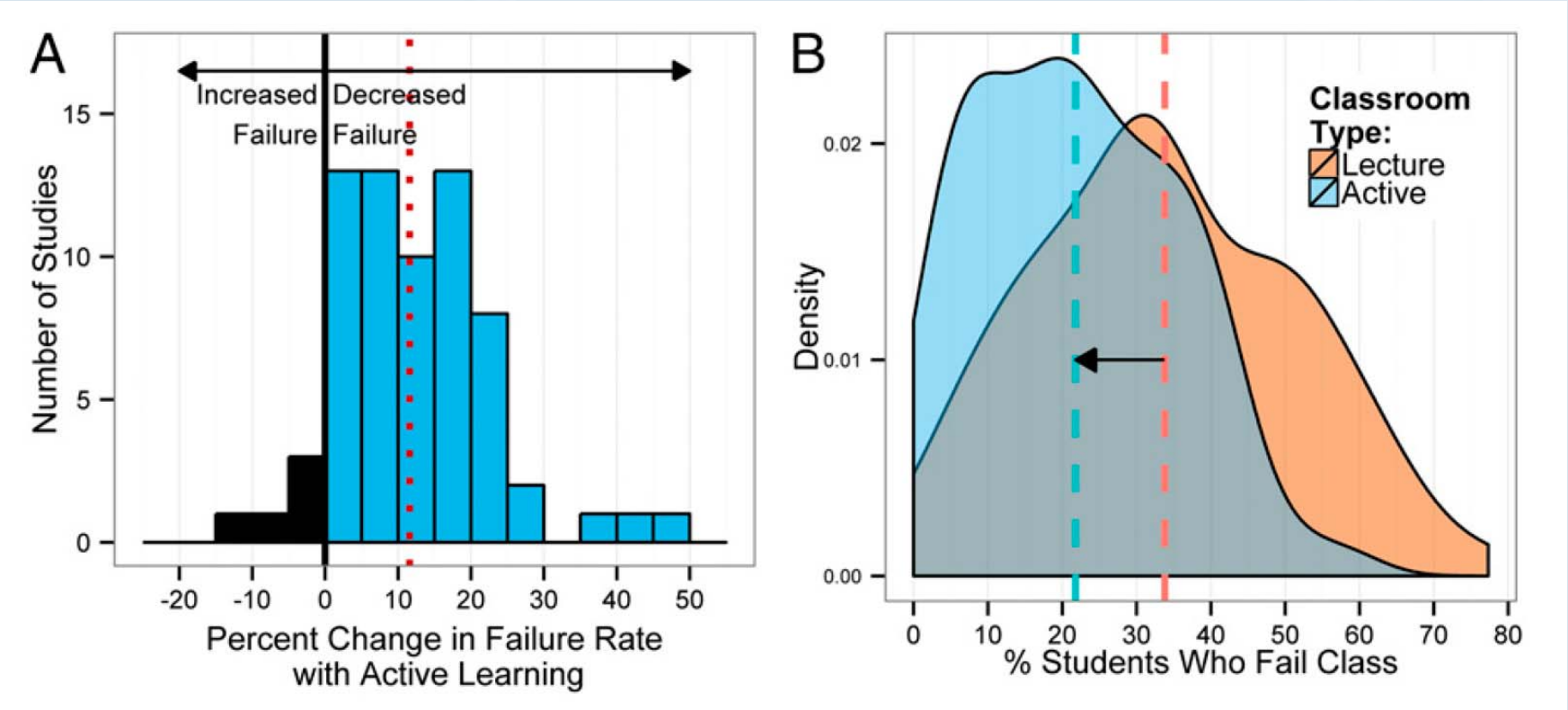
Freeman, S. et al, Proceedings of the National Academy of Sciences, June 10, 2014, vol. 111 no. 238410-8414.

Changes in Failure Rate Lecture vs Active Learning



(A) The mean change (12%) is indicated by the dashed vertical line.

(B) Kernel density plots of failure rates under active learning and under lecturing. The mean failure rates under each classroom type (21.8% and 33.8%) are shown by dashed vertical lines.

