Modeling Curiosity:
From Hmmm to Ohhhh I See

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High School: Modeling Standard

Mathematical models describe situations in the world, to the surprise of many. Albert Einstein wondered, "How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality?" This points to the basic reason to model with mathematics and statistics: to understand reality.

Modeling and the Standards for Mathematical Practice

One of the eight mathematical practice standards—MP4 Model with mathematics—focuses on modeling and modeling draws on and develops all eight. This helps explain why modeling with mathematics and statistics is challenging. It is a capstone experience, the proof of the pudding.

Curiosity – Questioning - Modeling

Applied Mathematics = Science

???

Applied mathematics focuses on the creation and study of mathematical and computational tools broadly applicable in science and engineering (harvard.edu)

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2. Developing and using models

3. Planning and carrying out investigations

4. Analyzing and interpreting data

5. Using mathematics and computational thinking

6. Constructing explanations (for science) and designing solutions (for engineering)

7. Engaging in argument from evidence

8. Obtaining, evaluating, and communicating information

Curiosity

- Modeling starts with curiosity about the world
- It begins with questions
  - Open questions
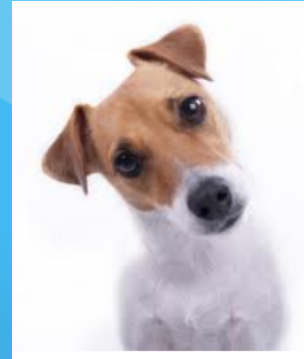 - Closed questions
  - Testable questions

Questions are guaranteed in life; Answers aren't.

As we struggle to figure out how this fountain works, take note of:

1. The interplay between observation and inference

2. How do analogies play a role in generating inferences?

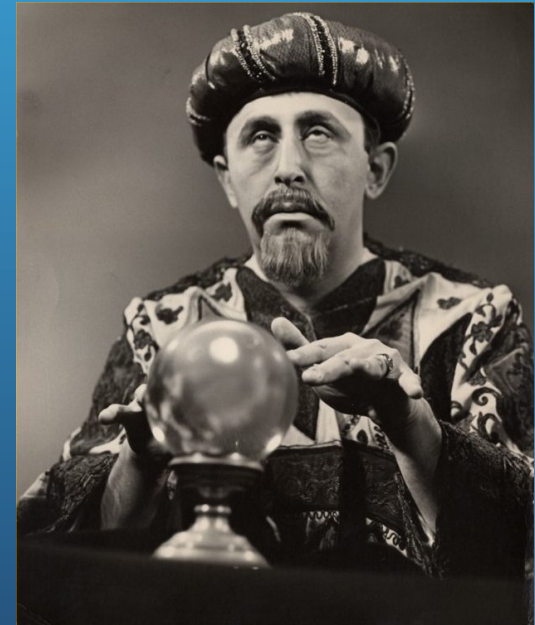3. Are the questions that are generated testable?
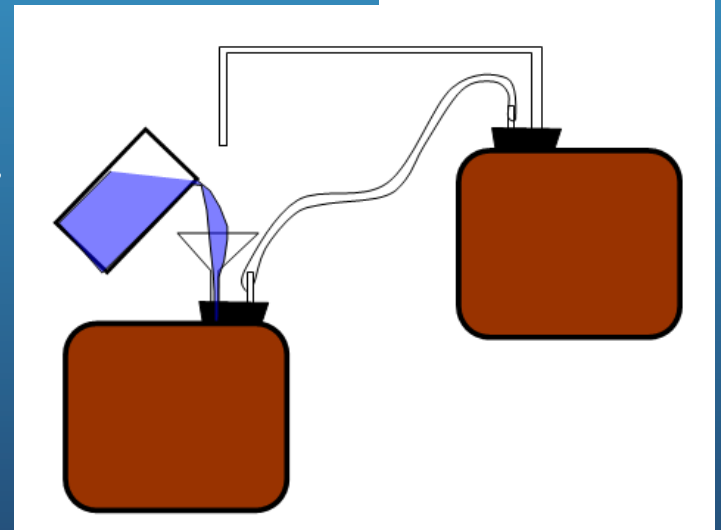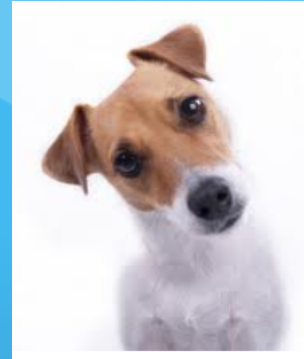
ARE WE DONE YET?

