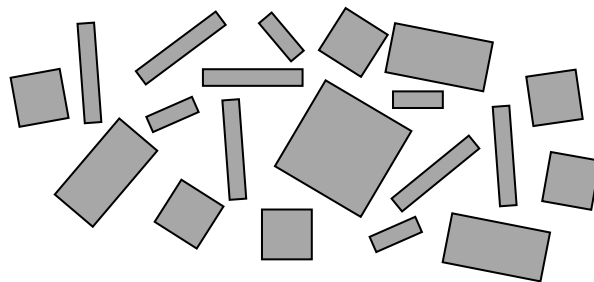# CPM

# Using Algebra Tiles from Polynomials to Factoring

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1-4.    DIAMOND PROBLEMS
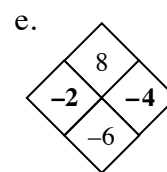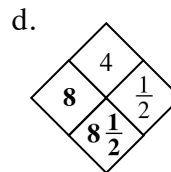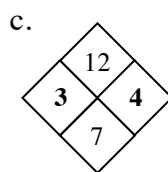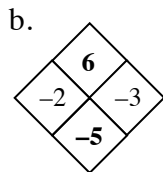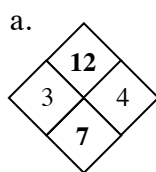
Finding and using a pattern is an important problem-solving skill you will use in algebra. The patterns in Diamond Problems will be used later in the course to solve other types of algebraic problems.

Look for a pattern in the first three diamonds below. For the fourth diamond, explain how you could find the missing numbers (?) if you know the two numbers (#).

| 10 | 6 | 4 | ? |
|---|---|---|---|
| 5  2 | 2  3 | −1  −4 | #  # |
| 7 | 5 | −5 | ? |
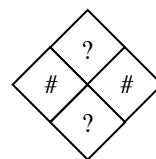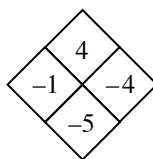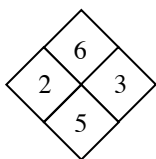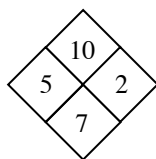
Copy the Diamond Problems below onto your paper. Then use the pattern you discovered to complete each one.

a.

| **12** |
|---|
| 3  4 |
| **7** |

b.

| **6** |
|---|
| −2  −3 |
| **−5** |

c.

| 12 |
|---|
| **3**  **4** |
| 7 |

d.

| 4 |
|---|
| 8  $\frac{1}{2}$ |
| **8**$\frac{1}{2}$ |

e.

| 8 |
|---|
| **−2**  **−4** |
| **−6** |

2-1.    Your teacher will distribute a set of algebra tiles for your team to use during this course. As you explore the tiles, address the following questions with your team. Be prepared to share your responses with the class.

- How many different shapes are there? What are all of the different shapes?

- How are the shapes different? How are they the same?

- How are the shapes related? Which fit together and which do not?
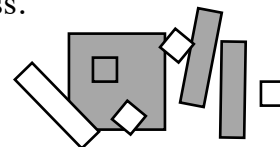
2-4.    Build each collection of tiles represented below. Then name the collection using a simpler algebraic expression, if possible. If it is not possible to simplify the expression, explain why not.

a.    $3x + 5 + x^2 + y + 3x^2 + 2$

b.    $2x^2 + 1 + xy + x^2 + 2xy + 5$

c.    $2 + x^2 + 3x + y^2 + 4y + xy$

d.    $3y + 2 + 2xy + 4x + y^2 + 4y + 1$

3-65.    Write the area as a **product** and as a **sum** for the composite rectangle shown at right.



