“A Common Core Approach to Meaningful Operations with Integers”

National Council of Teachers of Mathematics
Regional Conference: Philadelphia, Pennsylvania
November 1, 2016, 1:30-2:45 p.m. (Session 92)

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Key Shifts in Mathematics[1]:

- Greater focus on fewer topics
- Coherence: linking topics and thinking across grades
- Rigor: Pursue conceptual understanding, procedural skills and fluency, and application with equal intensity
- *Not a set of mnemonics or discrete procedures*

Workshop Goals:

- To apply and extend understandings of whole-number operations to rational number operations
- To gain knowledge, tools, and mindsets for strengthening the child’s conceptual understanding of positive and negative numbers
- To better understand how to incorporate the Mathematical Practice Standards into integer presentations
- To identify powerful learning tasks for practice and application

Hands-On Concept Development:

- Set Model (bead bars or integer chips)
- Number Line Model (horizontal or vertical number line diagrams)
- Number Patterns
- Fact Families
- Properties of Operations
- Real-World Contexts

Concrete to Abstract:

- Review the meaning(s) of the operation using whole numbers.
- Model the first case for integers using concrete materials.
- Students practice / guide observes.
- Present and practice other cases of the operation.
- Facilitate pattern seeking.
- Collaboratively write and revise an algorithm.
- Facilitate abstract practice and synthesis activities.

Varied Opportunities to Develop Mastery:

- Games and Puzzles for Individuals and Pairs, many with a built-in control of error
- Real-World and Mathematical Problems, including problems that incorporate other mathematical domains

[1] 1http://www.corestandards.org/other-resources/key-shifts-in-mathematics/

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CCSS.Math.Content.6.NS.C.5

Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

CCSS.Math.Content.6.NS.C.6

Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

The History of Negative Numbers (NRICH Mathematics)

Sea Level (NRICH Mathematics)

What is Your Sign? (Georgia Performance Standards Framework: Grade 7, Unit 3, 2nd ed., p. 8)

CCSS.Math.Content.6.NS.C.7

Understand ordering and absolute value of rational numbers.

Before Me / After Me (CRCT Study Guide, Grade 7, Version 10, p. 41)

Secret Message (North Carolina Middle Grades Resources: 2003 SCS, p. 26)

CCSS.Math.Content.6.NS.C.8

Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Helicopters and Submarines (Georgia Performance Standards Framework: Grade 7, Unit 3, 2nd ed., p. 18)

Taxi Cab (CRCT Study Guide, Grade 7, Version 10, p. 51)

CCSS.Math.Content.7.NS.A.1

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Concept Development for Integer Addition and/or Subtraction: Positive Second Term

A Cold Day in Fairbanks (North Carolina Middle Grades Resources: 2003 SCS, p. 24)

Using an Elevator to Evaluate Signed Number Expressions: Elevator Arithmetic (NCTM Illuminations)

James Bond Game (North Carolina Middle Grades Resources: 2003 SCS, p. 23)

Tug Harder (NRICH Mathematics)

First Connect Three (NRICH Mathematics)

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Concept Development for Integer Addition and/or Subtraction

Up, Down, Flying Around (NRICH Mathematics)

Modeling Signed Numbers with Heaps and Holes (North Carolina Middle Grades Resources: 2003 SCS, P. 9)

Adding and Subtracting Positive Numbers Article (NRICH Mathematics)

Adding and Subtracting Directed Numbers (Mathematics Assessment Project: Lessons)

Zip, Zelch, Zero (NCTM Illuminations)

Decontextualized Practice of Integer Addition and/or Subtraction

Sum Fun (North Carolina Middle Grades Resources: 2003 SCS)

Counting Beans – Addition of Integers (North Carolina Middle Grades Resources: 2003 SCS)

Addition Integer Logic (North Carolina Middle Grades Resources: 2003 SCS)

Lining Up Dominoes—Subtraction of Integers (North Carolina Middle Grades Resources: 2003 SCS)[2]

Connect Three (NRICH Mathematics)

I Have, Who Has—Integer Addition, Subtraction (North Carolina Middle Grades Resources: 2003 SCS, p. 44)

Using Integer Addition and Subtraction to Solve Real-World and Mathematical Problems

Consecutive Numbers (NRICH Mathematics)

Consecutive Negative Numbers (NRICH Mathematics)

Using Positive and Negative Numbers in Context (Mathematics Assessment Project: Lessons)

Power Up (NCTM Illuminations: Investigation) / (NCTM Illuminations: Game)

Temperature on the Beach (North Carolina Middle Grades Resources: 2003 SCS, p. 25)

