Strategies and Tasks to Build Procedural Fluency from Conceptual Understanding

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2016 NCTM Phoenix Regional Conference
October 27, 2016

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FYI

Electronic copies of slides are available
by request
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and will be posted at
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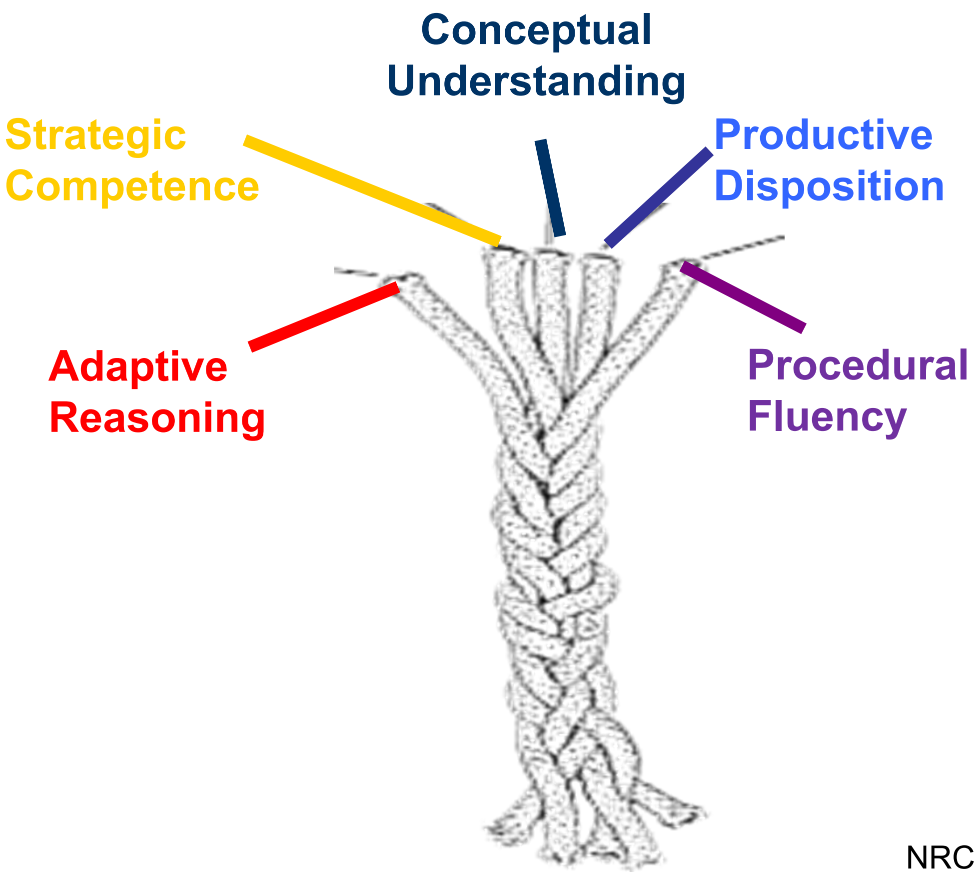
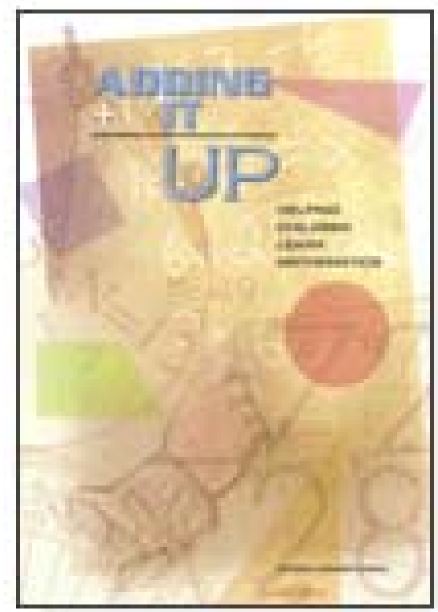
Key Questions

- What is procedural fluency?
- What instructional practices promote students' development of procedural fluency?

Strands of Mathematical Proficiency





NRC (2001). *Adding It Up*. Washington, D.C.: National Academies Press.

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Key Features of CCSS-M

- **Focus:** Focus strongly where the standards focus.
- **Coherence**: Think across grades, and link to major topics
- **Rigor:** In major topics, pursue conceptual understanding, procedural skill and fluency, and application
- **Standards for Mathematical Practice**

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- **Standards for Mathematical Practice**

Discuss with a Shoulder Partner

What does it mean to be fluent with procedures?

What is Procedural Fluency?

Which students demonstrate procedural fluency? Evidence for your answer?

- Alan
- Ana
- Marissa

Alan

1000 - 98

100 - 18

https://mathreasoninginventory.com/Home/VideoLibrary

Source: The Marilyn Burns Math Reasoning Inventory

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Ana

1000 - 98 | **99 + 17**

https://mathreasoninginventory.com/Home/VideoLibrary

Source: The Marilyn Burns Math Reasoning Inventory

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What is Procedural Fluency?

Which students demonstrate procedural fluency? Evidence for your answer?

- Alan
- Ana

Procedural Fluency

- **Efficiency**—can carry out easily, keep track of subproblems, and make use of intermediate results to solve the problem.
- **Accuracy**—reliably produces the correct answer.
- **Flexibility**—knows more than one approach, chooses appropriate strategy, and can use one method to solve and another method to double-check.

NCTM, 2014; Russell, 2000

Marissa

295 students, 25 on each bus

Source: The Marilyn Burns Math Reasoning Inventory

Procedural Fluency

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- **Appropriately**—knows when to apply a particular procedure.

Adapted from NCTM, 2014; Russell, 2000

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Procedural Fluency Beyond Whole Number Computation

Order each set of fractions from least to greatest.

