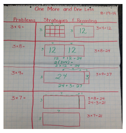
Presentation 44

Welcome to

Building Computational Fluency
Through Conceptual Understanding

NCTM Regional
Phoenix
2016



Beth Newman

Building Computational Fluency
Through Conceptual Understanding

A Mathematical Teaching Practice

for Effective Teaching and Learning

NCTM's *Principles to Actions*

NCTM's Principles to Action:

Computational fluency is strongly related to number sense and involves so much more than the conventional view of it encompasses

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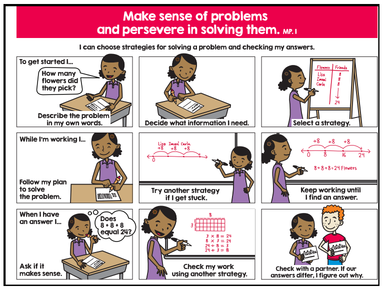
NCTM's Principles to Action:

Fluency Progression:

- Initial exploration and discussion of number concepts
- Informal reasoning strategies based on meanings and properties of operations
- General methods as tools in solving problems

NCTM's Principles to Action:

- Fluency is not a simple idea
  - Students are able to choose flexibly among strategies
  - Students understand and are able to explain their approaches
  - Students produce accurate answers efficiently



NCTM's Principles to Actions

Although they provide a vehicle for all students to demonstrate understanding and to extend thinking, math drawings and visual supports are of particular importance

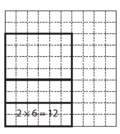
- for ELL
- learners with special needs
- struggling learners

because they allow more students to participate meaningfully in the math discourse in the classroom.

Models of Thinking and for Thinking

Area Model

Array



$$\begin{aligned}8 \times 6 &= 2 \times 4 \times 6\\ &= 2 \times 2 \times 2 \times 6\\ &= 2 \times 2 \times 12\\ &= 2 \times 24\\ &= 48\end{aligned}$$

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Models of Thinking and for Thinking

Ratio Table

Ratio Table

Number of Cats	Number of Legs
1	4
2	8
4	16
8	32
10	40

2 + 8

8 + 32

$1 \times 4 = 4$
$2 \times 4 = 8$
$4 \times 4 = 16$
$8 \times 4 = 32$
$10 \times 4 = 40$

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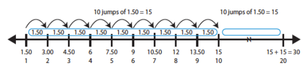
Models of Thinking and for Thinking

Number Line

Number Line



3 Components

What Is Computational Fluency?

Efficiency

Efficiency implies that the student does not get bogged down in many Steps or lose track of the logic in the strategy. An efficient strategy is one that the student can carry out easily, keeping track of sub-problems and making use of intermediate results to solve the problem.

Accuracy

Accuracy depends on several aspects of the problem-solving process, among them careful recording, the knowledge of basic number combinations and other important number relationships, and concern for double-checking results.

Flexibility

Flexibility requires the knowledge of more than one approach to solving a particular kind of problem. Students need to be flexible and choose an appropriate strategy for solving the problem at hand. They can use one method to solve a problem and another method to double-check results.

Fluency

The CCSS-M reinforce problem solving and conceptual understanding, and they place greater emphasis on Number and Operations.

- automaticity of facts
- proficiency with efficient computational strategies
- define grade levels where proficiency should be achieved

Where must students show fluency at each grade level?

Third Grade

3.OA.5	Understand properties of multiplication and the relationship between multiplication and division. Apply properties of operations as strategies to multiply and divide. *Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$ (Distributive property).*
3.OA.6	Understand division as an unknown-factor problem. *For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.*
3.OA.7	Multiply and divide within 100. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of grade 3, know from memory all products of two 1-digit numbers.
3.NBT.2	Use place value understanding and properties of operations to perform multi-digit arithmetic. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Fourth Grade

4.OA.3	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
4.NBT.4	Use place value understanding and properties of operations to perform multi-digit arithmetic. Fluently add and subtract multi-digit whole numbers using the standard algorithm.
4.NBT.5	Multiply a whole number of up to four digits by a 1-digit whole number, and multiply two 2-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
4.NBT.6	Find whole-number quotients and remainders with up to 4-digit dividends and 1-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Fifth Grade

