Make a Mistake? Now What?

Eliciting Evidence of Student Thinking

Barbara Lynch
NCTM Institute Facilitator

Lakewood City Schools
Lakewood, Ohio

Lakewood, Ohio

NCTM Principles to Action

As teachers plan lessons, key components for them to consider are **student struggles** and **misconceptions** that might arise.

Teachers acknowledge that struggle is an important and natural part of learning and doing mathematics.

Principles to Action, Page 48

John Dewey

~~Failure~~ is instructive. The person who *really thinks* learns quite as much from their ~~failures~~ as from the successes.

Making Mistakes

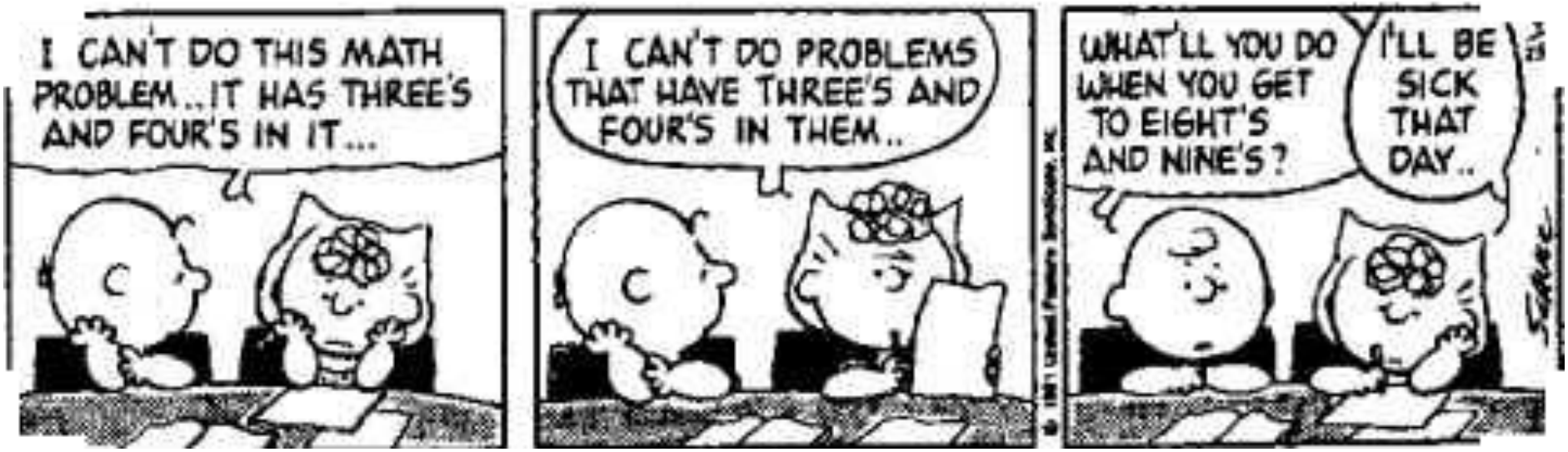
STRUGGLE

Misconceptions

My intentions:

- Errors are an important, natural part of learning mathematics.
- Mistakes are valuable to a teacher
- Mistakes are valuable to a student
- Student Error Analysis
- Teacher Favorite No

Learned Helplessness



I don't know what to do, and will wait for the teacher explanation.

Classroom Poster

1. Dividing by zero is troublesome.
2. Take risks, make mistakes.
3. Be a pattern sniffer
4. Questions are really important.
5. Don't be a spectator.
   Try something. Do something.
   Don't let others do your thinking.

Examples of errors/misconceptions

Data were collected on the **depth of a dive** of penguins and the **duration of the dive**. The following linear model is a fairly good summary of the data, where $\boldsymbol{t}$ is the duration of the dive in minutes and $\boldsymbol{d}$ is the depth of the dive in yards.