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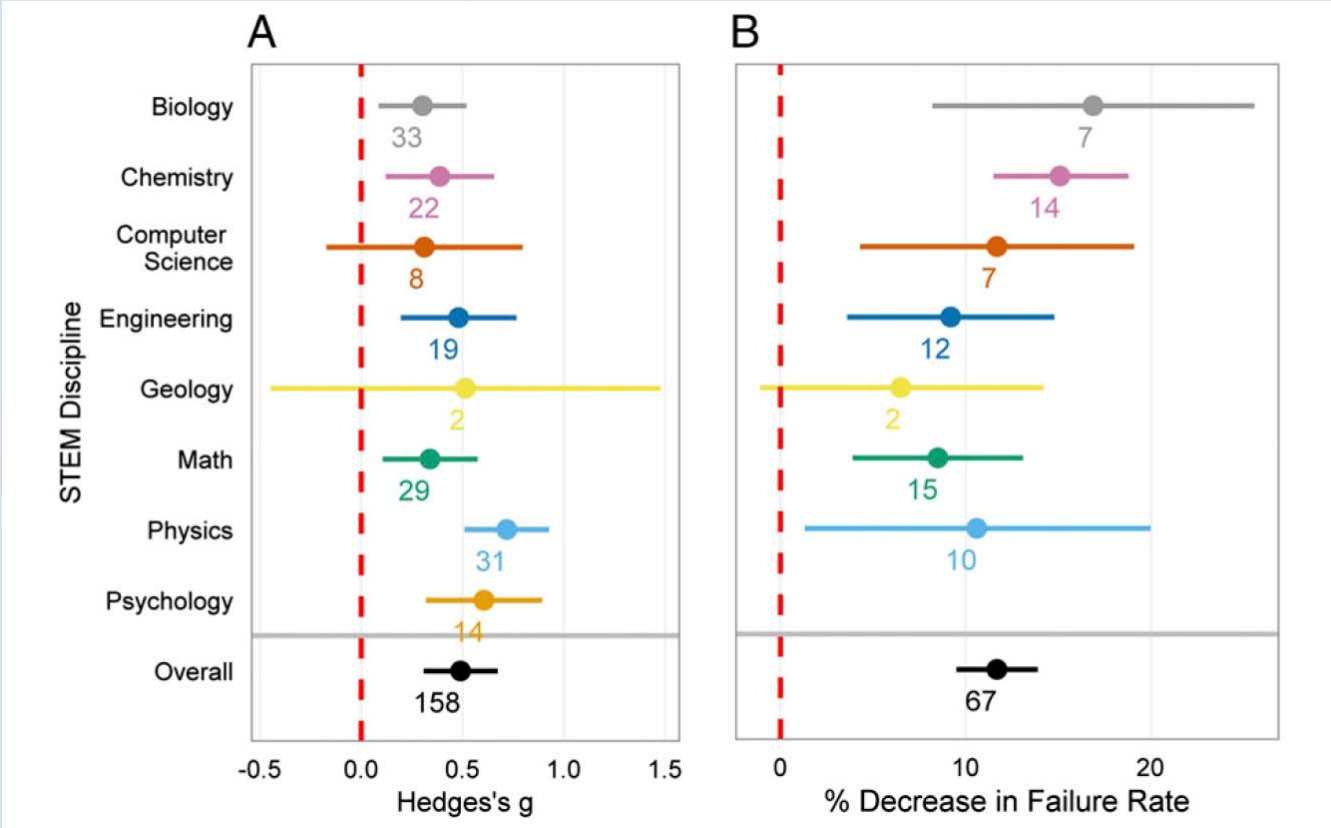
Freeman et al., 2014

Changes in Assessment Performance and Failure Rate
Lecture vs Active Learning



(A) Data on examination scores, concept inventories or other assessments

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Freeman et al., 2014

Active Learning Increases Student Performance in Science, Engineering, and Mathematics

- Active learning confers disproportionate benefits for STEM students from disadvantaged backgrounds and for female students in male-dominated fields;

- The results raise questions about the continued use of traditional lecturing as a control in research studies, and support **active learning as the preferred, empirically validated teaching practice in regular classrooms**.

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Freeman et al, 2014

Effective Mathematics Teaching Practices

1. Establish mathematics **goals** to focus learning.
2. Implement **tasks** that promote reasoning and problem solving.
3. Use and connect mathematical **representations.**
4. Facilitate meaningful mathematical **discourse**.
5. Pose purposeful **questions**.
6. Build **procedural fluency** from conceptual understanding.
7. Support **productive struggle** in learning mathematics.
8. **Elicit and use evidence** of student thinking.

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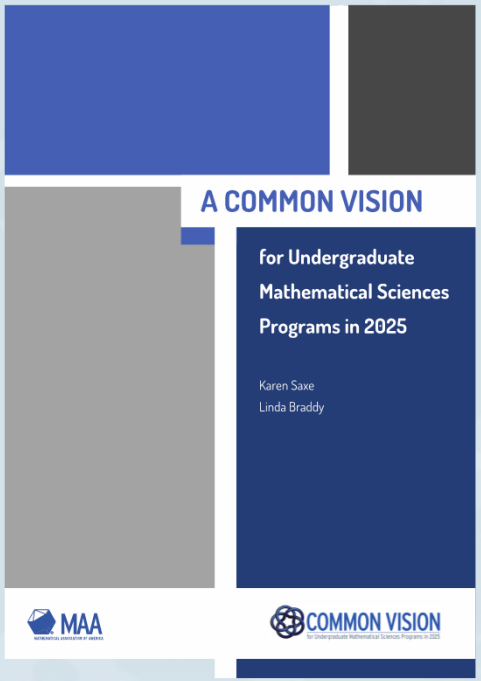
A Common Vision for Undergraduate Mathematical Sciences in 2025

"The status quo is unacceptable."



