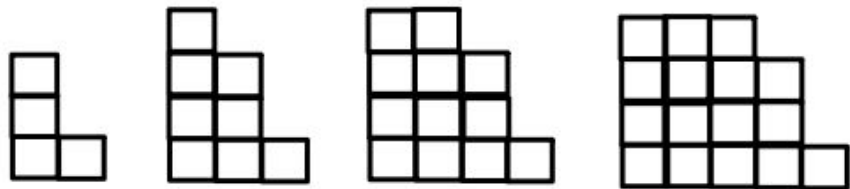
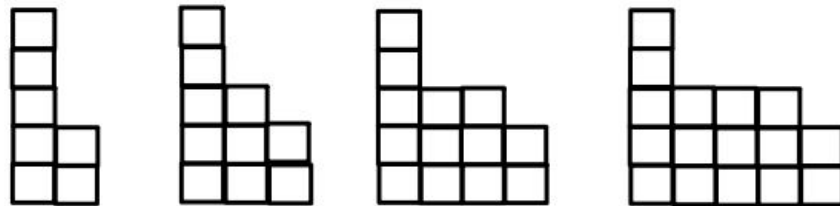
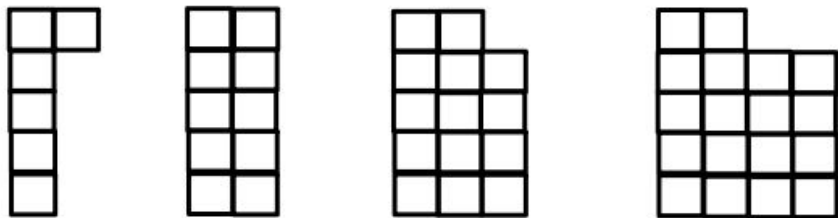


Which one doesn't belong?



9, 13, 17, 21



Describing patterns algebraically

Finding the next or finding the n th?

2016 NCTM Regional - Philadelphia, PA

Belinda Thompson and Erin McDonough

11/01/2016



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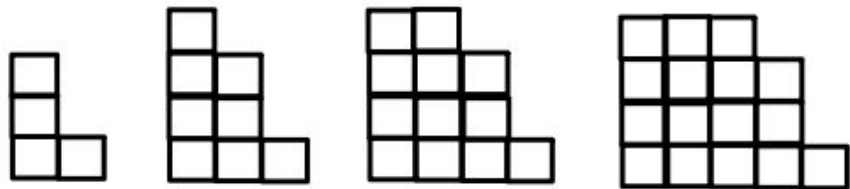
Belinda Thompson

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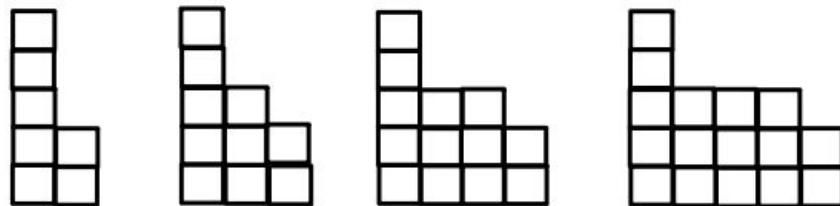
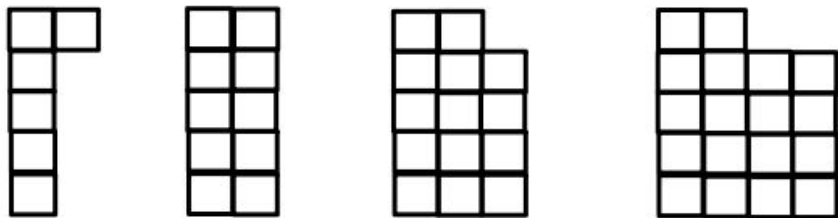
@teachingtweaks

belindathompson@learnzillion.com

Which one doesn't belong?