The equation for the model is $\boldsymbol{d = 2.915t + 0.015}$

In context to this problem, explain the meaning of $\mathbf{2.915}$

Student answers

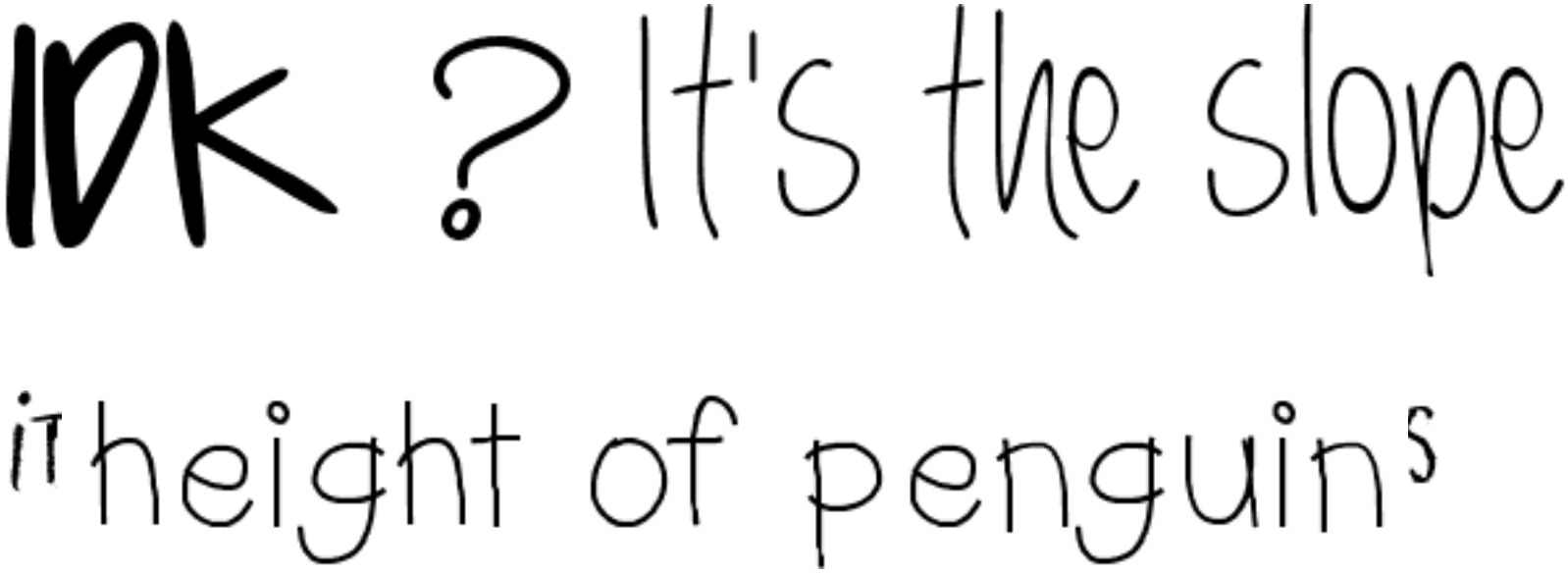
In context to this problem, explain the meaning of **2.915**

The deeper the dive, the longer the penguin stays under water.

For about every 3 yards below the surface, the penguin in under water for 1 more minute.

Student answers

In context to this problem, explain the meaning of **2.915**



Student Misconceptions

Think about this problem as a middle school math student

Dane and Quinn collect sports cards. Dane has 4 cards for every 3 cards that Quinn has. If Dane gives Quinn ½ his cards, what will be the new ratio of Dane's cards to Quinn's?

What if Dane originally had 60 cards? How many does Quinn have after the switch?

