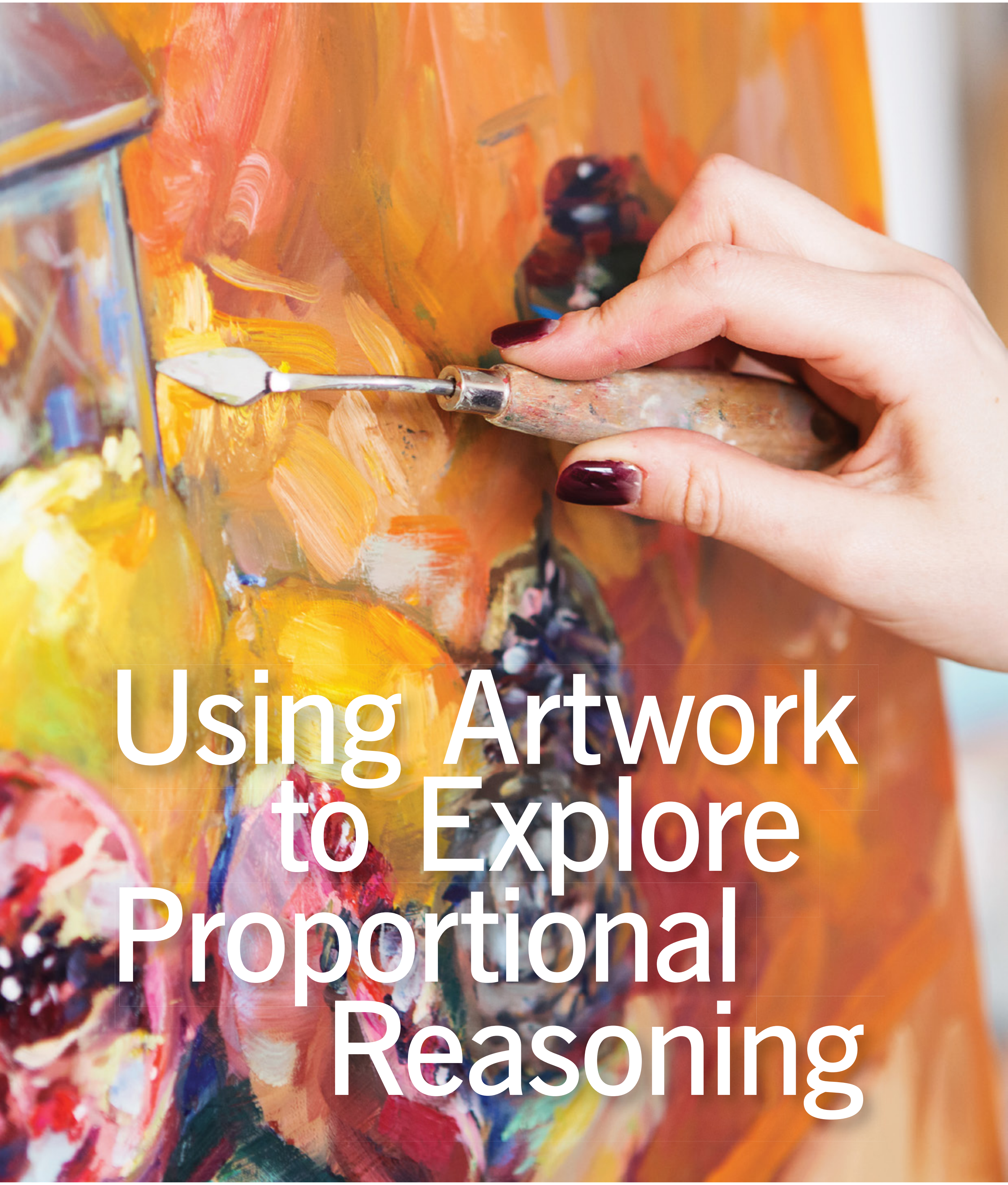


Using Artwork to Explore Proportional Reasoning

NATA_ZHEKOVA/THINKSTOCK

By examining ratios in paintings and using a free educational app, students can size up artists' use of proportional reasoning in their creations.