What strategies did you use?

1. $\frac{1}{8} \quad \frac{1}{4} \quad \frac{1}{5}$

2. $\frac{3}{5} \quad \frac{3}{4} \quad \frac{3}{8}$

3. $\frac{3}{12} \quad \frac{1}{12} \quad \frac{8}{12}$

4. $\frac{7}{8} \quad \frac{3}{4} \quad \frac{4}{5}$

5. $\frac{17}{25} \quad \frac{3}{10} \quad \frac{4}{8}$

Comparing & Ordering Fractions

- Common denominators
- Common numerators
- Benchmark fractions, e.g. 1, ½
- Other equivalent representations, e.g., decimals, percents

Procedural Fluency Beyond Whole Number Computation

Solve for *x*.

$$3(x + 5) = 45$$

$$\begin{aligned} 3x + 15 &= 45 \\ 3x &= 30 \\ x &= 10 \end{aligned}$$

$$\begin{aligned} x + 5 &= 15 \\ x &= 10 \end{aligned}$$

Procedural Fluency

- **Efficiency**—can carry out easily, keep track of subproblems, and make use of intermediate results to solve the problem.
- **Accuracy**—reliably produces the correct answer.
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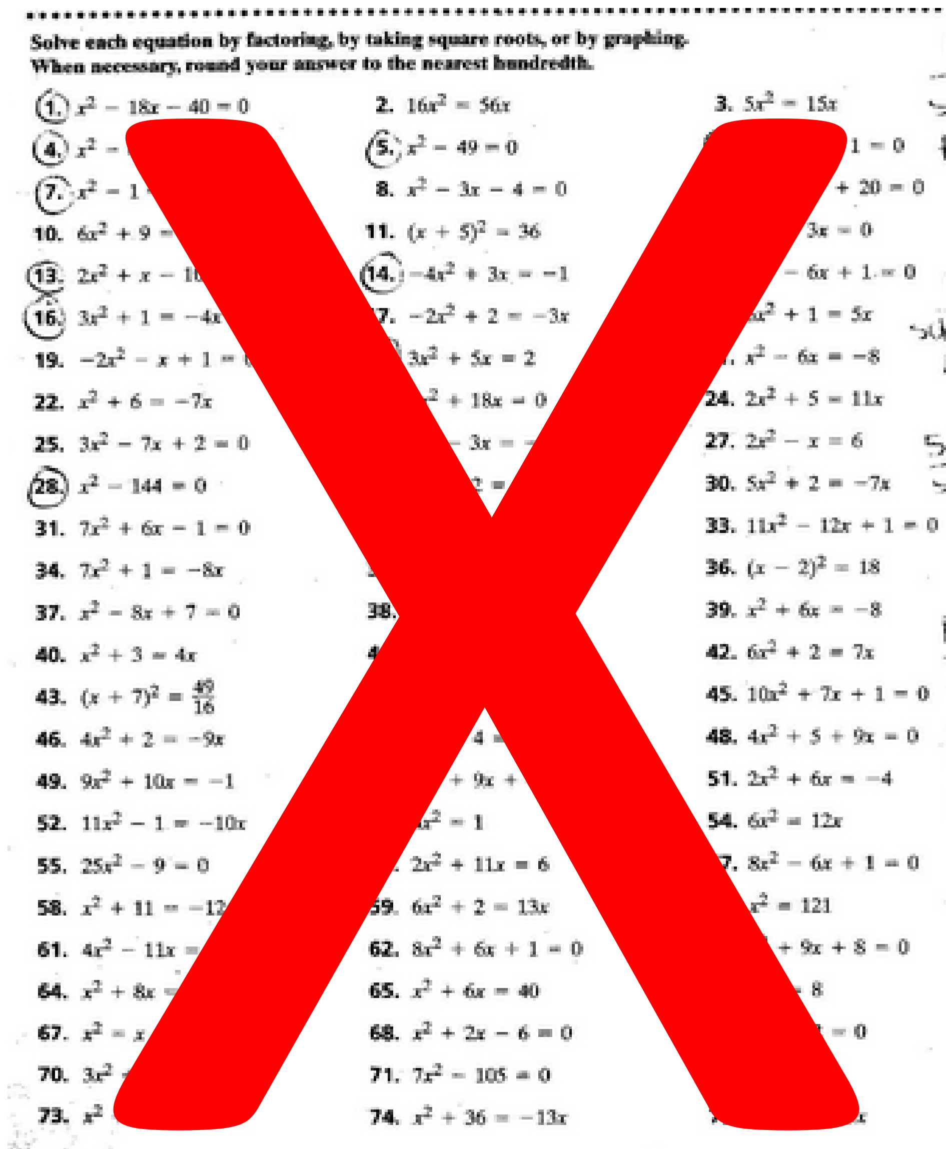
- **Appropriately**—knows when to apply a particular procedure.

How Can We Develop Students' Proficiency?



Solve each equation by factoring, by taking square roots, or by graphing. When necessary, round your answer to the nearest hundredth.

