

Focus on Visual Representations: A Goldmine for Reasoning and Sense Making

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Eight Effective Mathematics Teaching Practices —NCTM's *Principles to Action*

- Establish mathematics goals to focus learning
- Implement tasks that promote reasoning and problem solving
- Use and connect mathematical representations
- Facilitate meaningful discourse

Eight Effective Mathematics Teaching Practices

- Pose purposeful questions
- Build procedural fluency from conceptual understanding
- Support productive struggle in learning mathematics
- Elicit and use evidence of student thinking

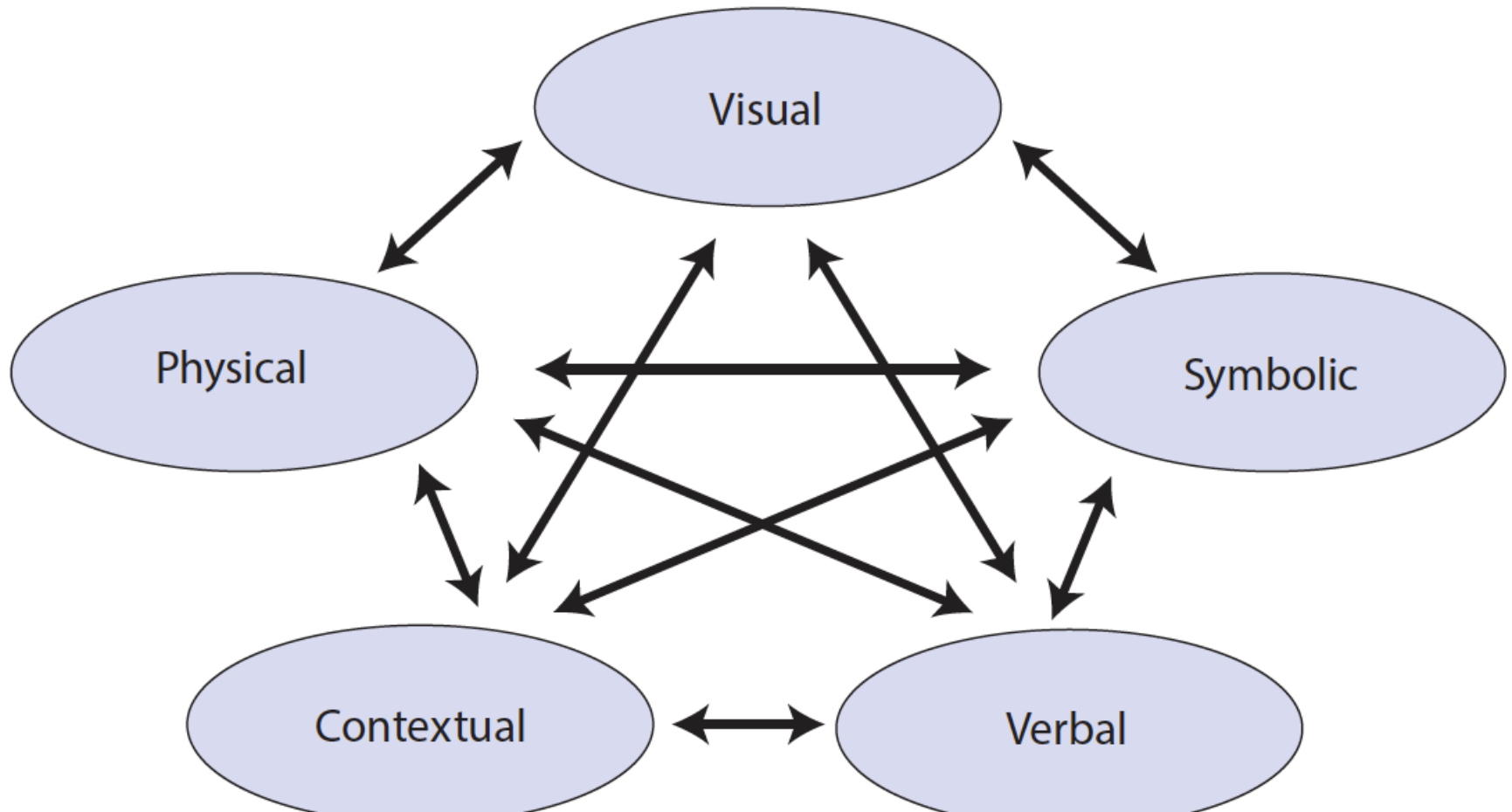
Focus Effective Practices for our Session Today

- Implement tasks that promote reasoning and problem solving
- Use and connect mathematical representations

Also we hope to:

- Facilitate mathematical discourse
- Elicit and use evidence of student thinking

Representations of Mathematics



Importance of Visual Representations

--from *Principles to Action*

Visual representations are of particular importance in the mathematics classroom, helping students to advance their understanding of mathematical concepts and procedures, make sense of problems, and engage in mathematical discourse (Arcavi 2003; Stylianou and Silver 2004). Visuals support problem solving as students consider relationships among quantities when they sketch diagrams or make tables and graphs. The visual representations also support discourse because the diagrams or drawings leave a trace of student problem solving that can be displayed, critiqued, and discussed. Math drawings and other visual supports are of particular importance for English language learners, learners with special needs, or struggling learners, because they allow more students to participate meaningfully in the mathematical discourse in the classroom (Fuson and Murata 2007). The visuals assist students in following the reasoning of their classmates and in giving voice to their own explanations as they gesture to parts of their math drawings and other visual representations.