Sarah B. Bush, Karen S. Karp, Jennifer Nadler, and Katie Gibbons

Having an answer to "When are we ever going to use this in real life?" is important to middle school mathematics teachers. The activity we describe awakened sixth graders' understanding of how artists use mathematics. By exploring ratio and proportionality in different paintings, students realized the use of proportional reasoning in artistic compositions.

In this article, we share our lesson created for middle-grades students using a new game app called Keys to the Collection, developed by the Barnes Foundation in partnership with Drexel University's School of Education. This lesson was implemented in an urban sixth-grade class as well as during a sixth-grade field trip to the Barnes Foundation.

The catalyst for this activity was an ongoing partnership formed between mathematics educators and art educators at the Barnes Foundation in Philadelphia, Pennsylvania. This partnership began with an email in which we asked the Barnes Foundation for access to diagrams used when the Barnes Foundation relocated from a suburban home to a large building in Philadelphia (Bush et al. 2013). Since that time, we have collaborated with art educators to develop field trip lessons and conduct workshops integrating mathematics with art in ways that align to the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010).

EDUCATING THROUGH ART: THE BARNES FOUNDATION

The Barnes Foundation is an educational institution dedicated to teaching art in an integrated way. Originally established by Dr. Albert C. Barnes, a scientist, the Barnes Foundation houses a diverse art collection including impressionist, postimpressionist, early modern, and old master paintings. African sculpture, Pennsylvania German chests, Native American ceramics, jewelry, textiles, and metalwork are also represented. Barnes decided to display his collection by assembling works from different artists, periods, and geographic regions in wall groupings he called *ensembles* so that the viewer would compare the way artists composed their work using the same artistic elements.

John Dewey, the well-known educator and philosopher, and Barnes collaborated on an educational method for interpreting art. Barnes's ultimate goal was to use his collection of artwork for educational purposes, focusing on artists' use of color, light, line, and space—all of which

Fig. 1 These paintings (a–e) and one other available online (f) were analyzed for their various proportional attributes.

(a) Jules Pascin, *Landscape with Figures and Carriage*

(b) Henri Rousseau, *View of Montsouris Park, the Kiosk*

(c) Paul Cézanne, *Gardanne*

(d) El Greco, *Apparition of the Virgin and Child to Saint Hyacinth*

(e) Édouard Manet, *Laundry*

(f) Giorgio de Chirico, *The Arrival*

See this link to view online: http://www.barnesfoundation.org/collections/art-Collection/object/5697/the-arrival-la-meditazione-del-pomeriggio?artistID=46&rNo=0

directly connect to mathematics through such concepts as proportionality and ratio. These ensemble displays provide an opportunity to look at how mathematics and art are integrated by exploring how artists use proportionality in creating the illusion of three dimensions on a two-dimensional canvas.

ACTIVATING PRIOR KNOWLEDGE WITH WHOLE-CLASS DISCUSSION

Students were placed in study teams (groups of four) and given the following warm-up question,

> What ratio is formed when you compare the length of a large paper clip to the length of the eraser on your pencil?

Students immediately started by gathering a paper clip, their pencil, and a metric ruler. We asked students to use millimeters as their unit of measure because inches or centimeters would not have been precise enough to measure the artwork replications. We were surprised when some students were unsure which markings on their ruler represented millimeters. We used this opportunity to discuss the size of a millimeter and its relation to centimeters (1 cm = 10 mm). Even though we asked students to measure in millimeters, some initially offered solutions in inches. Solutions for the ratio of the length of the paper clip to the length of the eraser generated by the teams included the following ratios:

2 in. : 0.5 in.
2.5 in. : 0.5 in.
24 mm : 6 mm
30 mm : 10 mm
31 mm : 11 mm
29 mm : 5 mm

When asked why solutions might vary, even within the same measurement unit, students agreed that it could be due to the length of the paper clip or eraser being measured. They also mentioned the possibility of measurement error.