Verify understanding:

What predictions can be posed <u>and tested</u> regarding this system?
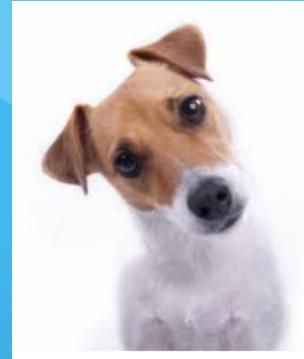
ARE WE DONE YET?

Verify understanding:

What predictions can be posed and tested regarding this system?

What questions remain unanswered?

ARE WE DONE YET?

If a man will begin with certainties, he will end in doubts; but if he will be content to begin with doubts, he will end in certainties.
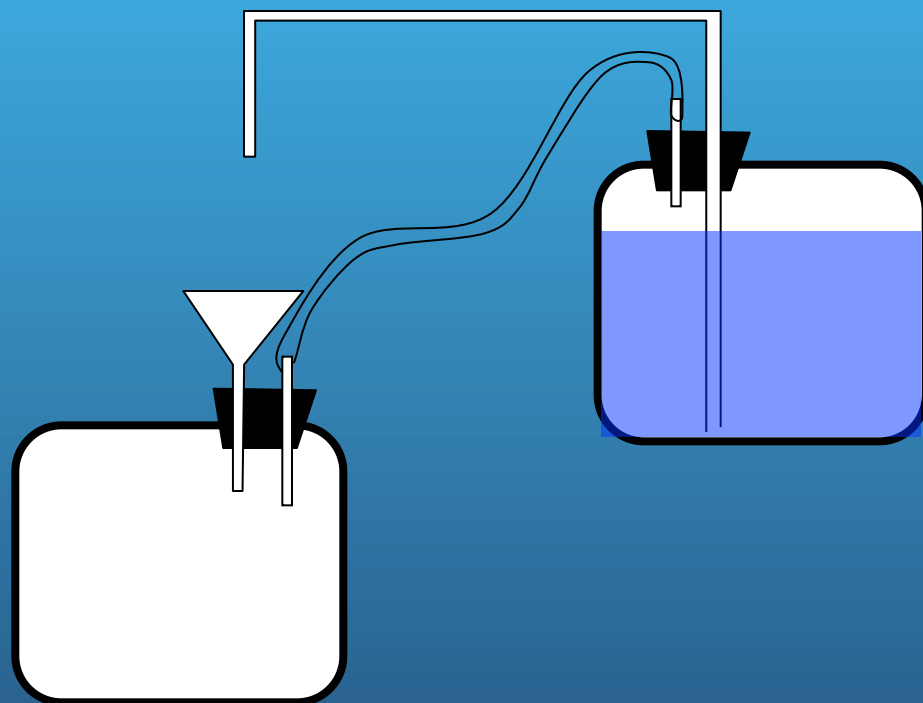-Francis Bacon (1601)

I have yet to see any problem, however complicated, which, when you looked at it the right way, did not become still more complicated.
-- Paul Alderson (1926)