In Summary, Visual Representations of Mathematics:

- Help students make sense of concepts and procedures
- Support mathematical discourse
- Support problem solving
- Are important for English language learners
- Are important for special needs students
- Allow access to the mathematics for more students

Thinking Visually—accessing our Mind's Eye

- Using diagrams and sketches to Represent mathematical concepts and procedures is something ***we can all learn to do***
- To develop our repertoire of visual approaches to mathematics we need to challenge ourselves to solve mathematical problems visually

Solving Problems using Visual Representations

Today's negotiated agreement: We Will--

- Solve these problems using only diagrams and sketches and a modicum of arithmetic.
- *Resist all impulses to use algebra, or techniques such as guess and check.*
- Appropriately label our diagrams and sketches.

Puzzle Problem 1

- There are 29 more boys than girls in the senior class. The class has 533 students. How many boys are there?

Structured Discourse Protocol 1

- Start with 3 min PRT (Private Reasoning Time) to develop diagrams/sketches for the task
- Elbow partner share—30 sec each Partner A talks, Partner B listens, then switch roles
- Whole group share out

Puzzle Problem 2

- Each side of an equilateral triangle is 2 feet more than each side of a square. Their perimeters are the same. Find the length of each side of the triangle.

Puzzle Problem 3

- Mike has three times as many nickels as Larry has dimes. Mike has 45c more than Larry.
- How much money does Mike have?

Structured Discourse Protocol 2

- Half of us will start with Problem 2, the other half start with Problem 3. (Try the other problem if you finish)
- Start with 5 minutes PRT
- (Different) elbow partner share—swap your solution approaches. Read over your partners' diagrams/sketches (1 minute)
- 1 minute each--Take turns explaining to your partner what you think they are trying to do in their solution process—then work together
- Whole group share.....

Puzzle Problem 4

- In a town, $\frac{2}{3}$ of the men are married to $\frac{3}{5}$ of the women. What fraction of the people in the town are married?

Puzzle Problem 5

- One man can dig a trench in 4 days. His son can dig a similar trench in 6 days. How long would it take them to dig a trench working together?

Structured Discourse Protocol 3

- Half of us will start with Problem 4, the other half start with Problem 5. (Try the other if you finish).
- Start with PRT (Private Reasoning Time) to develop diagrams/sketches for the tasks (5-7 min)
- Take turns explaining to a partner what you are trying to do in your solution process (1 minute each)—then continue working together (3 minutes)
- Share diagrams and sketches in your whole group—Once Around protocol (5 minutes)
- Whole group share out....

Water Buckets

- You have a 9 liter pail and a 4 liter pail. What whole number amounts of water can you bring back from a river using only these pails?

(Note: Bring exact amounts. There are no other markings on the pails).

Grouchy Customers

- Every early morning at a local café customers stumble in for their morning coffee. Typically early morning customers do not want to engage in conversation. The café has 8 counter seats.
- How many different ways can the first three morning customers sit in the 8 seats so that no two of them are sitting adjacent?

Puzzle Problem 6

- Elizabeth is placing rabbit in cages. She notices that if she place 6 rabbits in each cage, she has 3 cages left over. If she places 4 rabbits in each cage, she has 2 rabbits left over.
- How many cages, and how many rabbits does she have?

Structured Discourse Protocol 4

- Pick a problem of your choice—Water Buckets, Grouchy Customer, or Puzzle Prob 6
- PRT (5 minutes)
- Stand up—find others who also chose your problem—form group (or groups) and do a Once Around share of your approaches
- Whole Group Share....

Visual Representations of Mathematics Concepts, Procedures, and Solutions to Problems

Are important to our students for:

- SENSE MAKING of procedures and concepts
- DISCOURSE—explain, critique, debate, generalize, justify
- ENHANCING PROBLEM SOLVING SKILLS and STRATEGIES
- ACCESS—for ELA, for Special Needs, FOR ALL!

THANK YOU FOR COMING!!

- REMEMBER TO INCLUDE YOUR MIND'S EYE IN ALL YOUR MATHEMATICAL ENDEAVORS!!
- TO BOOST STUDENTS' MATHEMATICAL ENTERPRISE,
HAVE THEM VISUALIZE!!

MIKE

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