

## The Value of Pairing Pedagogical Hooks with Meaningful Learning

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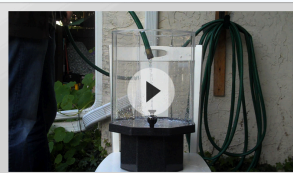
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## Question

- How might student conjecture be used to engage students and advance mathematics learning in a lesson?
- Question was part of a larger study that explored the relationship between teachers' mathematical goals for teaching and their understandings of the key mathematical ideas of lessons they taught.
- Conjecture
  - An educated guess, based on previous experience or learning.

## Regarding conjecture

- An Example (Dan Meyer's Three Acts Math)



1. How long will it take the tank to fill up?
2. Guess as close as you can.
3. Give an answer you know is too high.
4. Give an answer you know is too low.

(Source: <http://mrmeier.com/threeracts/watertank/>)

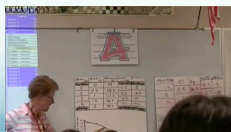
## Regarding Lesson Planning

- Lesson Planning and Debrief Protocols Developed
  - Influenced by
    - Thinking through a Lesson Protocol (Smith, Bill, Hughes)
    - Professional Developmental Spiral (Thompson)
    - Pathways Professional Development Workshops (Carlson, Oehrtman, Moore; Carlson, O'Bryan, Joyner)

## Methods

### Profile of this study

- Carolyn at Atlas High School (Fall 2014)
  - Experienced teacher (20 years)
  - Used a conceptual Precalculus curriculum (*Pathways*)
  - Teaching third time with conceptual curriculum
  - Attended professional development trainings



## Methods

### Study

- Lesson planning sessions
  - Lesson Preflection Questionnaire
  - Classroom Observations
- Lesson debrief sessions
  - Lesson Debrief Questionnaire
- Post Study Interview

**LESSON PREFLECTION QUESTIONNAIRE**

To use when thinking about an initially planned lesson

1. What are the primary ideas being developed in this lesson?
  - What are the key ideas of this lesson that you find important?
2. What are 3 other questions you think will be useful to pose to your students as they complete this investigation?
  - What ways of thinking do you hoping emerges from these interactions?
3. What are your mathematical goals for student learning as you plan to teach this lesson?
  - Are there other goals you have for this lesson?
  - What criteria will you use to see your goals (for student learning, teaching, ...)

**LESSON DEBRIEF QUESTIONNAIRE**

To use when reflecting on a lesson that was recently taught

1. How do you think today's lesson went?
2. Was there something about the lesson or what a student said in class today that you found noteworthy, interesting, or surprising?
  - Are there other goals you had for today's lesson?
3. What were your goals for student learning in the lesson you led today?
  - What methods did you envision students would use?
  - What ways of thinking did you hope emerge?
4. How might the understandings that are suggested by your goals develop or be supported for students?

## Question

- How might student conjecture be used to engage students and advance mathematics learning in a lesson?
- Curriculum already established.
  - Conceptual curriculum contained rich tasks that promoted quantitative reasoning and sense meaning
  - Supportive of eight Standards for Mathematical Practice
- Lesson Collaboration resulted in refinements to existing lessons.
  - One of these possible avenues for refinement included a mini-task promoting student conjecture used as a segue into the key ideas of the primary task.

## An example

- Original mathematics task from conceptual curriculum:

Hannah, Kristi, and Jolene discover a secret that no one else knows. On Day 1 each of them tells two other people. After that, everyone who learns the secret on a given day tells the secret to two new people the next day. This pattern continued for 15 days.



- Later in same lesson....

| $x$ = the number of days | $y$ = the number of people who learned the secret on day $x$ |
|--------------------------|--|
| 1                        | 3  |
| 2                        | 6  |
| 3                        | 12   |
| 4                        | 24   |

- Define a function  $f$  that determines the number of people  $P$  who learn the secret on day  $x$ .
- Use the function defined in part (a) to determine the number of people who learn the secret on day 15.

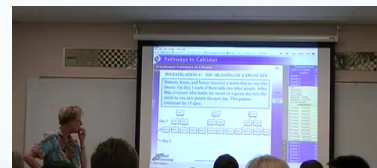
## Staging the conjecture

| $x$ = the number of days | $y$ = the number of people who learned the secret on day $x$ |
|--------------------------|--|
| 1                        | 3  |
| 2                        | 6  |
| 3                        | 12   |
| 4                        | 24   |



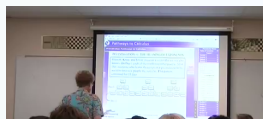
The question that would be asked to the students: If this pattern continues, how many days do you predict it will take for everyone in the United States to **know** the secret?

## Class implementation

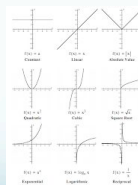


Students asked to Google the information of U.S. population with their cellphones. Class consensus is 314 million people in the United States.

## Conjectures creates new opportunities for discussion

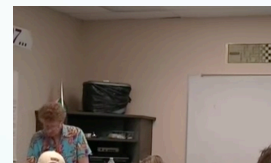


(Posters on back wall)  
Meaning of "up there"  
during discussion.



A rich discussion begins in comparing/contrasting linear and exponential functions.

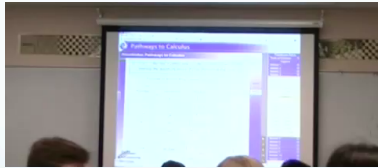
## Main lesson on exponential functions continues...



| $x$ = the day number | $y$ = the number of people who learned the secret on day $x$ |
|----------------------|--|
| 1                    | 3  |
| 2                    | 6  |
| 3                    | 12   |
| 4                    | 24   |

Students filled out a table that describes the pattern of people who **learn** the secret for the first 15 days. They use the pattern to write a function that describes the pattern in the table.

## Conjecture Resolved Meaningfully for class



The reasoning used in this interaction was that if 314 million people **knew** the secret, half of people (157 million) had **learned** the secret. Then Carolyn asks students to use the model to determine the numbers of days it takes.

Some students used guess and check, some continued the table, one used a graphing method (finding when  $y_1 = 3 \cdot 2^x$  intersects  $y_2 = 157,000,000$ ). More rich discussion.

## What makes an effective pedagogical hook?

- Piques interest and engages students.
- Creates an intellectual need.
- Relates to the key ideas of the mathematical task.
- Is resolved meaningfully.

## A note about collaboration

- Thoughts on collaboration and lesson planning.



Working and planning together fostered additional opportunities for students to engage in rich mathematical tasks. These opportunities emerged as a result of this collaboration.

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## Thank you for coming!

Questions?

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