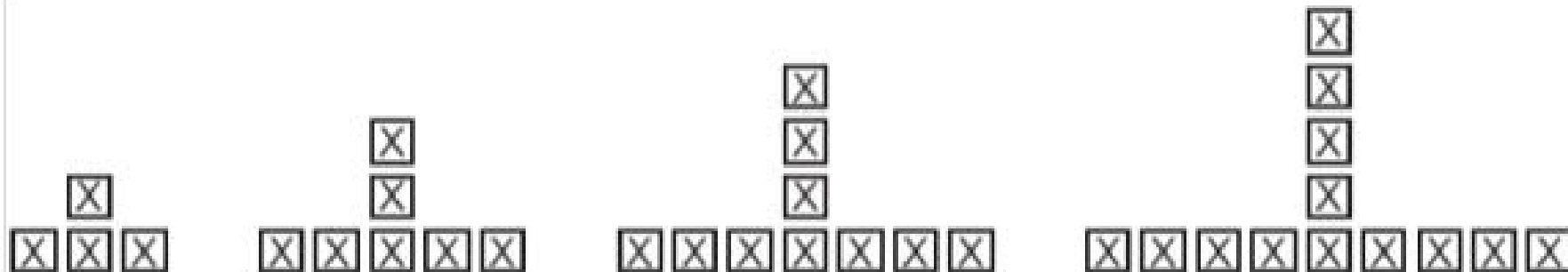
9, 13, 17, 21



Do math together

How do you see this pattern growing?

- Describe **two** different ways you see it growing.
- Write a bunch of stuff down because somebody will be looking at your work!



3-2-1 Sharing

3 People

- Work on the problem independently

2 Silent Passes

- Form a triad
- Pass your work to the right
- Jot notes: How is the approach the **same**?
How is it **different**?
- Pass the work to the right again and jot

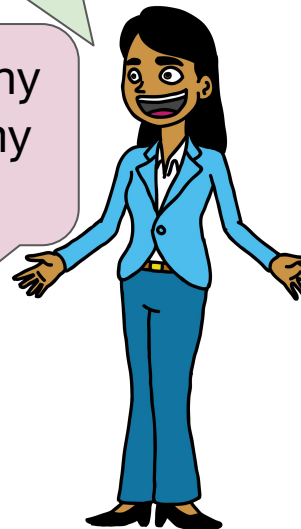
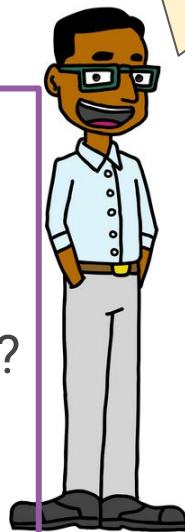
1 Discussion

- Where do you see “finding the next” and where do you see “finding the nth”?

I added each time to find the next one.

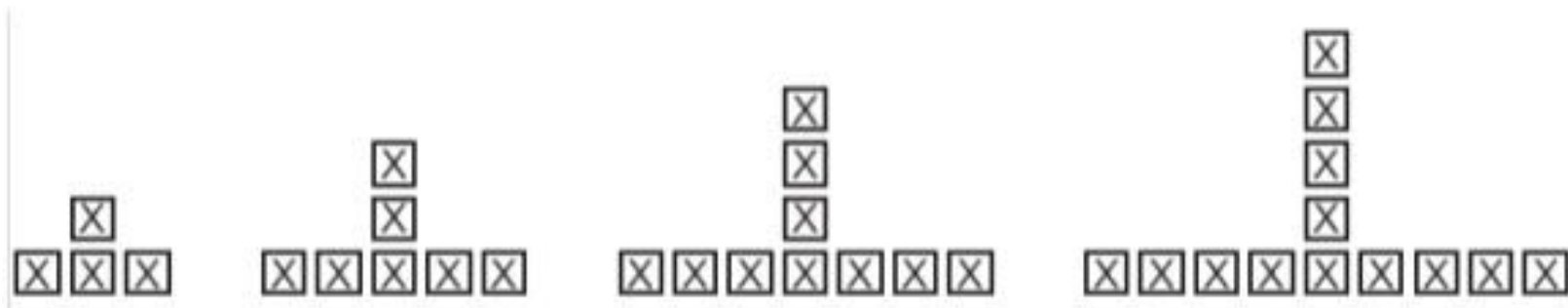
I'm not sure which one I used.