1. $x^2 - 18x - 40 = 0$
2. $16x^2 = 56x$
3. $5x^2 = 15x$
5. $x^2 - 49 = 0$
8. $x^2 - 3x - 4 = 0$
11. $(x + 5)^2 = 36$
14. $-4x^2 + 3x = -1$
16. $3x^2 + 1 = -4x$
19. $-2x^2 - x + 1 = 0$
22. $x^2 + 6 = -7x$
24. $2x^2 + 5 = 11x$
25. $3x^2 - 7x + 2 = 0$
27. $2x^2 - x = 6$
28. $x^2 - 144 = 0$
30. $5x^2 + 2 = -7x$
31. $7x^2 + 6x - 1 = 0$
33. $11x^2 - 12x + 1 = 0$
34. $7x^2 + 1 = -8x$
36. $(x - 2)^2 = 18$
37. $x^2 - 8x + 7 = 0$
39. $x^2 + 6x = -8$
40. $x^2 + 3 = 4x$
42. $6x^2 + 2 = 7x$
43. $(x + 7)^2 = \frac{49}{16}$
45. $10x^2 + 7x + 1 = 0$
46. $4x^2 + 2 = -9x$
48. $4x^2 + 5 + 9x = 0$
49. $9x^2 + 10x = -1$
51. $2x^2 + 6x = -4$
52. $11x^2 - 1 = -10x$
54. $6x^2 = 12x$
55. $25x^2 - 9 = 0$
57. $8x^2 - 6x + 1 = 0$
59. $6x^2 + 2 = 13x$
62. $8x^2 + 6x + 1 = 0$
65. $x^2 + 6x = 40$
68. $x^2 + 2x - 6 = 0$
71. $7x^2 - 105 = 0$
74. $x^2 + 36 = -13x$

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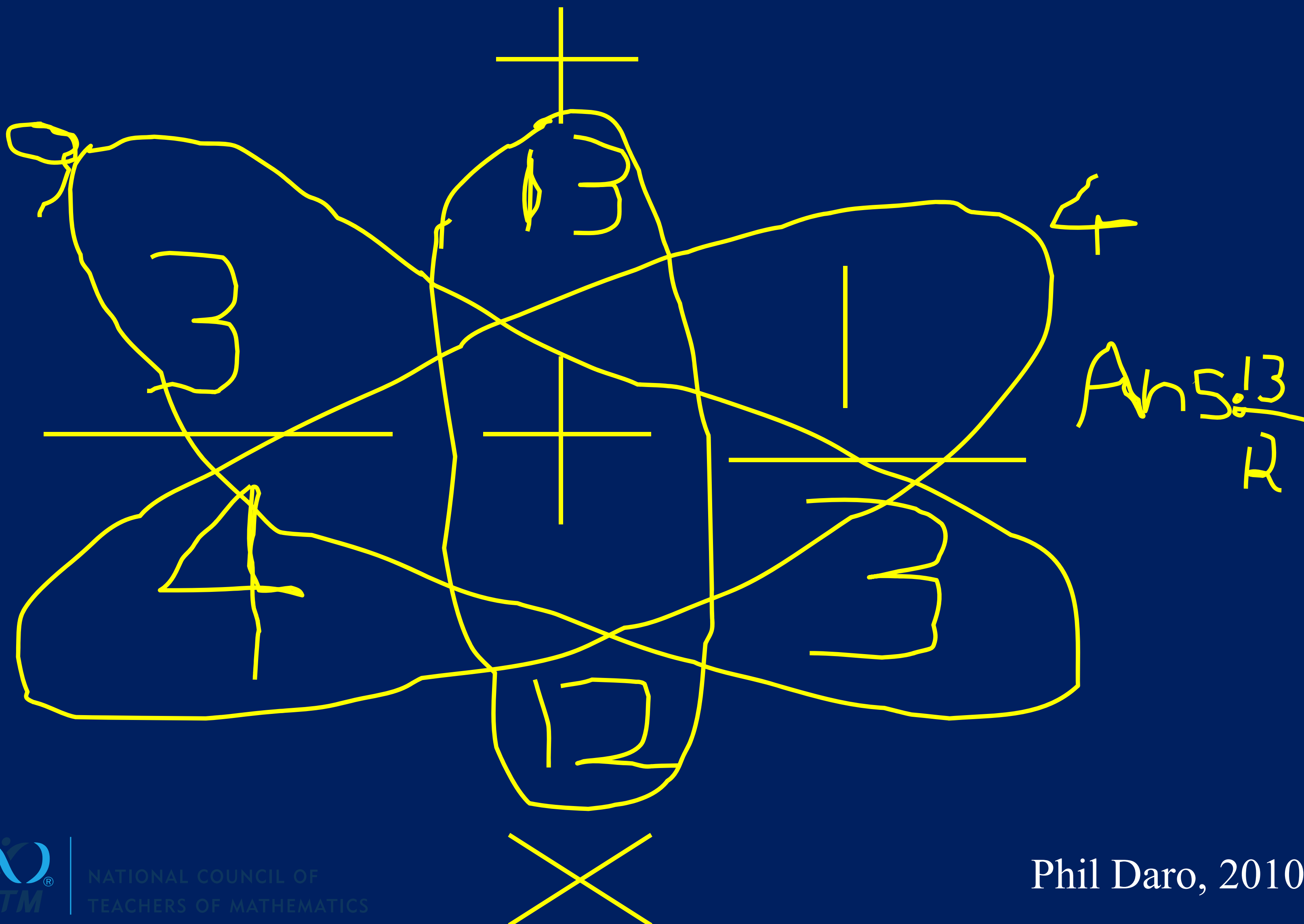
$$\frac{3}{4} + \frac{1}{3}$$

Phil Daro, 2010



Phil Daro, 2010

Other "Butterflies"?

Other “Butterflies”?