3-66.    Now examine the following diagram.  How is it similar to the set of tiles in problem 5-22?  How is it different?  Talk with your teammates and write down all of your observations.

|  | $12x$ | $15$ |
|---|---|---|
| $3$ | | |
| $2x$ | $8x^2$ | $10x$ |
|  | $4x$ | $5$ |

3-68.    Use a generic rectangle to multiply the following expressions.  Write each solution both as a sum and as a product.

a.  $(2x+5)(x+6)$

b.    $(m-3)(3m+5)$

c.  $(12x+1)(x-5)$

d.    $(3-5y)(2+y)$

8-2.    The process of changing a sum to a product is called **factoring**.  Can every expression be factored?  That is, *does every sum have a product that can be represented with tiles?*

Investigate this question by building rectangles with algebra tiles for the following expressions.  For each one, write the area as a sum and as a product.  If you cannot build a rectangle, be prepared to convince the class that no rectangle exists (and thus the expression cannot be factored

a.  $2x^2 + 7x + 6$

b.    $6x^2 + 7x + 2$

c.  $x^2 + 4x + 1$

d.    $2xy + 6x + y^2 + 3y$

8-3.    Work with your team to find the sum and the product for the following generic rectangles.  Are there any special strategies you discovered that can help you determine the dimensions of the rectangle?  Be sure to share these strategies with your teammates.

a.

| $2x$ | $5$ |
|---|---|
| $6x^2$ | $15x$ |

b.

| $-2y$ | $-6$ |
|---|---|
| $5xy$ | $15x$ |

c.

| $-9x$ | $-12$ |
|---|---|
| $12x^2$ | $16x$ |

8-4.    While working on problem 8-3, Casey noticed a pattern with the diagonals of each generic rectangle. However, just before she shared her pattern with the rest of her team, she was called out of class! The drawing on her paper looked like the diagram at right. Can you figure out what the two diagonals have in common?
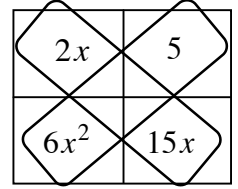
8-14.   FACTORING QUADRATIC EXPRESSIONS

To develop a method for factoring without algebra tiles, first study how to factor with algebra tiles, and then look for connections within a generic rectangle.

a. Using algebra tiles, factor $2x^2 + 5x + 3$; that is, use the tiles to build a rectangle, and then write its area as a product.

b. To factor with tiles (like you did in part (a)), you need to determine how the tiles need to be arranged to form a rectangle. Using a generic rectangle to factor requires a different process.

Miguel wants to use a generic rectangle to factor $3x^2 + 10x + 8$. He knows that $3x^2$ and 8 go into the rectangle in the locations shown at right. Finish the rectangle by deciding how to place the ten $x$-terms. Then write the area as a product.
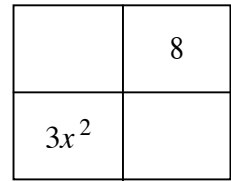
c. Kelly wants to find a shortcut to factor $2x^2 + 7x + 6$. She knows that $2x^2$ and 6 go into the rectangle in the locations shown at right. She also remembers Casey's pattern for diagonals. Without actually factoring yet, what do you know about the missing two parts of the generic rectangle?
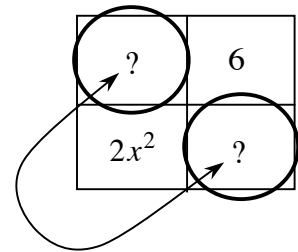
d. To complete Kelly's generic rectangle, you need two $x$-terms that have a sum of $7x$ and a product of $12x^2$. Create and solve a Diamond Problem that represents this situation.

**product**

**sum**

e. Use your results from the Diamond Problem to complete the generic rectangle for $2x^2 + 7x + 6$, and then write the area as a product of factors.
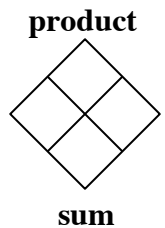
8-15.    Factoring with a generic rectangle is especially convenient when algebra tiles are not available or when the number of necessary tiles becomes too large to manage. Using a Diamond Problem helps avoid guessing and checking, which can at times be challenging. Use the process from problem 8-13 to factor $6x^2 + 17x + 12$. The questions below will guide your process.

a. When given a trinomial, such as $6x^2 + 17x + 12$, what two parts of a generic rectangle can you quickly complete?

b. How can you set up a Diamond Problem to help factor a trinomial such as $6x^2 + 17x + 12$? What goes on the top? What goes on the bottom

**product**

c. Solve the Diamond Problem for $6x^2 + 17x + 12$ and complete its generic rectangle.