Initial State

As water is poured into bottom tank, air is displaced through hose to top tank...
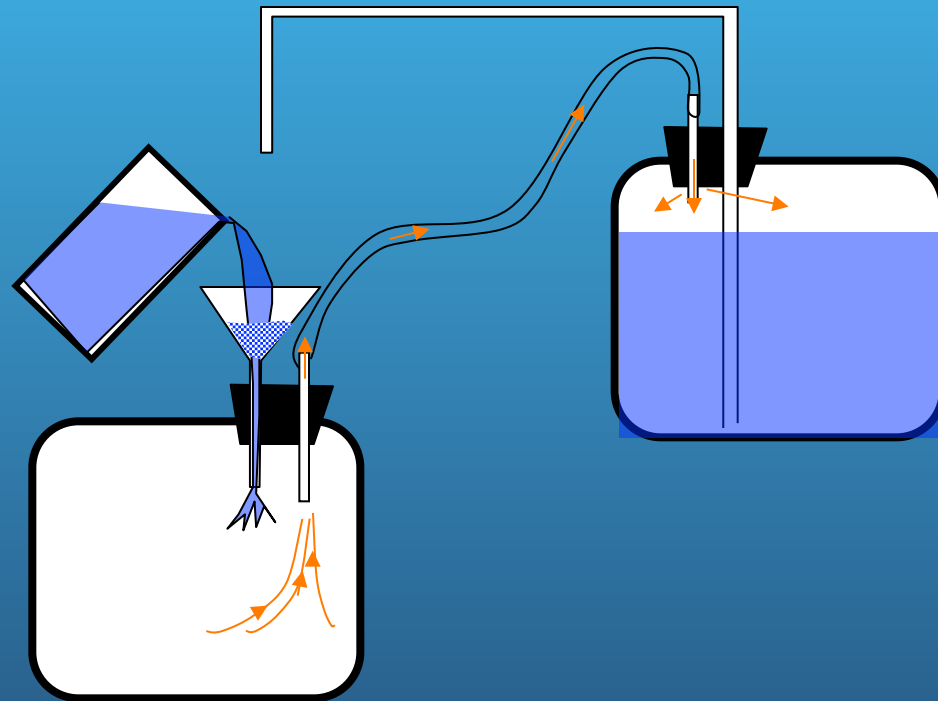
Pressure of incoming air forces water out of the top tank