- FOIL
- Cross multiplication
- Fraction division:
  - Keep-change-flip, KFC,
  - Yours is not to reason why, just invert and multiply
- Integer subtraction: Keep-change-change $(a - b = a + {}^{-}b)$
- Long Division:
  - Dad, Mother, Sister, Brother, Rover
  - Does McDonalds Sell Cheese Burgers?
- Key words

Key Instructional Shift

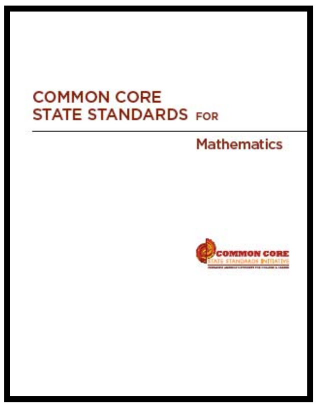
From emphasis on:

How to get answers

To emphasis on:

Understanding mathematics

High Quality Standards Are Necessary, but Insufficient, for Effective Teaching and Learning



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Principles to Actions: Ensuring Mathematics Success for All

Guiding Principles for School Mathematics

1. **Teaching and Learning**
2. **Access and Equity**
3. **Curriculum**
4. **Tools and Technology**
5. **Assessment**
6. **Professionalism**

→ Essential Elements of Effective Math Programs

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Teaching and Learning Principle

An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically.

Principles to Actions: Ensuring Mathematical Success for All

Effective **Mathematics Teaching Practices**

1. Establish mathematics **goals** to focus learning.
2. Implement **tasks** that promote reasoning and problem solving.
3. Use and connect mathematical **representations.**
4. Facilitate meaningful mathematical **discourse**.
5. Pose purposeful **questions**.
6. Build **procedural fluency** from conceptual understanding.
7. Support **productive struggle** in learning mathematics.
8. **Elicit and use evidence** of student thinking.

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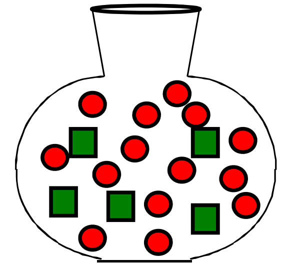
What Research Tells Us

- When procedures are connected with the underlying concepts, students have better retention of the procedures and are more able to apply them in new situations
- Informal methods → general methods → formal algorithms is more effective than rote instruction.
- Engaging students in solving challenging problems is essential to build conceptual understanding.

Candy Jar Problem

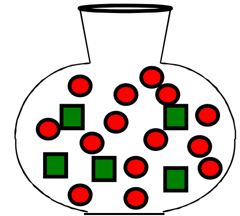


A candy jar contains 5 Jolly Ranchers (squares) and 13 Jawbreakers (circles). Suppose you had a new candy jar with the same ratio of Jolly Ranchers to Jawbreakers, but it contained 100 Jolly Ranchers. How many Jawbreakers would you have? Explain how you know.

- Please work this problem as if you were a seventh grader.
- When done, share your work with a neighbor.

Principles to Actions, NCTM, 2014.

Candy Jar Problem



Group 1 (incorrect, additive)

100 JRs is 95 more than the 5 we started with. So we will need 95 more JBs than the 13 we started with.

5 JRs + 95 JRs = 100 JRs
13 JBs + 95 JBs = 108 JBs

Groups 3 and 5 (scale factor)

You had to multiply the five JRs by 20 to get 100, so you'd also have to multiply the 13 JBs by 20 to get 260.

$$\begin{array}{ccc} & (\times 20) & \\ 5 \text{ JRs} & \longrightarrow & 100 \text{ JRs} \\ 13 \text{ JBs} & \longrightarrow & 260 \text{ JBs} \\ & (\times 20) & \end{array}$$

Groups 4 and 7 (scaling up)

JR	JB	JR	JB
5	13	55	143
10	26	60	156
15	39	65	169
20	52	70	182
25	65	75	195
30	78	80	208
35	91	85	221
40	104	90	234
45	117	95	247
50	130	100	260

Group 2 (unit rate)

Since the ratio is 5 JRs for 13 JBs, we divided 13 by 5 and got 2.6. So that would mean that for every 1 JR there are 2.6 JBs. So then you just multiply 2.6 by 100.

$$\begin{array}{ccc} & (\times 100) & \\ 1 \text{ JR} & \longrightarrow & 100 \text{ JRs} \\ 2.6 \text{ JBs} & \longrightarrow & 260 \text{ JBs} \\ & (\times 100) & \end{array}$$

Group 6 (scaling up)

JRs	5	10	20	40	80	100
JBs	13	26	52	104	208	260

We started by doubling both the number of JRs and JBs. But then when we got to 80 JRs, we didn't want to double it anymore because we wanted to end up at 100 JRs, and doubling 80 would give us too many. So we noticed that if we added 20 JRs:52 JBs and 80 JRs:208 JBs, we would get 100 JRs:260 JBs.

Group 8 (scaling up)

We drew 100 JRs in groups of 5. Then we put 13 JBs with each group of 5 JRs. We then counted the number of JBs and found we had used 260 of them.

Principles to Actions, NCTM, 2014.

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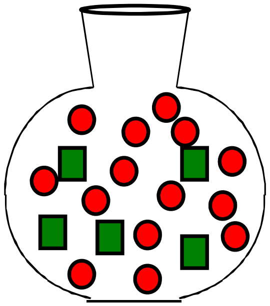
Procedural Fluency

Extending learning from the original Candy Jar Task:

$$\frac{5}{13} = \frac{127}{x}$$

Principles to Actions, NCTM, 2014.