5.NBT.5	Perform operations with multi-digit whole numbers and with decimals to hundredths. Fluently multiply multi-digit whole numbers using the standard algorithm.
5.NBT.6	Find whole-number quotients of whole numbers with up to 4-digit dividends and 2-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
5.NBT.7	Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

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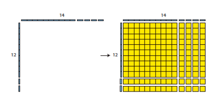
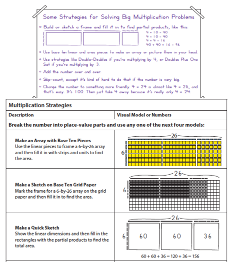
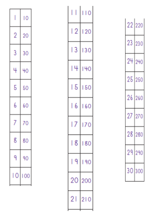
Multiplication Strategies

Models & Strategies for Multiplication





Building Ratio Tables



Work with a partner to build a ratio table with ***one*** of the following numbers to 20:

9, 12, 15, 20, 25, 34, 43, 52
13, 18, 27, 39, 48, 65, 72

This is part of a ratio table made by a fourth grade student.

3	45
4	60
5	75
6	90
7	105

What number was the student multiplying for this ratio table? ________

Raymond made a ratio table, but you can only see this part of it.

1	7
2	14
3	
	28
5	
	42

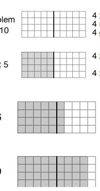
17	306
18	324
19	342
20	360

a What number did Raymond use to make his ratio table? ________

b What is the 10th row? ________

c What is the 22nd row? ________

Multiplying by Tens & Fives



Problem
4 x 10

4 x 10 = 40
4 rows of 10 = 40
4 groups of 10 is 40

4 x 5

4 x 5 is half of 4 x 10
4 x 5 is half of 40 so it's 20

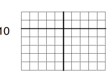
4 x 6

4 x 6 = 4 x 5 + 1 x 4
= 20 + 4 = 24

4 x 9

4 x 9 is 4 x 10 – 4 x 1
= 40 – 4 = 36

7 x 10



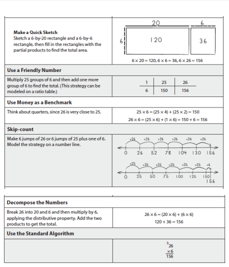
7 x 10 = 70
7 x 5 = 35
7 x 6 = 7 x 5
+ 1 x 5 = 42
7 x 9 = 7 x 10
– 1 x 7 =
70 – 7 = 63

Doubling & Halving

25 x 36 = (25 x 2) x (36 ÷ 2)
= 50 x 18
= (50 x 2) x (18 ÷ 2)
= 100 x 9

Five Is Half of Ten

1	36
10	360
5	180
20	720
25	900



NCTM's Principles to Action:

Hazards of a rush to fluency:

- Undermines student confidence
- Undermines interest
- Considered a cause of math anxiety
- Hinders sense making

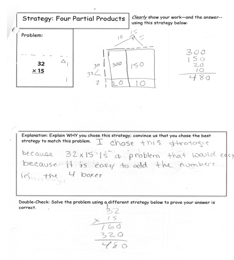
Strategies for Multiplication

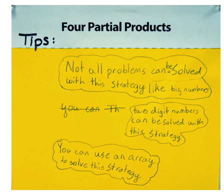
Cynthia Hockman-Chupp

NCTM's Principles to Actions:

Effective teaching of mathematics

- engages students in making connections among mathematical representations
- to deepen understanding of mathematical concepts and procedures
- and as tools for problem solving





NCTM's Principles to Actions:

- When procedures are connected to underlying concepts,
  - students have better retention of the procedures
  - students are more able to apply them in new situations.

NCTM's Principles to Actions:

Conceptual understanding

- comprehension and connection of concepts, operations and relations

establishes the foundation, and is necessary for developing procedural fluency

- meaningful and flexible use of procedures to solve problems