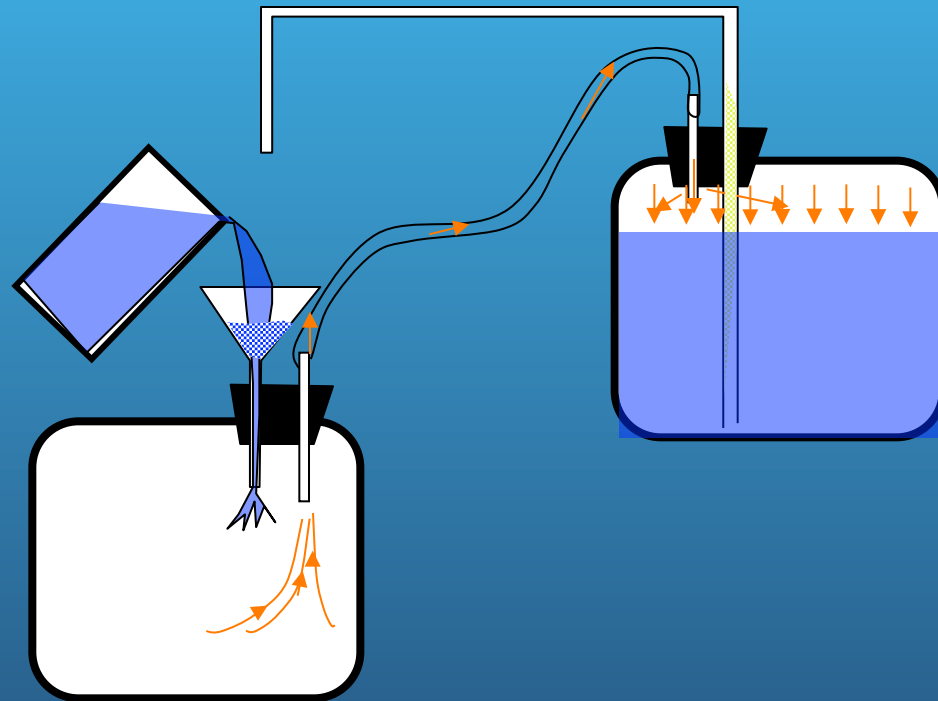
Pressure of incoming air forces water out of the top tank
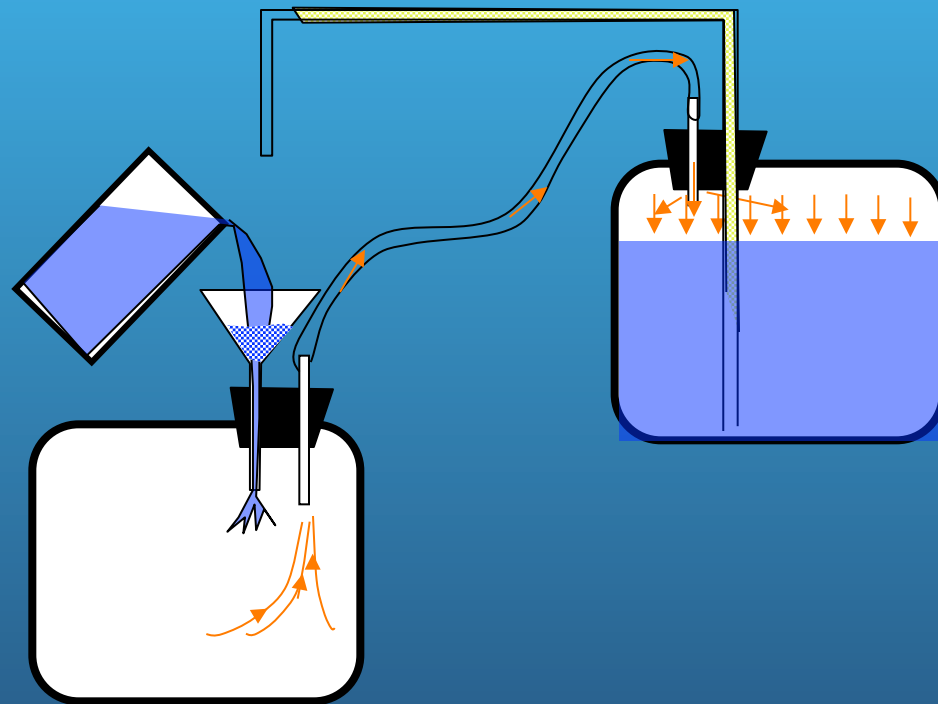
Water from top tank replenishes funnel