Procedural Fluency



$$\frac{5}{13} = \frac{127}{x}$$

$13 \div 5 = 2.6.$

$2.6 \cdot 127 = 330.2$

$\frac{5}{13} = \frac{127}{330.2}$

because

$127 \div = 25.4$

$330.2 \div 13 = 25.4$

×25.4

JR	5	10	15	20	127
JB	13	26	39	52	330.2

×25.4

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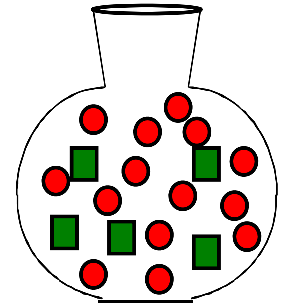
Procedural Fluency



$$\frac{5}{13} = \frac{127}{x}$$

Unit Rate

$13 \div 5 = 2.6.$
$2.6 \cdot 127 = 330.2$

Scale Factor

$$\frac{5}{13} = \frac{127}{330.2}$$

because
$127 \div = 25.4$
$330.2 \div 13 = 25.4$

Scaling Up

×25.4

JR	5	10	15	20	127
JB	13	26	39	52	330.2

×25.4

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Building Procedural Fluency

<u>Finding the Missing Value</u>

Find the value of the unknown in each of the proportions shown below.

$$\frac{5}{2} = \frac{y}{10} \qquad \frac{a}{24} = \frac{7}{8}$$

$$\frac{n}{8} = \frac{3}{12} \qquad \frac{30}{6} = \frac{b}{7}$$

$$\frac{5}{20} = \frac{3}{d} \qquad \frac{3}{x} = \frac{4}{28}$$

What might we expect students to be able to do when presented with a missing value problem, after they have had the opportunity to develop a set of strategies through solving a variety of contextual problems like the Candy Jar Task?

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Developing Procedural Fluency

1. Develop conceptual understanding building on students’ informal knowledge
2. Develop informal strategies to solve problems
3. Refine informal strategies to develop fluency with standard methods and procedures (algorithms)

Effective
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Implement Tasks that Promote Reasoning and Problem Solving

Mathematical tasks should:

- Provide opportunities for students to engage in exploration or encourage students to use procedures in ways that are connected to concepts and understanding;
- Build on students’ current understanding; and
- Have multiple entry points.

Building Procedural Fluency: Dividing Fractions

Problem 4.1 Dividing a Whole Number by a Fraction

Use written explanations or diagrams to show your reasoning for each part. Write a number sentence showing your calculation(s).

A. Naylah plans to make small cheese pizzas to sell at a school fundraiser. She has nine bars of cheese. How many pizzas can she make if each pizza needs the given amount of cheese?

1. $\frac{1}{3}$ bar **2.** $\frac{1}{4}$ bar **3.** $\frac{1}{5}$ bar

4. $\frac{1}{6}$ bar **5.** $\frac{1}{7}$ bar **6.** $\frac{1}{8}$ bar

B. Frank also has nine bars of cheese. How many pizzas can he make if each pizza needs the given amount of cheese?

1. $\frac{1}{3}$ bar **2.** $\frac{2}{3}$ bar **3.** $\frac{3}{3}$ bar **4.** $\frac{4}{3}$ bar

5. The answer to part (2) is a mixed number. What does the fractional part of the answer mean?

Building Procedural Fluency: Dividing Fractions

C. Use what you learned from Questions A and B to complete the following calculations.

1. $12 \div \frac{1}{3}$ **2.** $12 \div \frac{2}{3}$ **3.** $12 \div \frac{5}{3}$

4. $12 \div \frac{1}{6}$ **5.** $12 \div \frac{5}{6}$ **6.** $12 \div \frac{7}{6}$

7. The answer to part (3) is a mixed number. What does the fractional part of the answer mean in the context of cheese pizzas?

D. **1.** Explain why $8 \div \frac{1}{3} = 24$ and $8 \div \frac{2}{3} = 12$.

2. Why is the answer to $8 \div \frac{2}{3}$ exactly half the answer to $8 \div \frac{1}{3}$?

E. Write an algorithm that seems to make sense for dividing any whole number by any fraction.

F. Write a story problem that can be solved using $12 \div \frac{2}{3}$. Explain why the calculation matches the story.

The Band Concert

Make a quick sketch or diagram of the situation.

How might Grade 3 students approach this task?

The third-grade class is responsible for setting up the chairs for their spring band concert. In preparation, they need to determine the total number of chairs that will be needed and ask the school's engineer to retrieve that many chairs from the central storage area.

The class needs to set up 7 rows of chairs with 20 chairs in each row, leaving space for a center aisle.

How many chairs does the school's engineer need to retrieve from the central storage area?