Several interesting conversations emerged from the solutions that students proposed. First, students needed help with the idea that even if you measure using a different unit (e.g., inches or millimeters), the ratios are still equivalent. To address this, we asked, "Does measuring using a different unit change your ratio?" Because all the work with the paper clip and eraser comparisons proved to produce small measures in fractional units for the inches, we decided to model this idea with their pencils. All the pencil sizes differed, but, for example, one pair of students measured 120 millimeters for the pencil and 5 millimeters for the eraser when measuring in metric units and then measured 6 inches for the pencil and 0.25 inches for the eraser, each time getting a ratio equivalent to 24:1. Student responses included the following:

- "Inches are going to be a lot less because they represent more."
- "There are more millimeters than inches, but there are only more millimeters because they are smaller."
- "There are more millimeters than inches, but that is because the inches are longer."
- "No matter which unit I measure in, they will stay in the same range."

Because this last student response was on the path to discovering that the ratios would remain equivalent, we asked students, "Does a ratio represent an additive or a multiplicative relationship?" The class went back to the pencil example and agreed that it was multiplicative. One student said, "Even if you use different units of measure, they are proportional, which means a proportion of something in the same category that can be similar." Another student made the connection to art class when he exclaimed, "We did this in a drawing; we had to have the same proportion from our ear to our face as we made a self-portrait." Every study team offered a solution that was in the form of a ratio (e.g., 31 mm : 11 mm). Using the student response of 24 mm : 6 mm as an example, we asked the class, "How can you describe in words the relationship of the length of the paper clip to the length of the eraser?" One student responded, "Four multiplied by *x*," with *x* representing the length of the eraser. We asked if the ratio could be written in an inverse way when comparing the length of the eraser to the length of the paper clip. Although at first they found the idea a little tricky, students discussed it in study teams and determined that the ratio could be described as

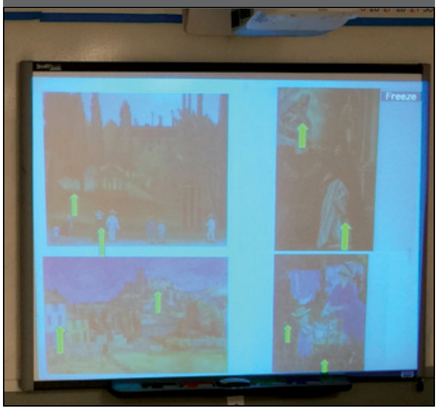
$$x \div 4 \text{ or also as } \frac{1}{4}x.$$

Students concluded that a ratio could be written in two ways, as either multiplication or division, depending on how the question was posed.

EXPLORING PROPORTIONAL REASONING THROUGH ART

Students were ready to be engaged in a meaningful task linking art and proportional reasoning. We shared information about our partnership with the Barnes Foundation and gave students a brief history of the foundation. Many students expressed a passion for art and were immediately excited about the activity. Each study team was given one of six selected color prints of paintings (see **fig. 1a–e**; note: only five paintings are shown; a link in **fig. 1f** will take readers to Giorgio de Chirico's *The Arrival*, which was also part of the class discussion) from the Barnes Foundation collection with the name of the artist and title of the painting on the back. The class shared two sets of the prints and passed them from group to group, meaning that groups were not working on the questions in the same order. Next, we gave each student an **activity sheet**, which asked two questions per painting: One question referred to the ratio between two objects in the painting, and the other question discussed an application that was designed to stretch students' thinking. This activity sheet contained several questions to challenge students' thinking about ratios and three-dimensional perspectives. It also asked about representations and relative sizes of objects and people. We projected presentation slides (see **fig. 2**) on the interactive whiteboard, with arrows to help direct students to the specific objects we were referring to in the questions.

Fig. 2 Presentation slides were projected on the interactive whiteboard.