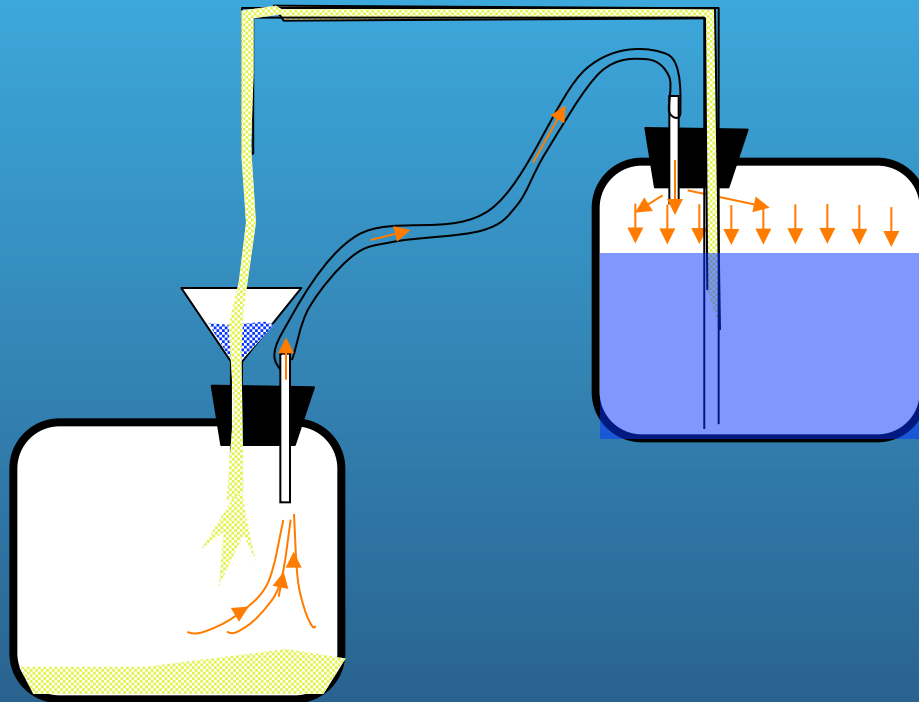
Water is transferred to bottom tank,
Air is transferred to top tank
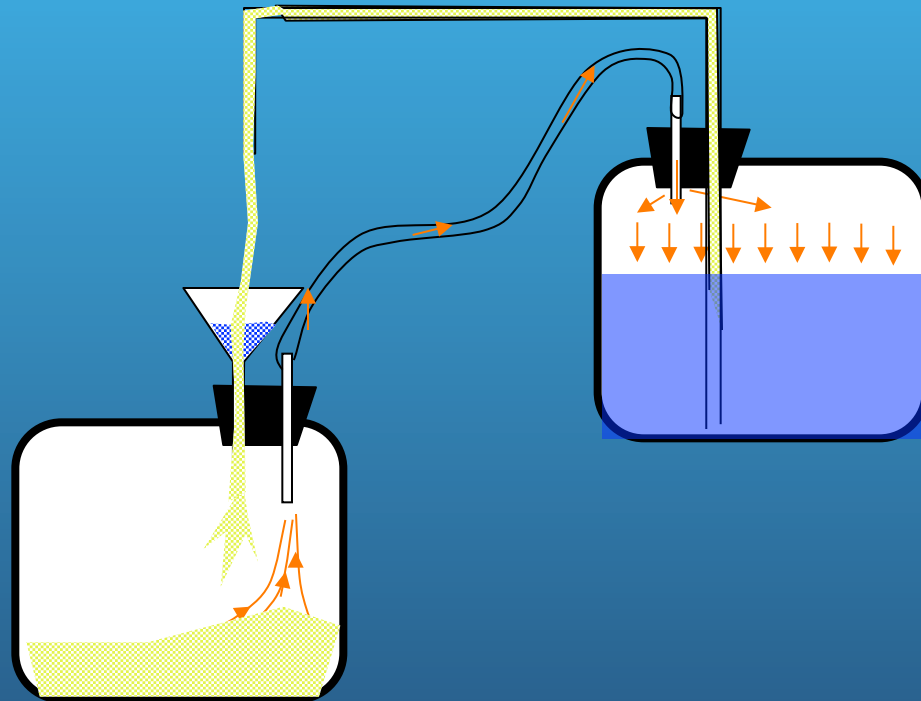
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