Principles to Actions, NCTM, 2014

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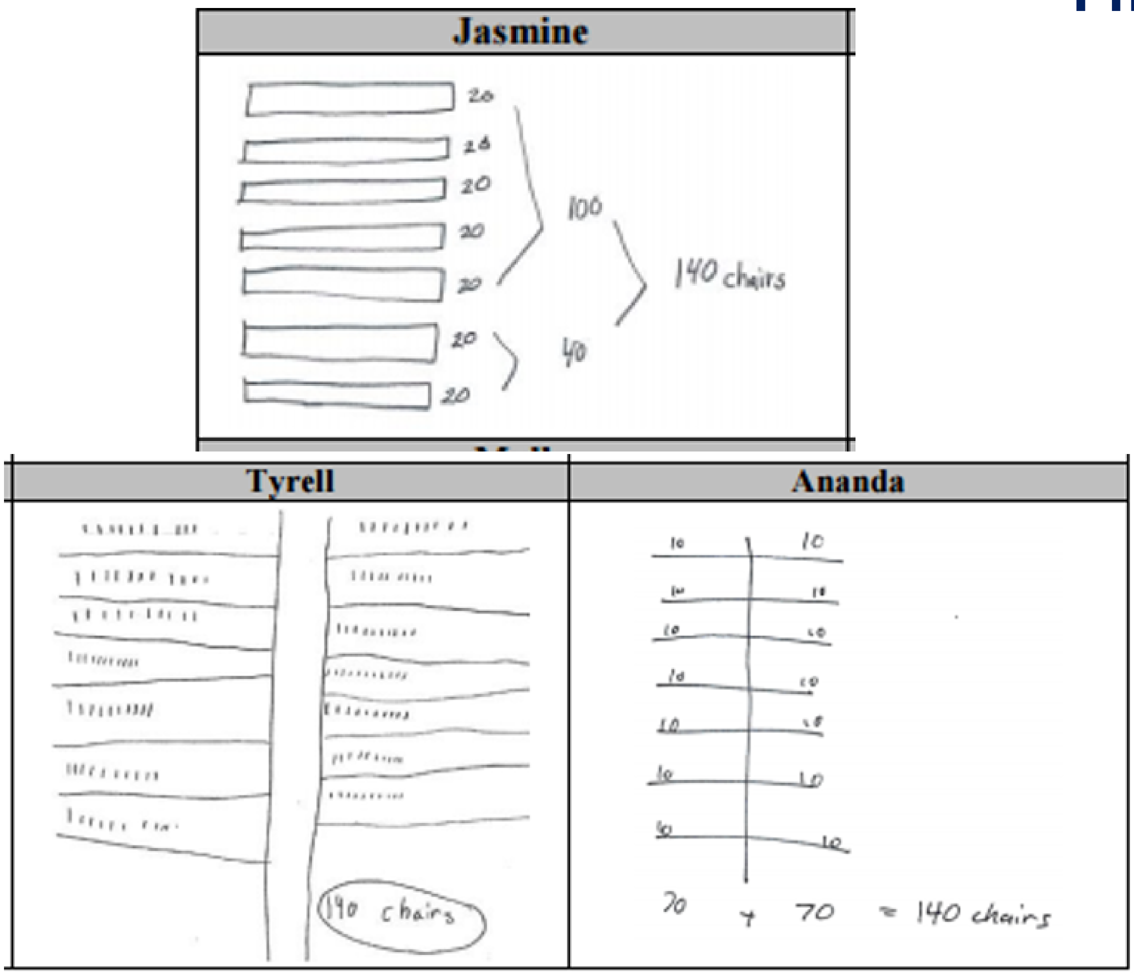
The Band Concert

Jasmine	Kenneth	Teresa
20 20 20 20 20 20 20 100 40 140 chairs	20+20+20+20+20+20+20 40+40=80 80+20=100 100+20=120 120+20=140 140 chairs	1 2 3 4 5 6 7 20, 40, 60, 80, 100, 120, 140
Molly	**Tyrell**	**Ananda**
160	140 chairs	10 10 10 10 10 10 10 10 10 10 10 10 10 10 70 + 70 = 140 chairs

Principles to Actions, NCTM, 2014

Starting Point to Build Procedural Fluency



Find the product

5 x 20 =

6 x 80 =

4 x 70 =

3 x 50 =

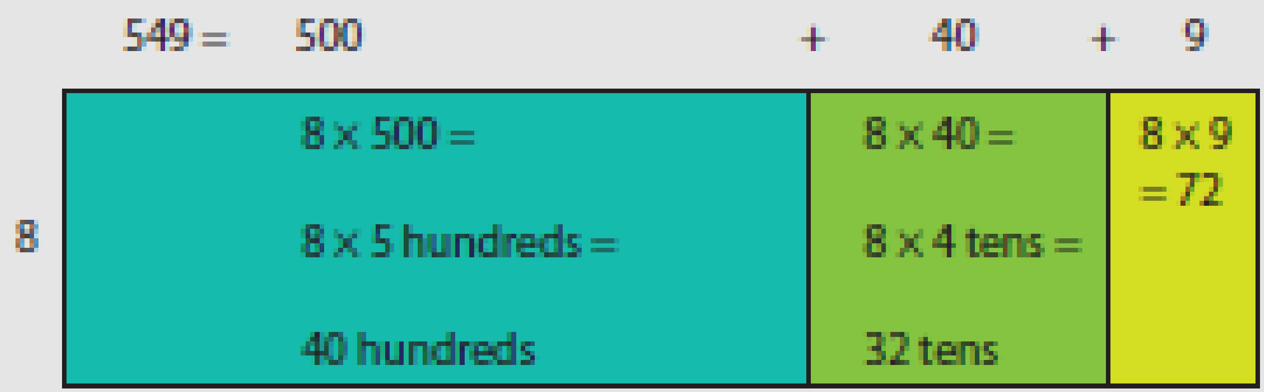
9 x 20 =

2 x 60 =

8 x 30 =

Multiplication Algorithms

Computation of 8×549 connected with an area model



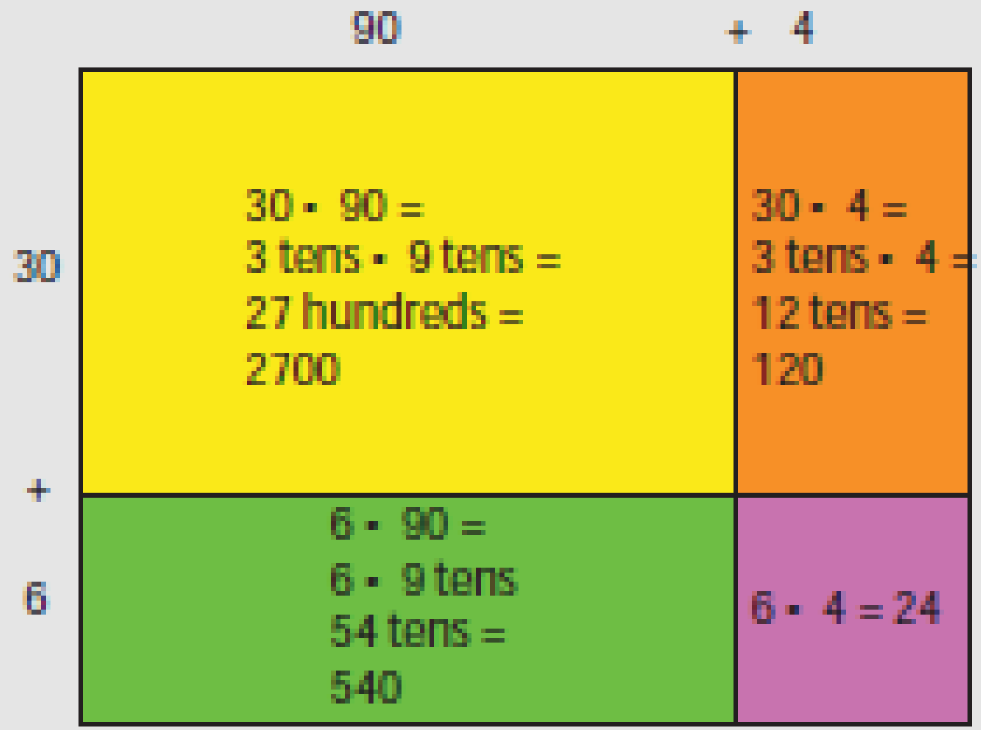
Each part of the region above corresponds to one of the terms in the computation below.