**sum**

d. Write the area of the rectangle as a product.
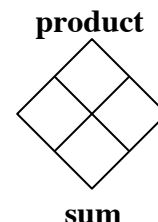

8-16.    Use the process you developed in problem 8-14 to factor the following quadratics, if possible. If a quadratic cannot be factored, justify your conclusion.

a. $x^2 + 9x + 18$

b.   $4x^2 + 17x - 15$

c. $4x^2 - 8x + 3$

d.   $3x^2 + 5x - 3$


8-24.    Factor each quadratic expression below, if possible. Use a Diamond Problem and generic rectangle for each one.

a. $x^2 + 6x + 9$

b.   $2x^2 + 5x + 3$

c. $x^2 + 5x - 7$

d.   $3m^2 + m - 14$


8-25.    SPECIAL CASES

Most quadratic expressions are written in the form $ax^2 + bx + c$. But what if a term is missing? Or what if the terms are in a different order? Consider these questions while you factor the expressions below. Share your ideas with your teammates and be prepared to demonstrate your process for the class.

a.   $9x^2 - 4$

b.   $12x^2 - 16x$

c.   $3 + 8k^2 - 10k$

d.   $40 - 100m$

8-26.    Now turn your attention to the quadratic expression below.  Use a generic rectangle and Diamond Problem to factor this expression.  Compare your answer with your teammates' answers.  Is there more than one possible answer?

$$4x^2 - 10x - 6$$

8-36.    c. *Without factoring*, predict which quadratic expressions below may have more than one factored form.  Be prepared to defend your choice to the rest of the class.

      *i*.  $12t^2 - 10t + 2$                 *ii*.  $5p^2 - 23p - 10$

      *iii*.  $10x^2 + 25x - 15$         *iv*.  $3k^2 + 7k - 6$

8-37.    FACTORING COMPLETELY

In part (c) of problem 8-36, you should have noticed that each term in $12t^2 - 10t + 2$ is divisible by 2.  That is, it has a **common factor** of 2.

a. An expression is considered **completely factored** if none of the factors can be factored any more.  Often it is easiest to remove common factors first, before factoring with a generic rectangle.  Rewrite this expression $10x^2 + 25x - 15$ with the common factor factored out.

b. Your result in part (a) is not completely factored if either factor can be factored.  Factor $10x^2 + 25x - 15$ completely.

8-100.    COMPLETING THE SQUARE

Jessica was at home struggling with her Algebra homework.  She had missed class and did not understand the new method called **completing the square**.  She was supposed to use it to change $y = x^2 + 8x + 10$ to graphing form.  Then her precocious younger sister, who was playing with algebra tiles, said, "*Hey, I bet I know what they mean.*"  Anita's Algebra class had been using tiles to multiply and factor binomials.
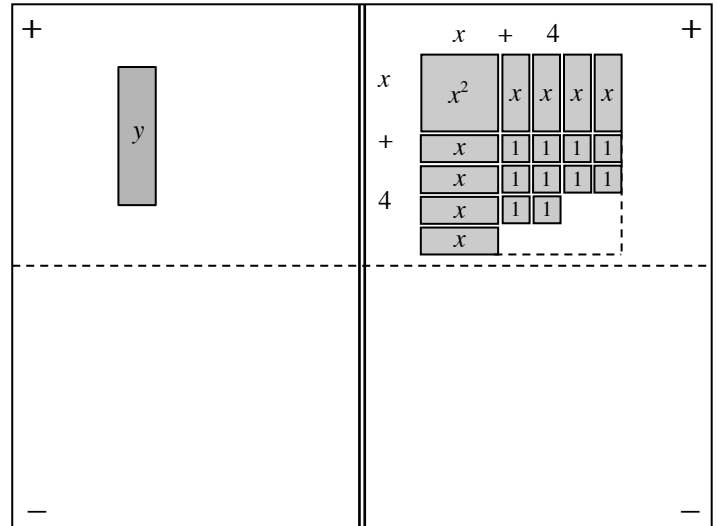
Anita explained:  "$x^2 + 8x + 10$ *would look like this*;"
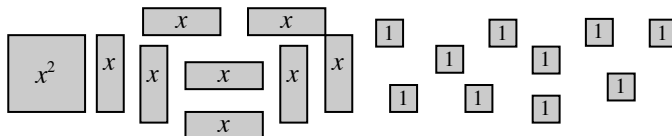