I can find any term with my rule.



How do you see this figure growing?

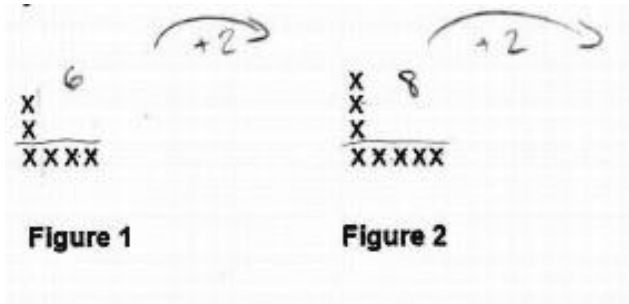
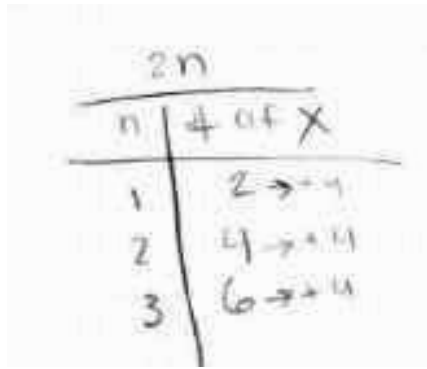
How many Xs in the 27th figure?



Let's Share!

| A recursive explanation (find the next) | A functional explanation (find the nth) | Another functional explanation |
|---|---|--|
| Add 3 to get the next | $3n + 1$ | $4 + 3(n-1)$ |

What's the difference?

| | Recursive rule | Functional rule |
|----------------|--|--|
| | Find the next | Find the n th |
| Sounds like... | "I added two to this one to get the next one" | "To find the 10th figure, I can multiply 2 times 10 and add 4" |
| Looks like... |  <p>Figure 1</p> <p>Figure 2</p> |  |

How do you see this figure growing?

X
X
XXXX

Figure 1

X
X
X
XXXXX

Figure 2

X
X
X
X
XXXXXX

Figure 3

Student work from Erin's class

- Where do you see recursive thinking?
- Where do you see functional thinking?

X
X
XXXX

Figure 1

X
X
X
XXXXX

Figure 2

X
X
X
X
XXXXXX

Figure 3

Figure = x $x = y$

$y + 2$

| x | y |
|---|----|
| 1 | 6 |
| 2 | 8 |
| 3 | 10 |
| 4 | 12 |

Arrows indicate a constant difference of +2 between consecutive y-values.

Student work from Erin's class

- Where do you see recursive thinking?
- Where do you see functional thinking?

7.) Write the rule to find the number of Xs in any figure number, n . Show your work.

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

| n | # of X |
|-----|--------|
| 1 | 5 |
| 2 | 8 |
| 3 | 11 |

Figure 7

| n | # of X |
|-----|--------------------|
| 1 | 2 $\rightarrow +4$ |
| 2 | 4 $\rightarrow +4$ |
| 3 | 6 $\rightarrow +4$ |

Handwritten notes:

- A circled formula $2n+4$ is written in the top right corner.
- A formula $3n+2$ is written above the table in Figure 6.

Student work from Erin's class

- Where do you see recursive thinking?
- Where do you see functional thinking?

Recursive:
"This one + 2 is
the next one."

Write the rule to find the number of Xs in any figure number, n . Show your work.