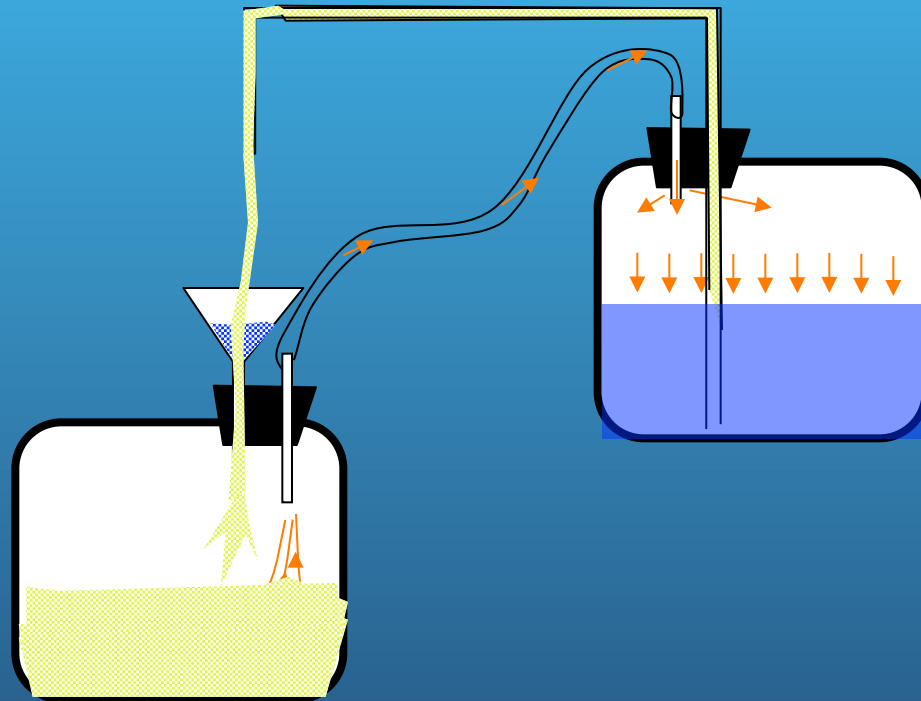
Water is transferred to bottom tank, Air is transferred to top tank
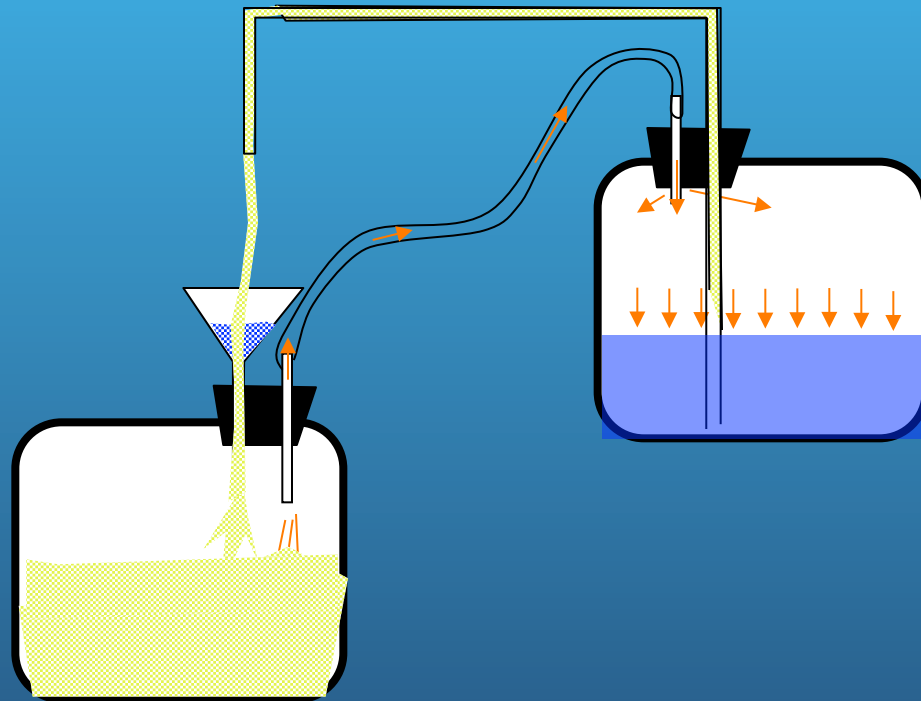
When water reaches the tube in the