"*Yes,*" said Jessica, "*I'm taking Algebra too, remember?*"

Anita continued, "*And you need to make it into a square!*"

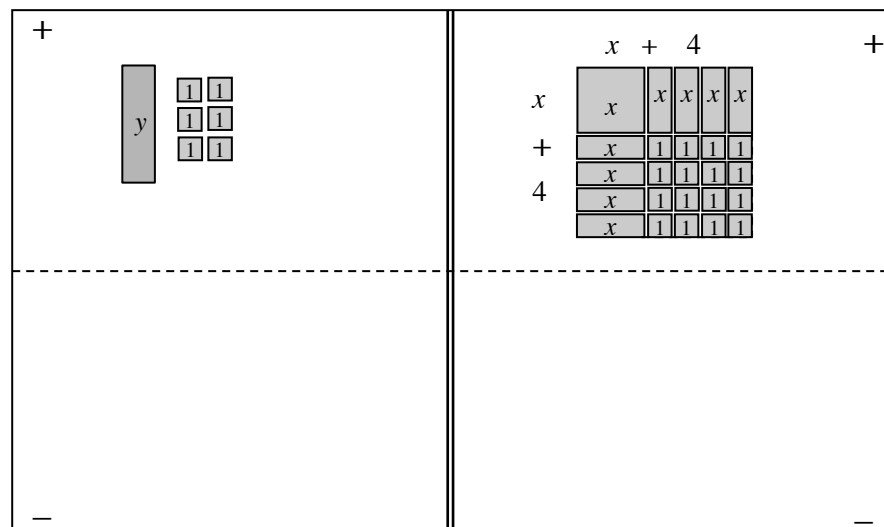*"OK,"* said Jessica, and she arranged her tiles on an equation mat as shown at right.

*"Oh,"* said Jessica. *"I need 16 small unit tiles to fill in the corner!"*

*"But you only have 10,"* Anita reminded her.

*"Right, I only have ten,"* Jessica replied. She drew the outline of the whole square and said:

*"Oh, I get it! To **complete the square**, I need to add six tiles to each side of the equation:"*

*"Oh, I see,"* said Anita. *"You started with $y = x^2 + 8x + 10$ , but now you can rewrite it as $y + 6 = (x + 4)^2$ ."*

*"Thank you so much, Anita! Now I can easily write the function in graphing form, $y = (x + 4)^2 - 6$ ."*

How can you use your graphing calculator to verify that $y = x^2 + 8x + 10$ and $y = (x + 4)^2 - 6$ are equivalent functions?

8-101.   Write each function in graphing form, then state the vertex and $y$-intercept of each parabola.

a.  $f(x) = x^2 + 6x + 7$         b.                    $f(x) = x^2 + 4x + 11$

8-102.   Help Jessica with a new problem.  She needs to complete the square to write $y = x^2 + 4x + 9$  in graphing form.

a. Draw tiles to help her figure out how to make this expression into a square.  Does she have too few or too many unit squares this time?  Write her function in graphing form.

b. Find the vertex and the $x$-intercepts.  What happened?  What does that mean?

c. Algebraically find the $y$-intercept.  Sketch the graph.