Common Misconceptions

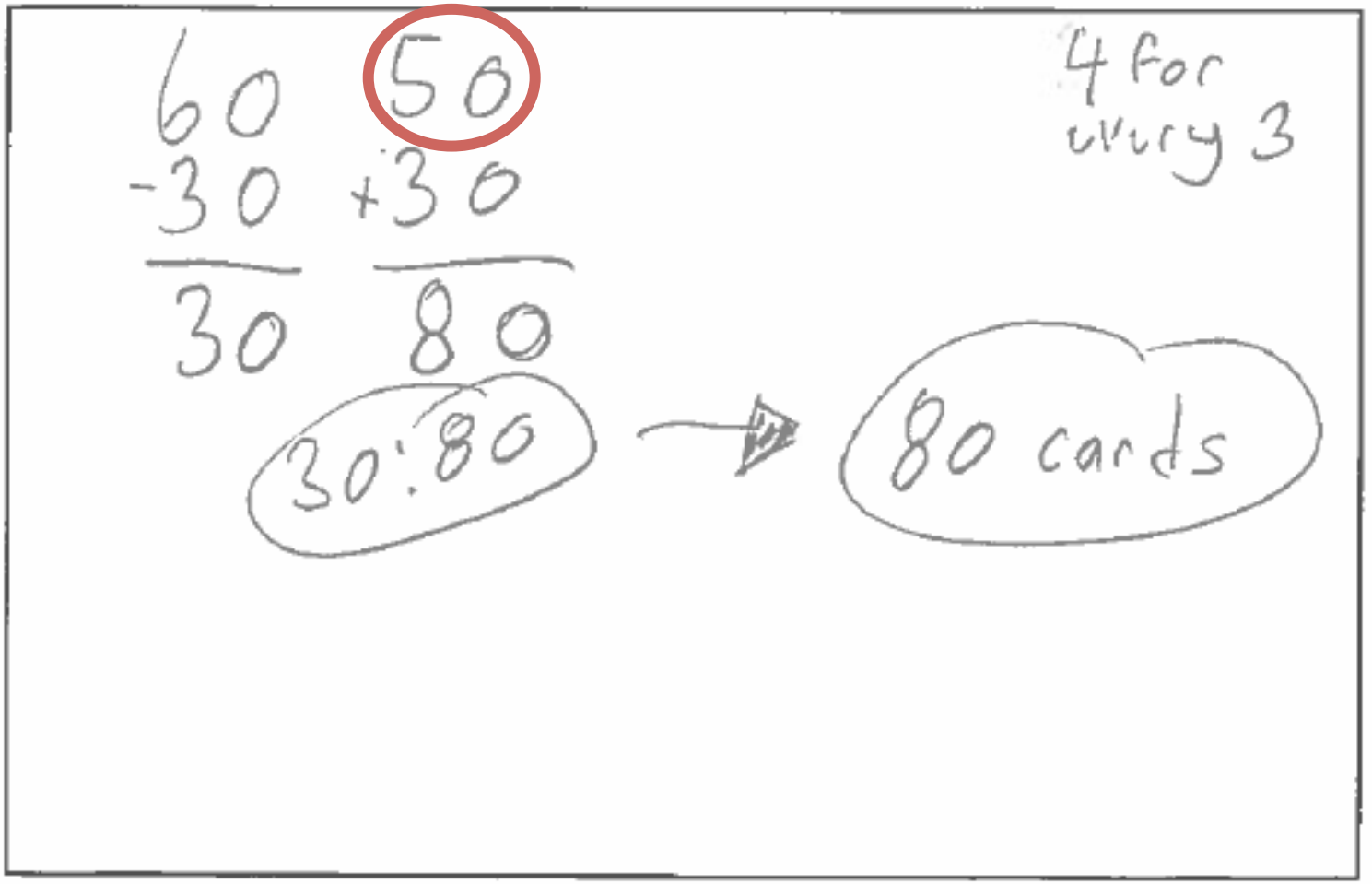
Ratio & Proportion

- Apply procedures incorrectly.
- Create incorrect ratio/proportion
- Unable to determine if answer makes sense.
- Rules for fractions don't necessarily apply to ratio
- Problem is not a routine problem.