Calls on the community to:

1. Update curricula
2. Articulate clear pathways between curricula driven by changes in K-12 and the first courses students take in college
3. Scale up the use of evidence-based pedagogical methods
4. Remove barriers at critical transition points
5. Establish stronger connections with other disciplines

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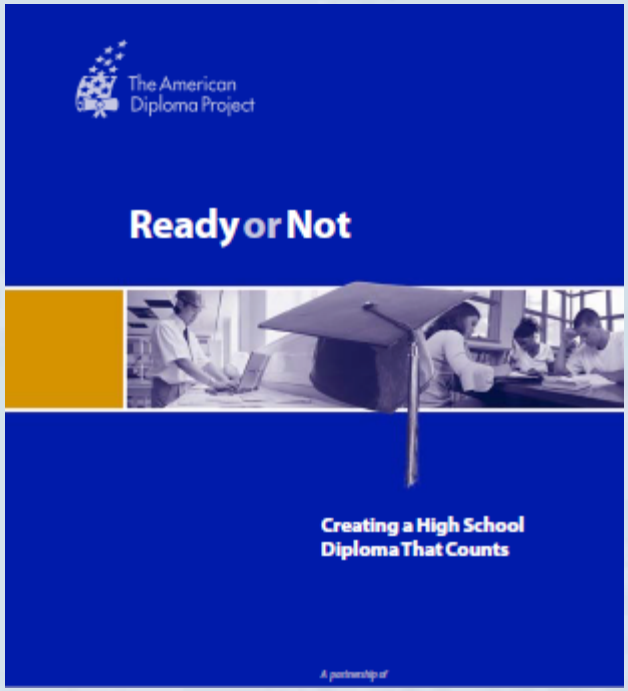
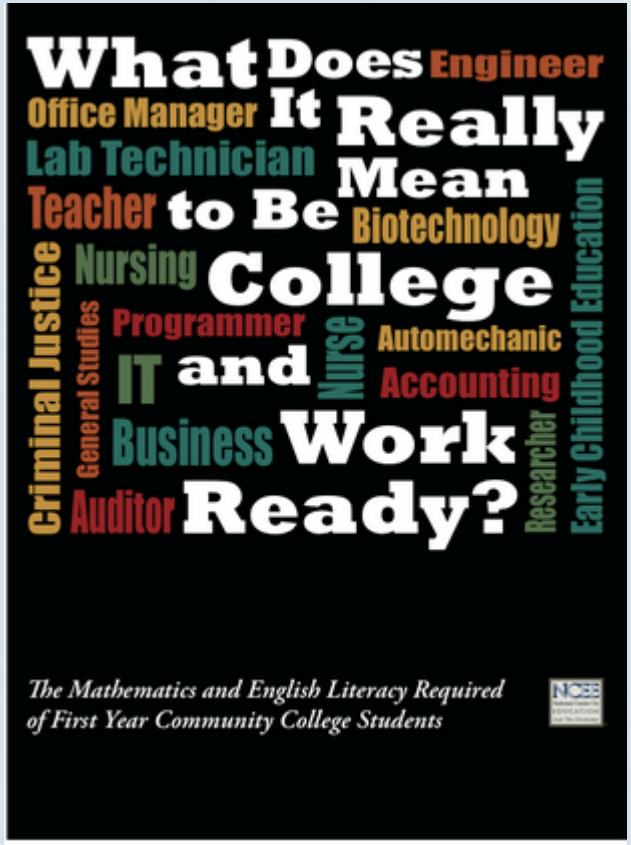
College and Career Readiness





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Systemic Improvement in High School Mathematics

- Increase emphasis on modeling, statistics and discrete mathematics.
- Increase emphasis on conceptual understanding, including ratios and proportional relationships.
- Ensure that the "rush to calculus" does not narrow the curriculum and decrease students' mathematical preparation.
- Clarify the mathematics required for college and career readiness based on research.
- Provide adequate time for students to develop proficiency in the Standards for Mathematical Practice/Process Skills.

Systemic Improvement in High School Mathematics

- Implement the 8 effective teaching practices in all high school courses
- Move to an integrated high school mathematics courses

Thank You!

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