8-103.   How could you complete the square to change  $f(x) = x^2 + 5x + 2$  into graphing form?  How would you split the five $x$-tiles into two equal parts?
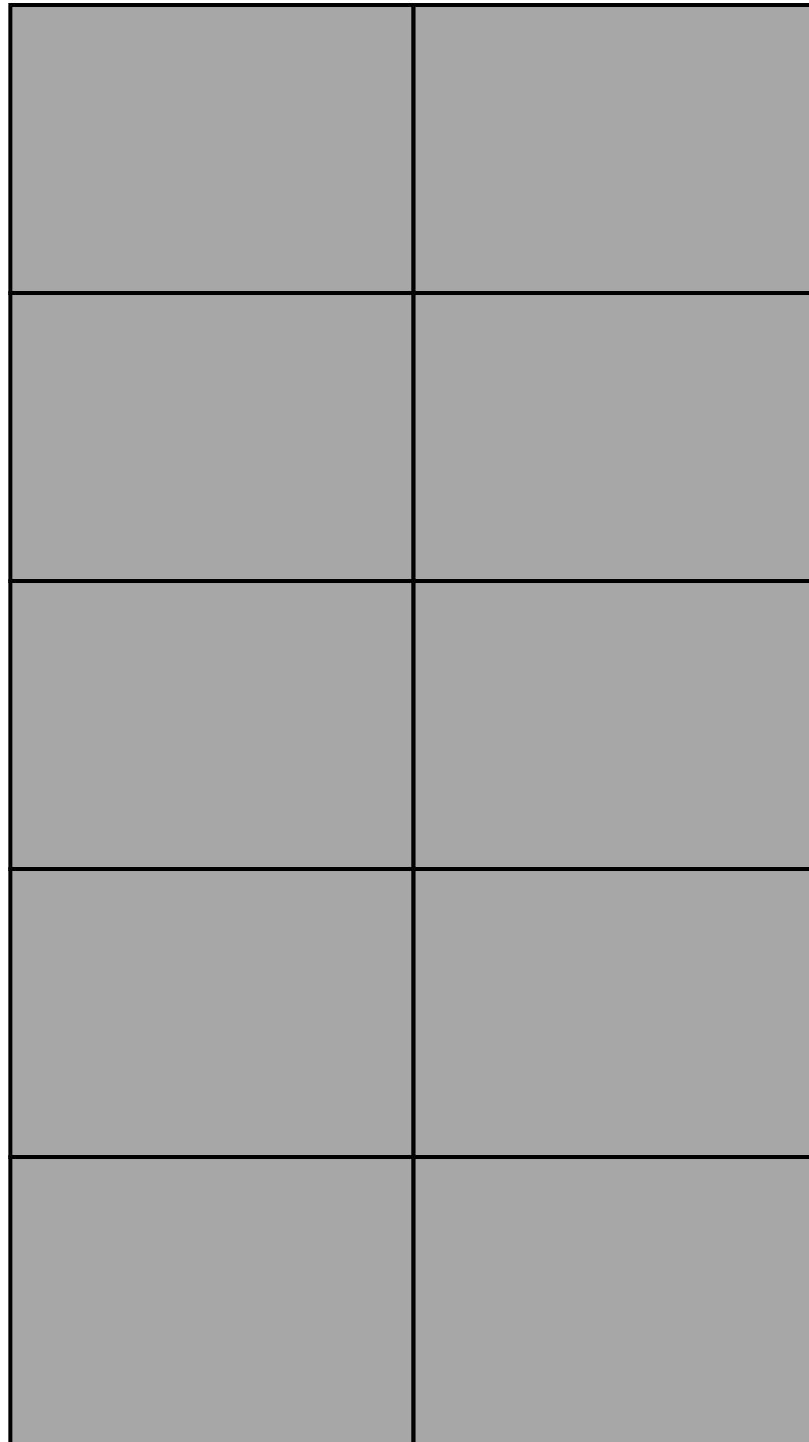