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7

Figure 8

Figure 9

Figure 10

Figure 11

Figure 12

Figure 13

Figure 14

Figure 15

Figure 16

Figure 17

Figure 18

Figure 19

Figure 20

Figure 21

Figure 22

Figure 23

Figure 24

Figure 25

Figure 26

Figure 27

Figure 28

Figure 29

Figure 30

Figure 31

Figure 32

Figure 33

Figure 34

Figure 35

Figure 36

Figure 37

Figure 38

Figure 39

Figure 40

Figure 41

Figure 42

Figure 43

Figure 44

Figure 45

Figure 46

Figure 47

Figure 48

Figure 49

Figure 50

Figure 51

Figure 52

Figure 53

Figure 54

Figure 55

Figure 56

Figure 57

Figure 58

Figure 59

Figure 60

Figure 61

Figure 62

Figure 63

Figure 64

Figure 65

Figure 66

Figure 67

Figure 68

Figure 69

Figure 70

Figure 71

Figure 72

Figure 73

Figure 74

Figure 75

Figure 76

Figure 77

Figure 78

Figure 79

Figure 80

Figure 81

Figure 82

Figure 83

Figure 84

Figure 85

Figure 86

Figure 87

Figure 88

Figure 89

Figure 90

Figure 91

Figure 92

Figure 93

Figure 94

Figure 95

Figure 96

Figure 97

Figure 98

Figure 99

Figure 100

Figure 101

Figure 102

Figure 103

Figure 104

Figure 105

Figure 106

Figure 107

Figure 108

Figure 109

Figure 110

Figure 111

Figure 112

Figure 113

Figure 114

Figure 115

Figure 116

Figure 117

Figure 118

Figure 119

Figure 120

Figure 121

Figure 122

Figure 123

Figure 124

Figure 125

Figure 126

Figure 127

Figure 128

Figure 129

Figure 130

Figure 131

Figure 132

Figure 133

Figure 134

Figure 135

Figure 136

Figure 137

Figure 138

Figure 139

Figure 140

Figure 141

Figure 142

Figure 143

Figure 144

Figure 145

<

Student work from Erin's class

- What's going on here?
- How does this fit or not fit?

7.) Write the rule to find the number of Xs in any figure number, n . Show your work.