SARAH B. BUSH

Fig. 3 Students measured one another to get started.

SARAH B. BUSH

For questions 1–12 on the **activity sheet**, students worked in study teams. Different teams approached the questions from different entry points, and overall solutions varied. Students engaged in meaningful mathematics discourse, justifying and explaining their reasoning, as well as working together with the shared reproductions. We will discuss questions 2 and 3 in detail to provide a sense of the discourse and learning that occurred as students worked to solve these contextual problems (see the **activity sheet**).

Study teams working on question 2 were at first perplexed by how they would estimate their own height in the painting (next to the horses and buggy) (see **fig. 1a**). We wanted students to struggle productively, as *Principles to Actions* recommends, so we knew to provide support in a way that kept "the thinking and reasoning at a high level" (NCTM 2014, p. 49). We asked students guiding questions such as these:

- Where could you start?
- What would you need to know to estimate your own height in the painting?
- Will your estimate be the same as the other students' on your study team? Why or why not?
- How could you use proportional reasoning to help you solve this problem?

Several study teams used the same general problem-solving plan. First, students determined that they had to measure themselves to get started and used rulers to measure one another (see **fig. 3**). They measured themselves in centimeters and then multiplied by 10 to obtain their actual height in millimeters, which for one student was 1500 millimeters. Then measuring the horse and using what the study team members believed to be true about the approximate height of an actual horse, they agreed that the ratio of print to actual size was about

$$\frac{1}{40}$$

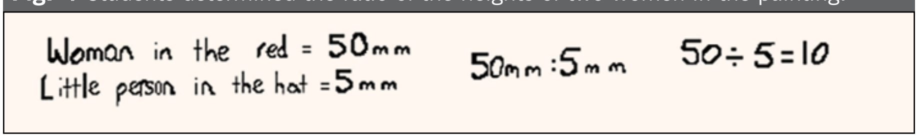
at the location of the horse and buggy. Students then took their own height of 1500 millimeters and divided by 40 or multiplied by

$$\frac{1}{40}$$

to obtain their estimated height in the print of 37.5 millimeters.

Students working on question 3 were asked to determine the ratio of the heights of two women in the painting, one in the foreground and one in the background. One group began by measuring the height of each and then wrote a ratio to represent the comparison. Next, students determined that the ratio was 10 mm : 1 mm (see **fig. 4**). To prompt one possible conversation with students, we asked what assumption was being made about the two women. We then guided students to the idea that the question assumed that the women had the same actual height, because if not, the ratio would not be representative of the actual height of both women.

Fig. 4 Students determined the ratio of the heights of two women in the painting.



Our favorite aspect of the **activity sheet** was the three culminating questions (13–15). We were impressed by the variety of students' ideas (see **fig. 5**) and the connections they made to real life, many of which were not previously mentioned. Notice that in some comments, students referred to the idea of proportion when they said that the artist "makes distances really show" by using the same ratio multiple times throughout the painting. Students noticed some big ideas about the art as well when they recognized a possible reason for circumventing the use of ratio. One stated, "Abstract artists might not use ratios because they don't want it [their painting] to be proportional." Students were excited to discuss the links between mathematics and art with the whole class, and doing so gave us the perfect transition to the next part of our activity—a closer look at Barnes Foundation artwork.

EXTENDING THE ACTIVITY WITH AN APP

As a culminating activity, we wanted students to further explore the collection of artwork at the Barnes Foundation and truly make this lesson an integrated experience with art. Because we could not take our students to Philadelphia, we introduced them to the free Keys to the Collection iPad® app created for students ages 7–14. We downloaded this app on a classroom set of iPads ahead of class time. Class members were excited to see how pieces of artwork they had just worked with fit within the bigger picture at the Barnes Foundation. With their new awareness of the proportional reasoning needed by artists, students were ready to explore.

Students opened the app and chose to start a new game. After designing their own avatar, students were ready to enter three different Barnes Foundation gallery rooms

Fig. 5 Students responded to culminating questions.