Jessica decided to use force!  She cut one tile in half, as shown below.  Then she added her two small unit tiles.
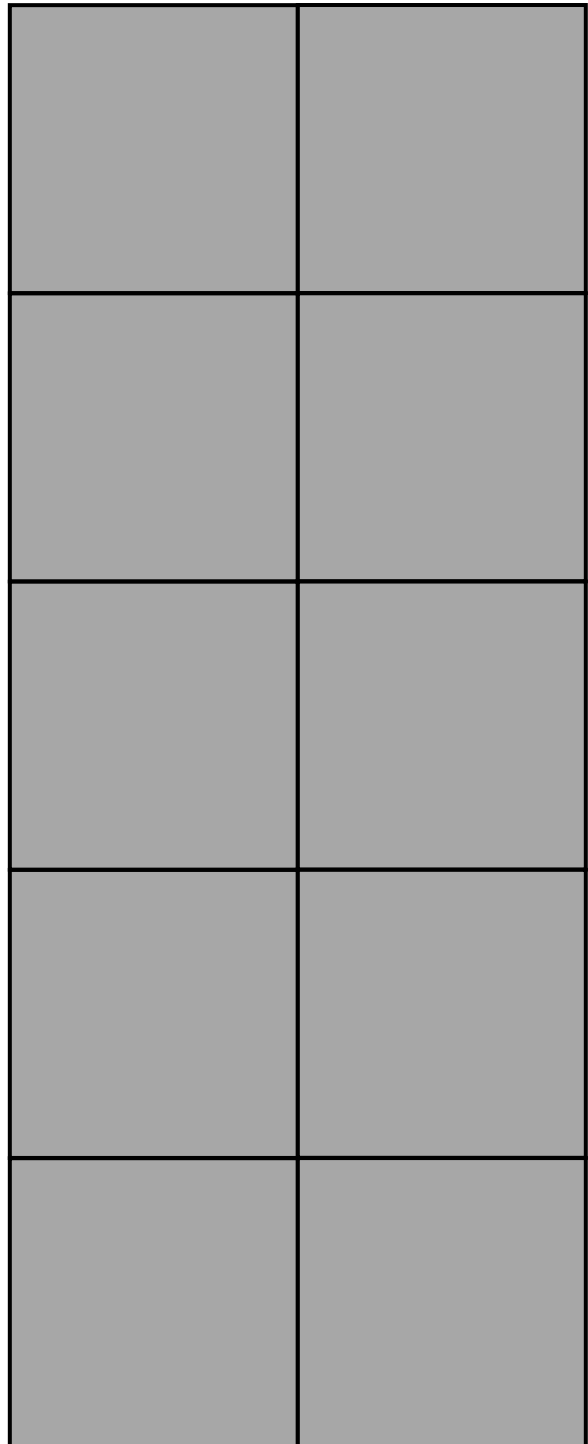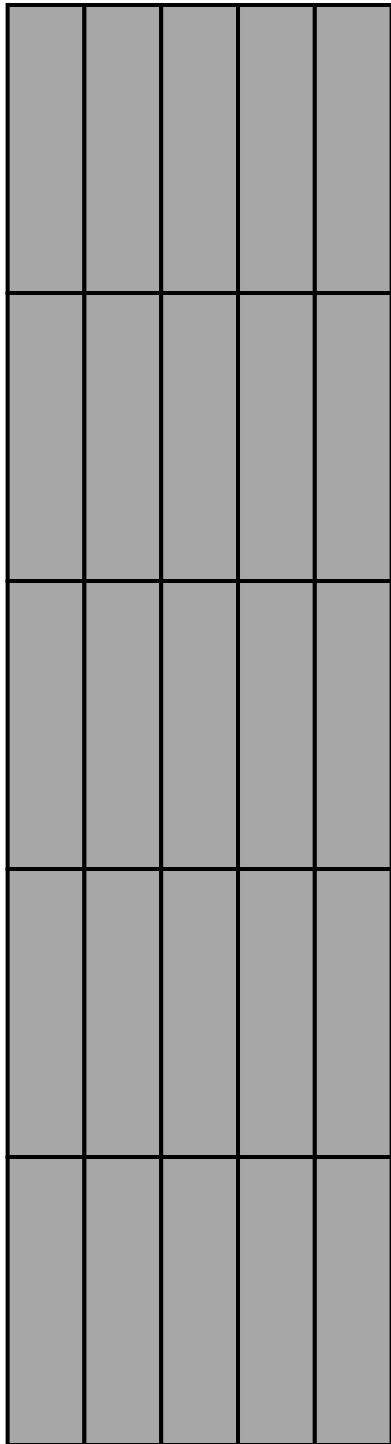