$$
\begin{aligned}
8 \times 549 &= 8 \times (500 + 40 + 9) \\
&= 8 \times 500 + 8 \times 40 + 8 \times 9.
\end{aligned}
$$

CCSS Numbers and Operations in Base-Ten Progression, April 2012

Multiplication Algorithms

Computation of 36×94 connected with an area model



The products of like base-ten units are shown as parts of a rectangular region.

CCSS Numbers and Operations in Base-Ten Progression, April 2012

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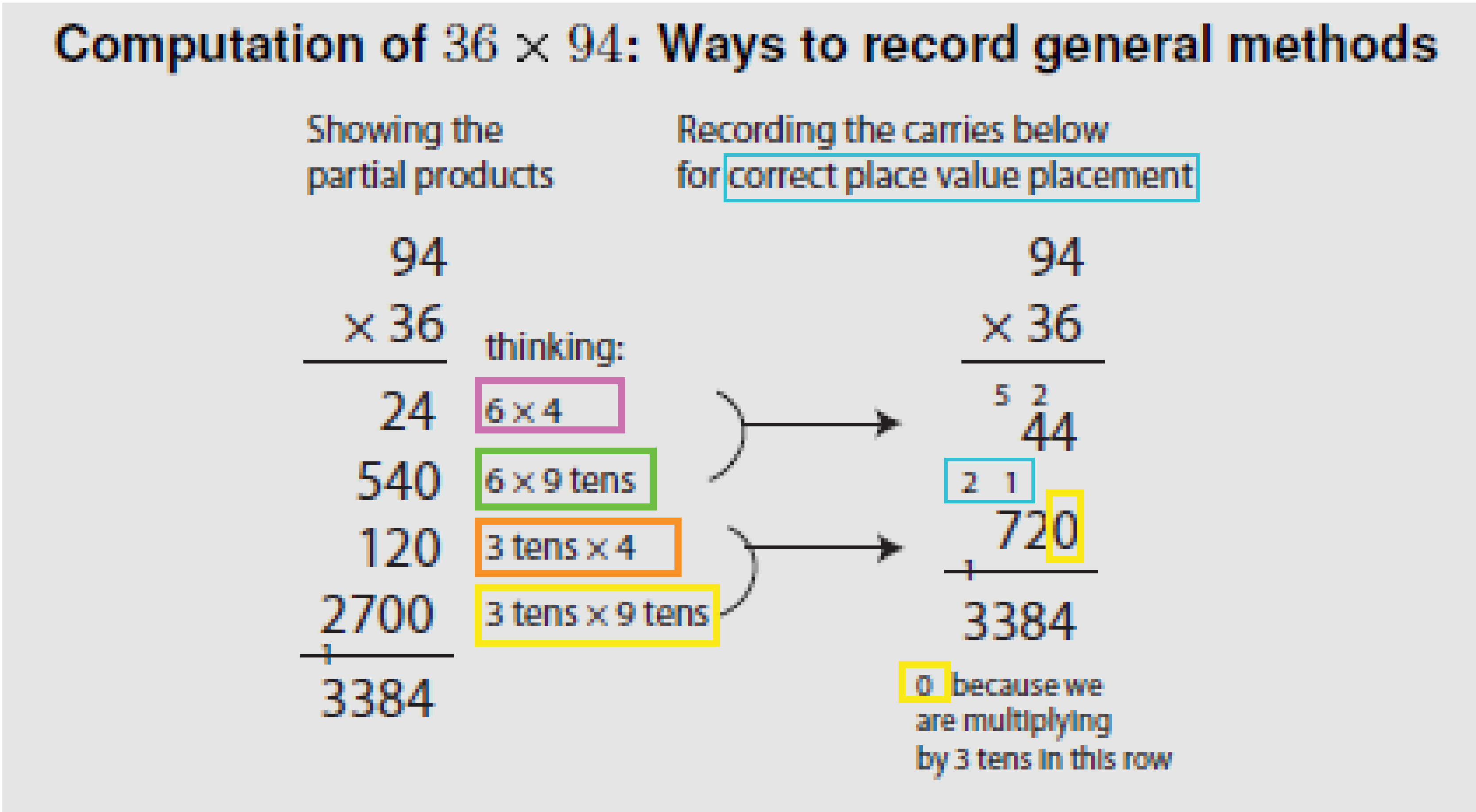
Multiplication Algorithms



CCSS Numbers and Operations in Base-Ten Progression, April 2012

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Providing a Basis for Future Learning