CCSS.Math.Content.7.NS.A.2

Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

Using Patterns to Develop the Rules for Multiplying and Dividing Integers (North Carolina Middle Grade Resources: 2003 SCS, p. 16)

Four in a Row—Multiplication of Integers (North Carolina Middle Grades Resources: 2003 SCS, p. 40)

Lining Up Dominoes—Multiplication of Integers (North Carolina Middle Grades Resources: 2003 SCS, p. 41)

Multiplication Integer Logic (North Carolina Middle Grades Resources: 2003 SCS)

Missing Multipliers (NRICH Mathematics)

[2] Please note that this task (Lining Up Dominoes—Subtraction of Integers) has an error in the first computation: -3 - (-7). I remedied the card layout by omitting dominoes 2-9, thus picking up again with the correct answer of 4.

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CCSS.Math.Content.7.NS.A.3

Solve real-world and mathematical problems involving the four operations with rational numbers.

Cumulative Review and Practice of Integer Operations

"A Poster" (Georgia Performance Standards Framework: Grade , Unit 3, 2nd ed., p. 32)

Integer Computation Square Puzzle (North Carolina Middle Grades Resources: 2003 SCS, p. 18)

Integer Computation Triangle Puzzle (North Carolina Middle Grades Resources: 2003 SCS, p. 19)

Number Tiles—Integer Operations (North Carolina Middle Grades Resources: 2003 SCS)

Sums and Products (Georgia Performance Standards Framework: Grade 7, Unit 3, 2nd ed., p. 21)

Using Integer Operations to Solve Real-World and Mathematical Problems

Weights (NRICH Mathematics)

Strange Bank Account and Strange Bank Account Part 2 (NRICH Mathematics)

Always, Sometimes, Never (Georgia Performance Standards Framework: Grade 7, Unit 3, 2nd ed., p. 25)

Making a Test (Georgia Performance Standards Framework: Grade 7, Unit 3, 2nd ed., p. 27)

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CCSS.Math.Content.6.NS.C.5
Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

CCSS.Math.Content.6.NS.C.6
Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

CCSS.Math.Content.6.NS.C.6.a
Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite.

CCSS.Math.Content.6.NS.C.6.b
Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

CCSS.Math.Content.6.NS.C.6.c
Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

CCSS.Math.Content.6.NS.C.7
Understand ordering and absolute value of rational numbers.

CCSS.Math.Content.6.NS.C.7.a
Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right.

CCSS.Math.Content.6.NS.C.7.b
Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write -3°C > -7°C to express the fact that -3°C is warmer than -7°C.

CCSS.Math.Content.6.NS.C.7.c
Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of -30 dollars, write |-30| = 30 to describe the size of the debt in dollars.

CCSS.Math.Content.6.NS.C.7.d
Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.

CCSS.Math.Content.6.NS.C.8
Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

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CCSS.Math.Content.7.NS.A.1
Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

CCSS.Math.Content.7.NS.A.1.a
Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.

CCSS.Math.Content.7.NS.A.1.b
Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

CCSS.Math.Content.7.NS.A.1.c
Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

CCSS.Math.Content.7.NS.A.1.d
Apply properties of operations as strategies to add and subtract rational numbers.

CCSS.Math.Content.7.NS.A.2
Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

CCSS.Math.Content.7.NS.A.2.a
Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

CCSS.Math.Content.7.NS.A.2.b
Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

CCSS.Math.Content.7.NS.A.2.c
Apply properties of operations as strategies to multiply and divide rational numbers.

CCSS.Math.Content.7.NS.A.3
Solve real-world and mathematical problems involving the four operations with rational numbers.[3]

[3] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

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CCSS.Math.Practice.MP1 ***Make sense of problems and persevere in solving them.***

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

CCSS.Math.Practice.MP2 ***Reason abstractly and quantitatively.***

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

CCSS.Math.Practice.MP3 ***Construct viable arguments and critique the reasoning of others.***

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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CCSS.Math.Practice **_MP4 Model with mathematics._**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

CCSS.Math.Practice.MP5 **_Use appropriate tools strategically._**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

CCSS.Math.Practice.MP6 **_Attend to precision._**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

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CCSS.Math.Practice.MP7 ***Look for and make use of structure.***

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression x2 + 9x + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 - 3(x - y)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

CCSS.Math.Practice.MP8 ***Look for and express regularity in repeated reasoning.***

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y - 2)/(x - 1) = 3. Noticing the regularity in the way terms cancel when expanding (x - 1)(x + 1), (x - 1)(x2 + x + 1), and (x - 1)(x3 + x2 + x + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.