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7

Figure 8

Figure 9

Figure 10

Figure 11

Figure 12

Figure 13

Figure 14

Figure 15

Figure 16

Figure 17

Figure 18

Figure 19

Figure 20

Figure 21

Figure 22

Figure 23

Figure 24

Figure 25

Figure 26

Figure 27

Figure 28

Figure 29

Figure 30

Figure 31

Figure 32

Figure 33

Figure 34

Figure 35

Figure 36

Figure 37

Figure 38

Figure 39

Figure 40

Figure 41

Figure 42

Figure 43

Figure 44

Figure 45

Figure 46

Figure 47

Figure 48

Figure 49

Figure 50

Figure 51

Figure 52

Figure 53

Figure 54

Figure 55

Figure 56

Figure 57

Figure 58

Figure 59

Figure 60

Figure 61

Figure 62

Figure 63

Figure 64

Figure 65

Figure 66

Figure 67

Figure 68

Figure 69

Figure 70

Figure 71

Figure 72

Figure 73

Figure 74

Figure 75

Figure 76

Figure 77

Figure 78

Figure 79

Figure 80

Figure 81

Figure 82

Figure 83

Figure 84

Figure 85

Figure 86

Figure 87

Figure 88

Figure 89

Figure 90

Figure 91

Figure 92

Figure 93

Figure 94

Figure 95

Figure 96

Figure 97

Figure 98

Figure 99

Figure 100

Figure 101

Figure 102

Figure 103

Figure 104

Figure 105

Figure 106

Figure 107

Figure 108

Figure 109

Figure 110

Figure 111

Figure 112

Figure 113

Figure 114

Figure 115

Figure 116

Figure 117

Figure 118

Figure 119

Figure 120

Figure 121

Figure 122

Figure 123

Figure 124

Figure 125

Figure 126

Figure 127

Figure 128

Figure 129

Figure 130

Figure 131

Figure 132

Figure 133

Figure 134

Figure 135

Figure 136

Figure 137

Figure 138

Figure 139

Figure 140

Figure 141

Figure 142

Figure 143

Figure 144

Figure 145

Figure 146

Figure 147

Figure 148

Figure 149

Figure 150

Figure 151

Figure 152

Figure 153

Figure 154

Figure 155

Figure 156

Figure 157

Figure 158

Figure 159

Figure 160

Figure 161

Figure 162

Figure 163

Figure 164

Figure 165

Figure 166

Figure 167

Figure 168

Figure 169

Figure 170

Figure 171

Figure 172

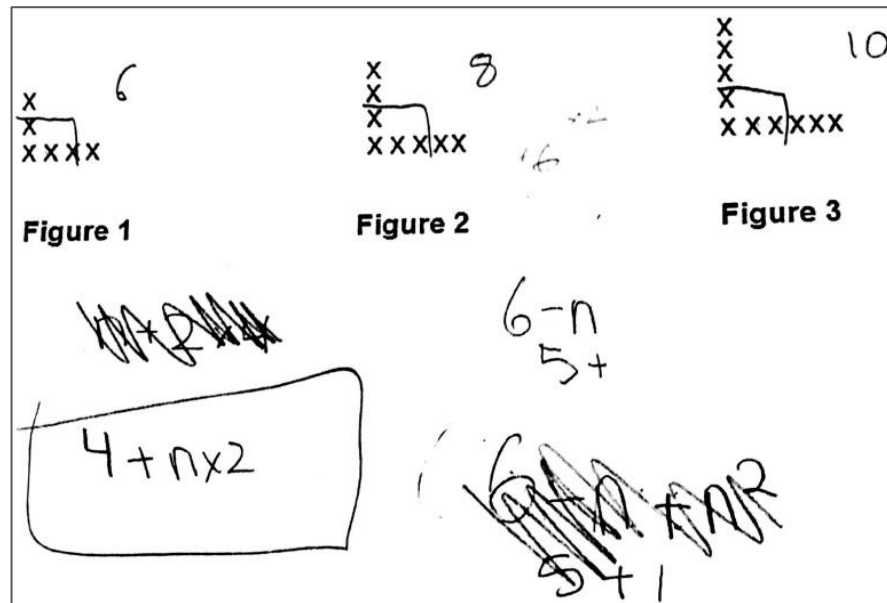
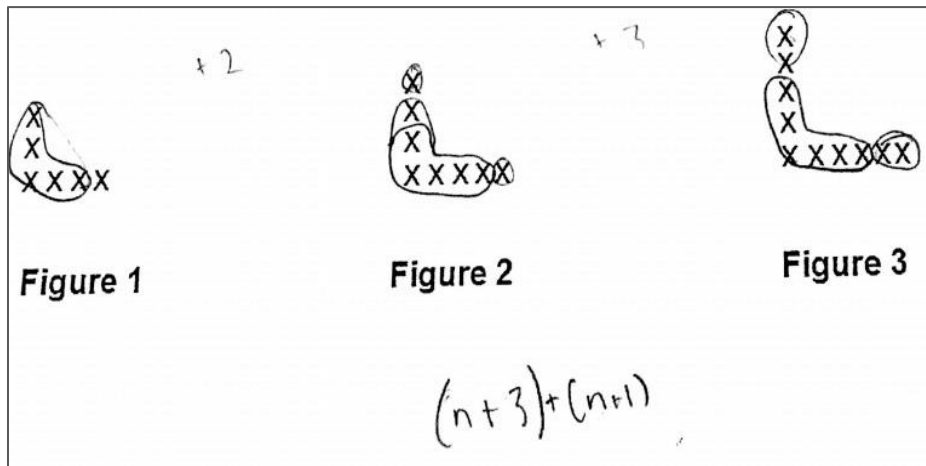
Figure 173

Figure 174

Figure 175

Figure 176

Student work from Erin's class



What's the same?

What's different?

The connection between recursive and functional.

| Figure # | Total Xs | |
|----------|----------|-------------------|
| 1 | 6 | 6 |
| 2 | 8 | $6 + 2$ |
| 3 | 10 | $(6 + 2) + 2$ |
| 4 | 12 | $((6+2) + 2) + 2$ |
| ... | | ... |
| n | $2n + 4$ | $6 + 2 (n - 1)$ |