36 x 94

	90	+ 4	
30	2700	120	→ 2820
+ 6	540	24	→ 564
			3384

$(x+6)(x+4)$

	x	+ 4	
x	x^2	$4x$	→ x^2+4x
+ 6	$6x$	24	→ $6x+24$
			$x^2+10x+24$

Developing Procedural Fluency
Whole Number Multiplication

1. Develop conceptual understanding building on students' informal knowledge
2. Develop informal strategies to solve problems
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	Numbers and Operations in Base Ten
1	Use place value understanding and properties of operations to add and subtract.
2	Use place value understanding and properties of operations to add and subtract.
3	Use place value understanding and properties of operations to perform multi-digit arithmetic. *A range of algorithms may be used.*
4	Use place value understanding and properties of operations to perform multi-digit arithmetic. *Fluently add and subtract multi-digit whole numbers using the standard algorithm.*
5	Perform operations with multi-digit whole numbers and with decimals to hundredths. *Fluently multiply multi-digit whole numbers using the standard algorithm.*
6	Compute fluently with multi-digit numbers and find common factors and multiples. *Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.*

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6	Compute fluently with multi-digit numbers and find common factors and multiples. ***Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.***

A Functions Approach to Equation Solving

$y = 12x + 10$

- Solve for y when $x = 3, 10, 100$.
- Solve $70 = 12x + 10$

U.S. Shirts charges $12 per shirt plus $10 set-up charge for custom printing.

- What is the total cost of an order for 3 shirts?
- What is the total cost of an order for 10 shirts?
- What is the total cost of an order for 100 shirts?
- A customer spends $70 on T-shirts. How many shirts did the customer buy?

Procedural Fluency

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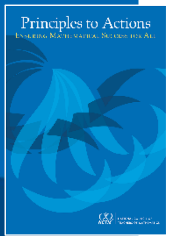

Evaluate and Compare Methods

<u>Finding the Missing Value</u>

Find the value of the unknown in each of the proportions shown below.

$$\frac{5}{2} = \frac{y}{10} \qquad \frac{a}{24} = \frac{7}{8}$$

$$\frac{a}{8} = \frac{3}{12} \qquad \frac{30}{6} = \frac{b}{7}$$

$$\frac{5}{20} = \frac{3}{d} \qquad \frac{3}{x} = \frac{4}{28}$$

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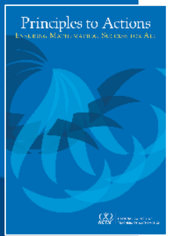

Evaluate and Compare Methods

Finding the Missing Value

Find the value of the unknown in each of the proportions shown below using both unit rates and scale factors. Which strategy do you prefer? Why?

$$\frac{5}{2} = \frac{y}{10} \qquad \frac{a}{24} = \frac{7}{8}$$

$$\frac{a}{8} = \frac{3}{12} \qquad \frac{30}{6} = \frac{b}{7}$$

$$\frac{5}{20} = \frac{3}{d} \qquad \frac{3}{x} = \frac{4}{28}$$

IES Algebra Practice Guide, 2015

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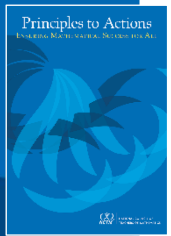

Evaluate and Compare Methods

	Unit Rate	Scale Factor	Which do you prefer? Why?
$\frac{5}{2} = \frac{y}{10}$			

IES Algebra Practice Guide, 2015

Analyze Worked Examples

Eliza solved this problem **correctly**. Here is her work:

$6 - k = -3$

$$\begin{aligned} 6 - K &= -3 \\ -6 \quad & \quad -6 \\ -K &= -9 \\ \div -1 \quad & \quad \div -1 \\ K &= 9 \end{aligned}$$

Why did Eliza subtract 6 FROM BOTH SIDES of the equation?

Why did Eliza divide by -1?

Your Turn:

$-6 - k = 3$

McGinn, Lange & Booth, MTMS, 2015

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Analyze Worked Examples

Helaina tried to simplify this expression, but she **didn't** do it correctly. Here is her first step:

$5 - 4x + 2$

$5 - 4x + 2$

$4x - 5 + 2$

What did Helaina do wrong in her first step?

Would it have been okay to write 5+2-4x? Explain why or why not.

Your Turn:

$12x + 4 - 5x$

McGinn, Lange & Booth, MTMS, 2015

Developing Procedural Fluency

1. Develop conceptual understanding building on students' informal knowledge
2. Develop informal strategies to solve problems
3. Refine informal strategies to develop fluency with standard methods and procedures (algorithms)
4. Explicitly compare/contrast different methods to solve the same problem to build fluency
5. Analyze worked examples to build conceptual understanding

Build Procedural Fluency from Conceptual Understanding

What are teachers doing?

Providing students with opportunities to use their own reasoning strategies and methods for solving problems.

Asking students to discuss and explain why the procedures that they are using work to solve particular problems.

Connecting student-generated strategies and methods to more efficient procedures as appropriate.

Using visual models to support students' understanding of general methods.

Providing students with opportunities for distributed practice of procedures.

What are students doing?

Making sure that they understand and can explain the mathematical basis for the procedures that they are using.

Demonstrating flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems.

Determining whether specific approaches generalize to a broad class of problems.

Striving to use procedures appropriately and efficiently.

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Effective
Mathematics Teaching Practices

1. Establish mathematics **goals** to focus learning.
2. Implement **tasks** that promote reasoning and problem solving.
3. Use and connect mathematical **representations.**
4. Facilitate meaningful mathematical **discourse**.
5. Pose purposeful **questions**.
6. Build **procedural fluency** from conceptual understanding.
7. Support **productive struggle** in learning mathematics.
8. **Elicit and use evidence** of student thinking.

Diane J. Briars, October, 2016

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The Title Is Principles to *Actions*

Your Actions?

Thank You!

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