Question 13: Describe, in your own words, how the mathematical concepts of ratios and proportional thinking are used in paintings.

"Ratios are used in paintings because in the 'Laundry' painting, the girl is a portion of the lady and we had to figure out how tall we would be so we had to use ratios."

"I think they are used in paintings to show the depth of the painting and to show contrast."

"Ratios are used in paintings to compare the size of different objects in certain paintings. Ratios are used to show how certain things stand out and how to make distance really show."

"I think that the concept of ratios are used to make the painting have deeper meaning and to give the painting more purpose."

"A painting could have more of a purpose than just looking at it for art, it can also have a mathematics purpose."

Question 14: Why does an artist need to know about ratios when creating a painting? Provide concrete examples from the paintings you worked with in this activity.

"Because if the proportions aren't right, they might make their drawing less realistic."

"They need to be able to enlarge. They also need to know ratios so they can make paintings symmetrical such as with ears and eyes."

"If the baby was the same size as the lady [in *Laundry*], in real life that would not look right and the ratio would be off."

"While painting, an artist needs to know ratios because when working with distances, such as in the painting *View of Montsouris Park, the Kiosk* the artist needed to use ratios because he had people far away and up close."

Question 15: Where else in real-life situations have you seen ratios used?

"In models, like globes, maps, model vehicles, and also buildings."

"I've seen ratios used when computers are being engineered."

"Abstract artists might not use ratios because they don't want it to be proportional."

"In architecture because you don't want the building to be too small or too big, you want it to be the exact size for your business, museum, or whatever you're building."

"In architectural design you wouldn't want the building to be the size of a penny or the size of a whole city."

"I went to an art museum in Washington, DC where everything was shown proportionally."

"I've seen them in drawings and using them to compare one object to others. I also use them in music to compose different rhythms."

JULENOCHEK/THINKSTOCK

Students were excited to discuss in a whole-class setting the links between mathematics and art, especially after authentically seeing the connections between math and art.

found in the app (see **fig. 6**). In each gallery room, students "jumped" into three different paintings. Our primary focus was on the first gallery room, in which students had to "restore" paintings by informally fixing incorrect proportions and sizes of objects in the paintings. Students used visualization and informal proportional reasoning to estimate and then used their fingers to "pinch" the figure on the screen to dilate it, which enlarged or shrank the object to restore the original artwork. **Figure 7** shows a statue restoration, which students thought was especially interesting because they had previously worked with this painting. Students found this technology connection to be engaging and novel. It also helped deepen their own understanding of proportional thinking and how ratios link art to mathematics. The app has other connections to mathematics that could be easily highlighted. For example, students could explore concepts of probability, sample space, and the counting principle during the creation of their avatar.

EXPLORING AN ON-SITE GALLERY

A school group of sixth graders from Philadelphia had the opportunity to try this activity on-site in the actual gallery rooms at the Barnes Foundation. As an alternative to the lesson sequence described above, students instead began with an exploration using the Keys to the Collection game app to "restore" paintings. Next, they used the **activity sheet** to engage in formalized thinking about ratios. With the paintings in front of them, students could see up close how artists like Jules Pascin, Giorgio de Chirico, and Henri Rousseau manipulated the size of objects using proportional reasoning to create a realistic sense of space. One student remarked, "I thought you just had to guess how big to make things in the background. I never realized that you could use math to figure that out."

REFLECTING ON LEARNING

This activity offered a fun, meaningful exploration of the mathematics knowledge that an artist needs. It also tied in with CCSSM's sixth-grade and seventh-grade Ratios and Proportional Relationships domain and SMP 4. Students also dealt with measurement, conversions, and attending to precision in their work (SMP 6). The critical component of the entire process was the group discussion held after the activity sheet was complete. During this time, the teacher strategically selected students who had ideas related to the big mathematical concepts she wished to highlight and reinforce. In this way, various strategies were shared, and any misunderstandings were brought to the forefront and addressed. At the end of this important conversation, we asked students to reflect on what they had learned. Student responses included the following:

- "I liked how I finally saw how much math is involved in paintings."
- "I thought it was a really fun activity, and I now see how you find math in art."
- "I thought it was fun because you got to go inside the paintings. . . . It was really cool to walk in the painting and fix [restore] stuff that was messed up."
- "I really liked that you were able to see the 3D version of each painting that you would hop in because some of them [from our class activity] were the same as the paintings on the app."
- "I like how when you were just looking at the pictures (2D), you could think about how they might look, but with the app, you actually could jump in[to] the painting."