bottom stopper, flow of gas stops and fountain stops.
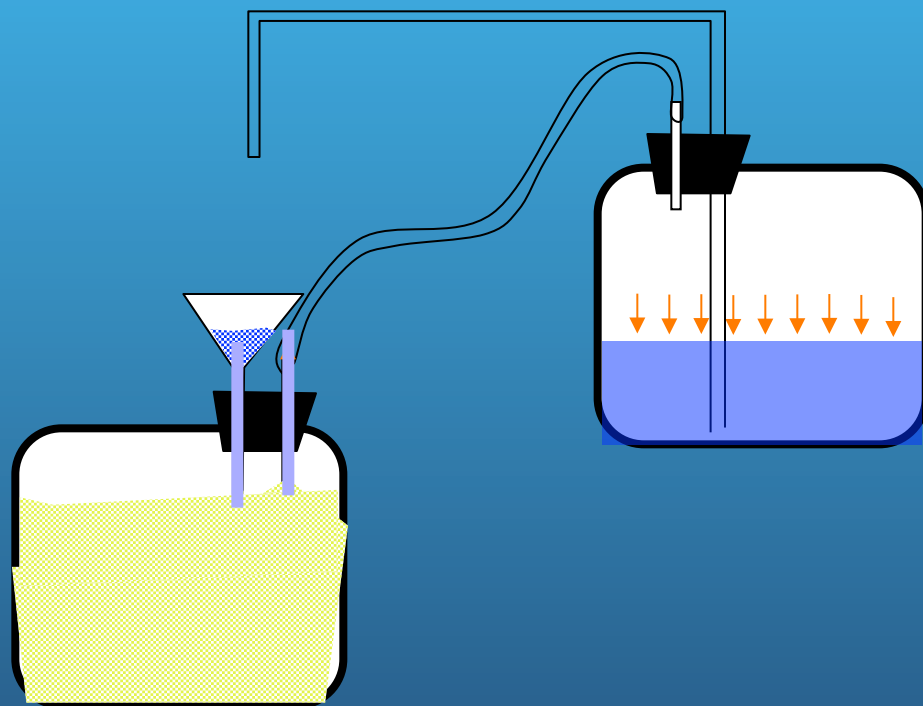
Concept Map of Gas Tank Fountain
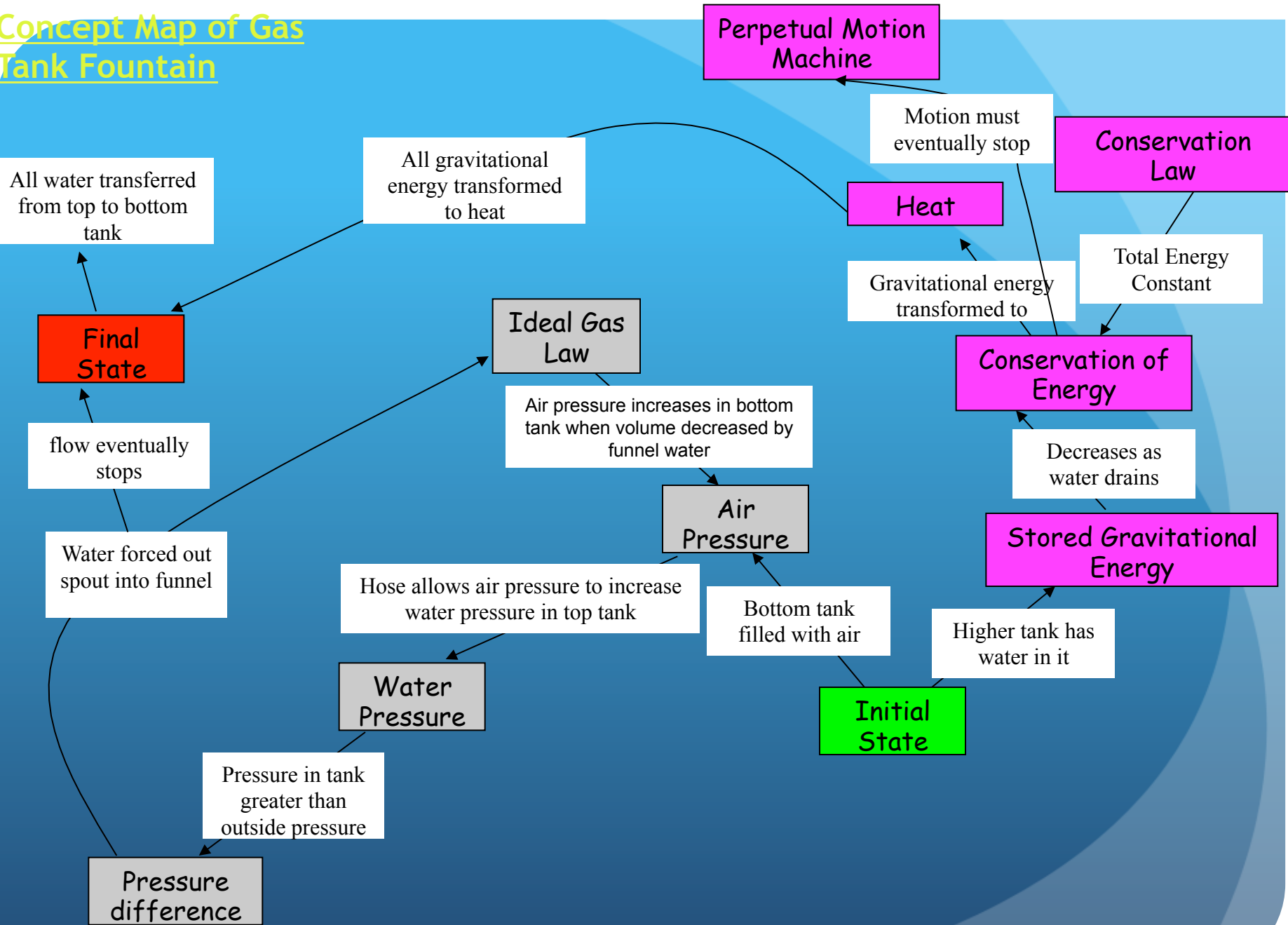
Perpetual Motion Machine