Student Work

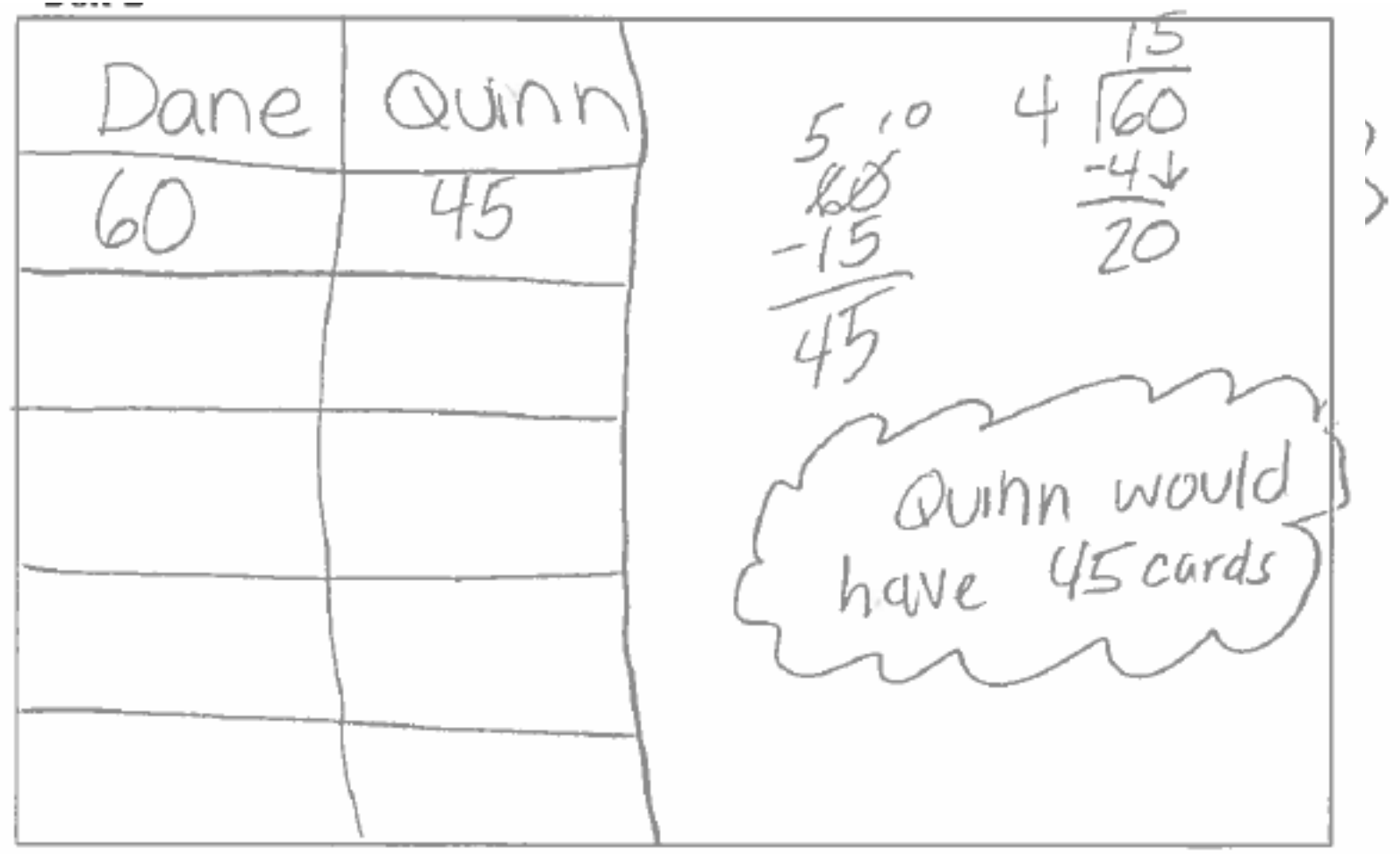
Careless Errors (Don't dismiss careless mistakes!)