Fig. 6 After opening the app and designing their own avatar, students were ready to enter three different Barnes Foundation gallery rooms.

SARAH B. BUSH

Fig. 7 This student rebuilt a statue.

SARAH B. BUSH

Students could authentically see the connections between art and mathematics. We hope that, as a result of reading about our work, other teachers are equally inspired to integrate art with mathematics, perhaps even using artwork from local museums or galleries to create new and exciting mathematical learning.

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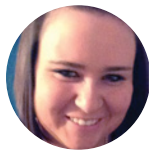
CCSSM Practices in Action

6.RP.1, 6.RP.2, and 6.RP.3
7.RP.1 and 7.RP.2
SMP 4 and SMP 6



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activity sheet

Name ______________________________

EXPLORING PROPORTIONAL REASONING THROUGH ARTWORK

Analyze the images provided by your teachers to answer the following questions. Use millimeters for all measurements.

Jules Pascin, *Landscape with Figures and Carriage*

1. What ratio is formed when you compare the height of the person on the front left of the painting to the height of the person near the far right (standing next to the child)?

2. Given your own height, estimate your height in millimeters if you were in the painting standing next to the horses and buggy.

Henri Rousseau, *View of Montsouris Park, the Kiosk*

3. What is the ratio of the height of the woman in the red dress and feathered hat in the front of the painting compared to the silhouette of the woman with a round hat to the left of the gazebo (two people to the left of the tree)?

4. If you were to be painted in the middle of the field (between the figures in the front and the gazebo), use the ratio found above to approximate your height in millimeters.

Paul Cézanne, *Gardanne*

5. What is the ratio of the height of the rectangular window on the left front building (above the door) compared to the height of a rectangular window in the very back row of buildings (on the building with four windows to the right of the tower)?

6. Given the relationship of the height of the two windows in question 9, to maintain the same ratio how tall should the door be for the back building?

El Greco, *Apparition of the Virgin and Child to Saint Hyacinth*

7. What is the ratio of the length of the hand of the man in the painting compared to the hand of the woman in the painting?

8. **a.** Compare the size of the man's hand to the size of his head. Do the same for your hand and your head. Is the relationship similar?

b. About how many heads tall is the man? What is the ratio?

Name ______________________________

c. How many heads tall are you? Is that approximately the same ratio as the man in the painting?

d. Compared to your own body proportions, how many millimeters long should the man's arm be in the painting?

e. Are this man's body proportions similar to your own? Why or why not?

f. Why do you think an artist might paint a person using proportions that are not the same as the average human?

Édouard Manet, *Laundry*

9. What is the ratio of the height of the red flower in the front of the painting compared to the red flower toward the back of the painting?

10. Given the height of the child and the height of the woman, how many millimeters tall would you need to be if you were painted right beside them in the painting?

Giorgio de Chirico, *The Arrival*

11. What is the ratio of the height of the archway in the front of the painting (facing the front) compared to the height of the archway in the back right of the painting?

12. If the statue were moved to the back of the painting near the back right archway, using the ratio above, approximately what would be the new height of the statue in the painting (in millimeters)?

Culminating Questions

13. Describe, in your own words, how the mathematical concepts of ratios and proportional thinking are used in paintings.

14. Why does an artist need to know about ratios when creating a painting? Provide concrete examples from the paintings you worked with in this activity.

15. Where else in real-life situations have you seen ratios used?