Conservation Law

Motion must eventually stop

All gravitational energy transformed to heat

Heat

Total Energy Constant

All water transferred from top to bottom tank

Gravitational energy transformed to

Conservation of Energy

Final State

Ideal Gas Law

Air pressure increases in bottom tank when volume decreased by funnel water

Decreases as water drains

flow eventually stops

Stored Gravitational Energy

Water forced out spout into funnel

Air Pressure

Hose allows air pressure to increase water pressure in top tank

Bottom tank filled with air

Higher tank has water in it

Water Pressure

Initial State

Pressure in tank greater than outside pressure

Pressure difference

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High School: Modeling Standard

Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions.

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Practice 3
Planning and Carrying Out Investigations

Students should have opportunities to plan and carry out several different kinds of investigations during their K-12 years. At all levels, they should engage in investigations that range from those structured by the teacher—in order to expose an issue or question that they would be unlikely to explore on their own (e.g., measuring specific properties of materials) – to those that emerge from students' own questions. (NRC Framework, 2012, p. 61)

Let's Try It!

https://www.vernier.com/products/sensors/

There are many Vernier probes spread out around the room. Please explore them and discuss how they can be used to collect data. What would the independent and dependent variables be? What kind of questions could be framed? What mathematics and/or statistics would, therefore, be appropriate to utilize to answer those questions with evidence?

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Practice 4
Analyzing and Interpreting Data

Once collected, data must be presented in a form that can reveal any patterns and relationships and that allows results to be communicated to others. Because raw data as such have little meaning, a major practice of scientists is to organize and interpret data through tabulating, graphing, or statistical analysis.

High School: Statistics & Probability

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread.

Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Mathematics Standards

Build a function that models a relationship between two quantities.

CCSS.Math.Content.HSF.BF.A.1
Write a function that describes a relationship between two quantities.[*]

Interpret functions that arise in applications in terms of the context.

CCSS.Math.Content.HSF.IF.B.4
For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*[*]

CCSS.Math.Content.HSA.CED.A.2
Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

CCSS.Math.Content.HSF.LE.A.1.b
Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

CCSS.Math.Content.HSF.LE.A.1.c
Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

CCSS.Math.Content.HSF.LE.B.5
Interpret the parameters in a linear or exponential function in terms of a context.

CCSS.Math.Content.HSF.TF.B.5
Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.[*]

High School: Modeling Standard

The basic modeling cycle involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them.

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Modeling with Mathematics to better understand the science of our world

http://www.nextgenscience.org/sites/default/files/ Appendix F Science and Engineering Practices in the NGSS-FINAL060513.pdf

http://www.nextgenscience.org/sites/default/files/ Appendix-L_CCSS Math Connections 06_03_13.pdf

http://commoncoretools.me/wp-content/uploads/ 2013/07/ccss_progression_modeling_2013_07_04.pdf

Discussion?

More examples…

Curiosity – Questioning – Modeling and Statistics

Things that make you say hmmm

Connections between Mathematical Modeling and Statistical Analyses – Matching Methods to Questions

Use existing data sets – ask testable questions that lend to Modeling and to Hypothesis Testing (we have interest in further developing classroom research across the globe)

Things that make you say hmmmm

What you or your students might be curious about...

Testable Research Questions

In the form of Null and Alternative Hypotheses

Common Hypothesis Tests

t-tests (1 sample, 2 sample paired and unpaired, ANOVA, Chi-Square)

Memorathon!

Investigating Quantitative Data via Hypothesis Testing and Mathematical Modeling

The questions you can ask will be constrained by your curiosity and what data you can find. Your ability to understand contexts where quantitative data can be found and analyzed should improve as you do so.

http://www.ncdc.noaa.gov/data-access
https://www.data.gov/
http://www.healthdata.gov/dataset/search
http://datahub.io/dataset
http://www.eia.gov/countries/data.cfm
http://data.nasa.gov/
http://earthexplorer.usgs.gov/
http://water.usgs.gov/data/