X
X
XXXX

Figure 1

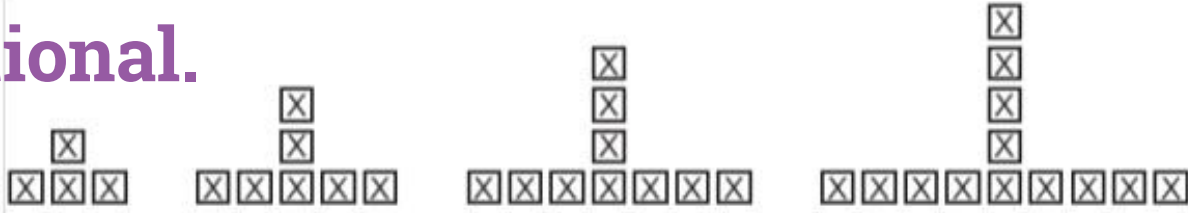
X
X
X
XXXXX

Figure 2

X
X
X
X
XXXXXX

Figure 3

The connection between recursive and functional.



| Figure # | Total Xs | |
|----------|----------|-------------------|
| 1 | 4 | 4 or (1 + 3) |
| 2 | 7 | 4 + 3 |
| 3 | 10 | (4 + 3) + 3 |
| 4 | 13 | ((4 + 3) + 3) + 3 |
| ... | | ... |
| n | $3n + 1$ | $4 + 3(n - 1)$ |

So what?

- Engagement
 - Students enjoy them and often want more challenging ones
- Discourse
 - Students have to make and defend claims
- Low floor, high ceiling
 - Multiple entry points usually begin with counting
- Many perspectives on “How does this pattern grow?”
 - Recursive thinking is allowed and
 - Functional expressions all simplify to the same expression

Connections to other topics

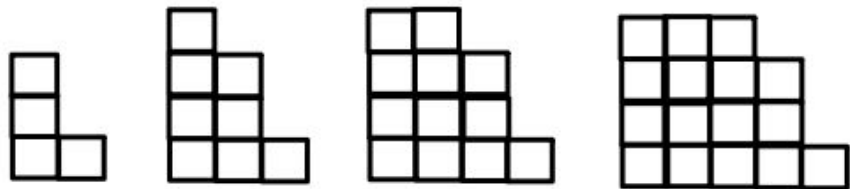
Not a lot of explicit mention in CCSS G6-8 about patterns so we may think it's not important or useful :(

- Proportional relationships
- Slope
 - $y=mx+b$
 - Easier for students to get a sense of slope as “for every change of 1 in x, y changes by m (we can see this in the recursive relationship
- Functions
 - Functions are defined by expressions
 - This might make for a smoother transition to functions

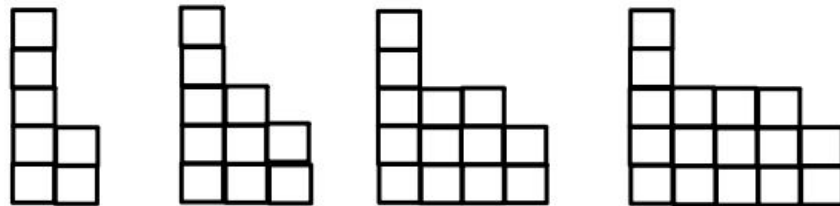
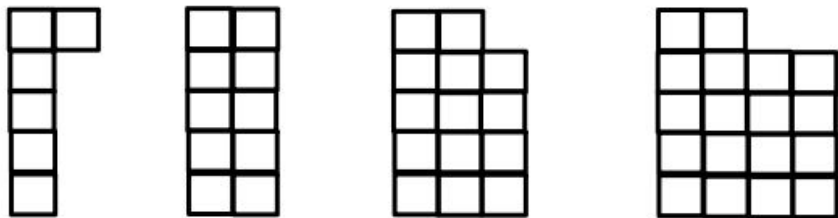
Practical tips for the transition from recursive thinking to functional thinking

- Don't discourage recursive thinking. It's the entry point!
- Do lots of drawing!
 - Draw the next and draw the 27th
 - What's changing and what's not?
- Teach them how to organize their thinking when they become disorganized.
- Look for and describe the connections
 - Between recursive rule and functional rule
 - Between different forms of the functional rule
- Do these often!

Which one doesn't belong?



9, 13, 17, 21



Thank you! Please keep in touch!



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