Figure A                    Figure B
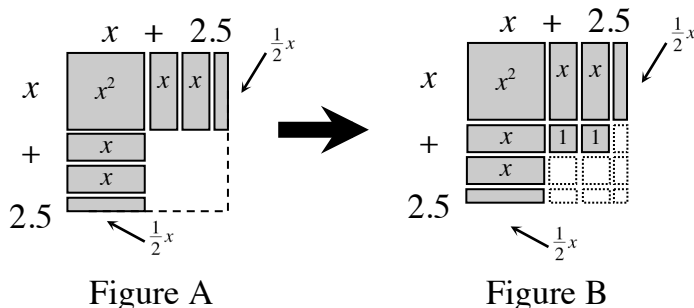
a.   How many small unit tiles are missing from Jessica's square?

b.   Write the graphing form of the function, name the vertex and $y$-intercept, and sketch the graph.
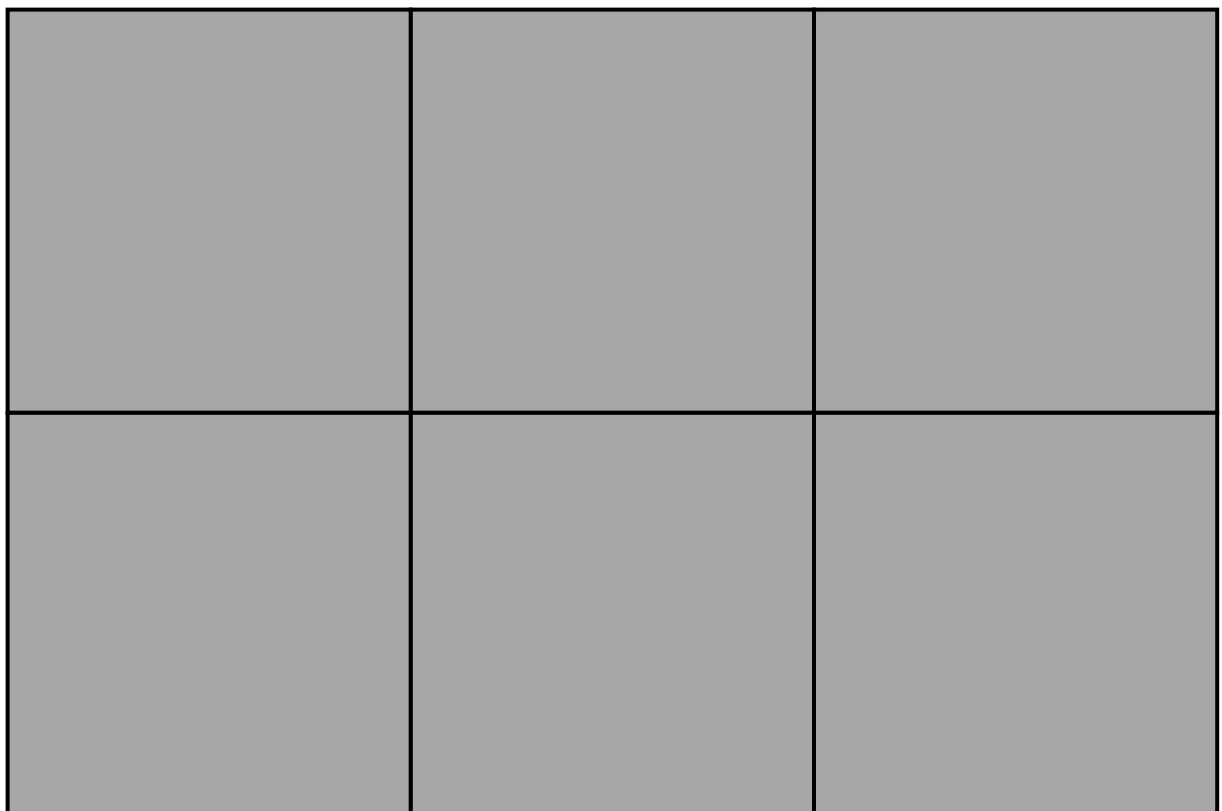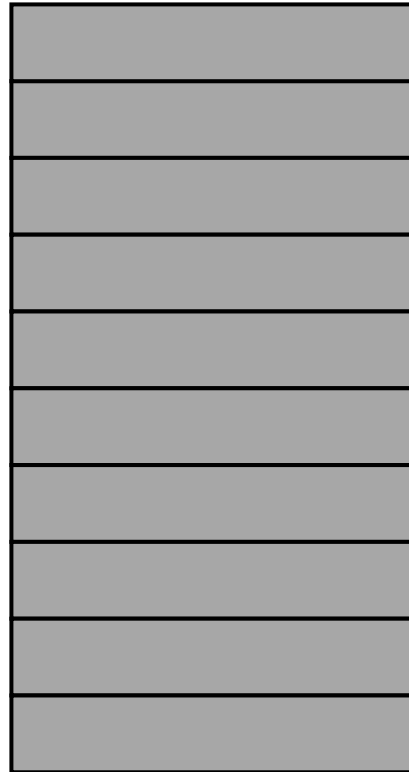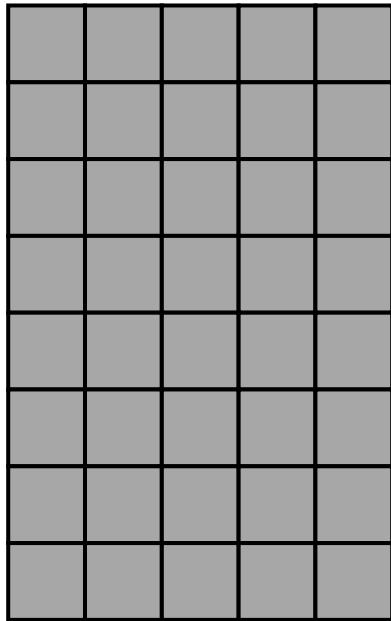
# Equation Mat

# Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

**1    Make sense of problems and persevere in solving them.**
- Find meaning in problems
- Look for entry points
- Analyze, conjecture and plan solution pathways
- Monitor and adjust
- Verify answers
- Ask themselves the question: "Does this make sense?"

**2    Reason abstractly and quantitatively.**
- Make sense of quantities and their relationships in problems
- Learn to contextualize and decontextualize
- Create coherent representations of problems

**3    Construct viable arguments and critique the reasoning of others.**
- Understand and use information to construct arguments
- Make and explore the truth of conjectures
- Recognize and use counterexamples
- Justify conclusions and respond to arguments of others

**4    Model with mathematics.**
- Apply mathematics to problems in everyday life
- Make assumptions and approximations to simplify a complicated situation
- Identify quantities in a practical situation
- Interpret results in the context of the situation and reflect on whether the results make sense

**5    Use appropriate tools strategically.**
- Consider the available tools when solving problems
- Are familiar with tools appropriate for their grade or course ( pencil and paper, concrete models, ruler, protractor, calculator, spreadsheet, computer programs, digital content located on a website, and other technological tools)
- Make sound decisions of which of these tools might be helpful

**6    Attend to precision.**
- Communicate precisely to others
- Use clear definitions, state the meaning of symbols and are careful about specifying units of measure and labeling axes
- Calculate accurately and efficiently

**7    Look for and make use of structure.**
- Discern patterns and structures
- Can step back for an overview and shift perspective
- See complicated things as single objects or as being composed of several objects

**8    Look for and express regularity in repeated reasoning.**
- Notice if calculations are repeated and look both for general methods and shortcuts
- In solving problems, maintain oversight of the process while attending to detail
- Evaluate the reasonableness of their immediate results