Computational Errors



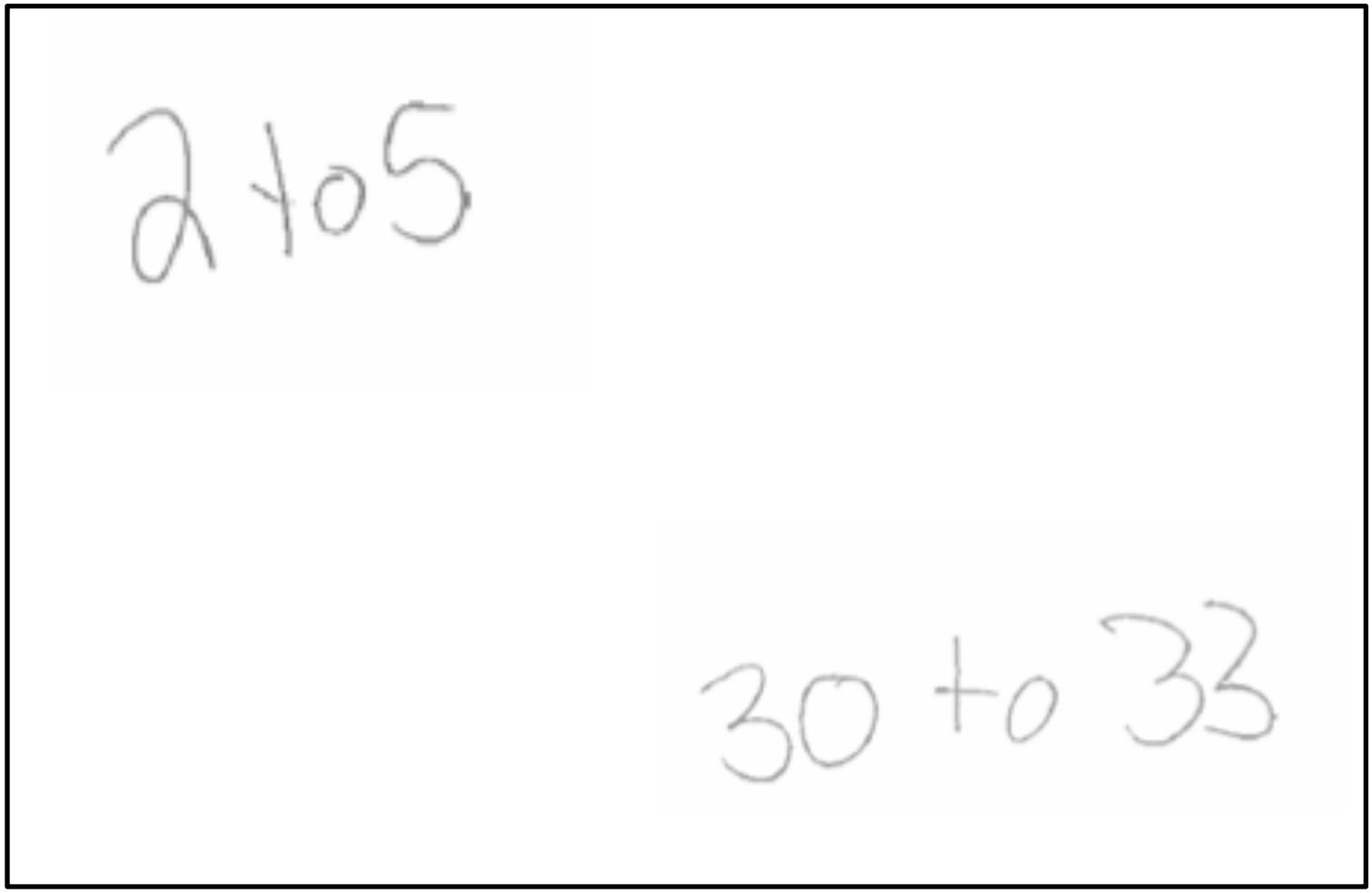
Student Work

Procedural Errors (Did you answer the question?)



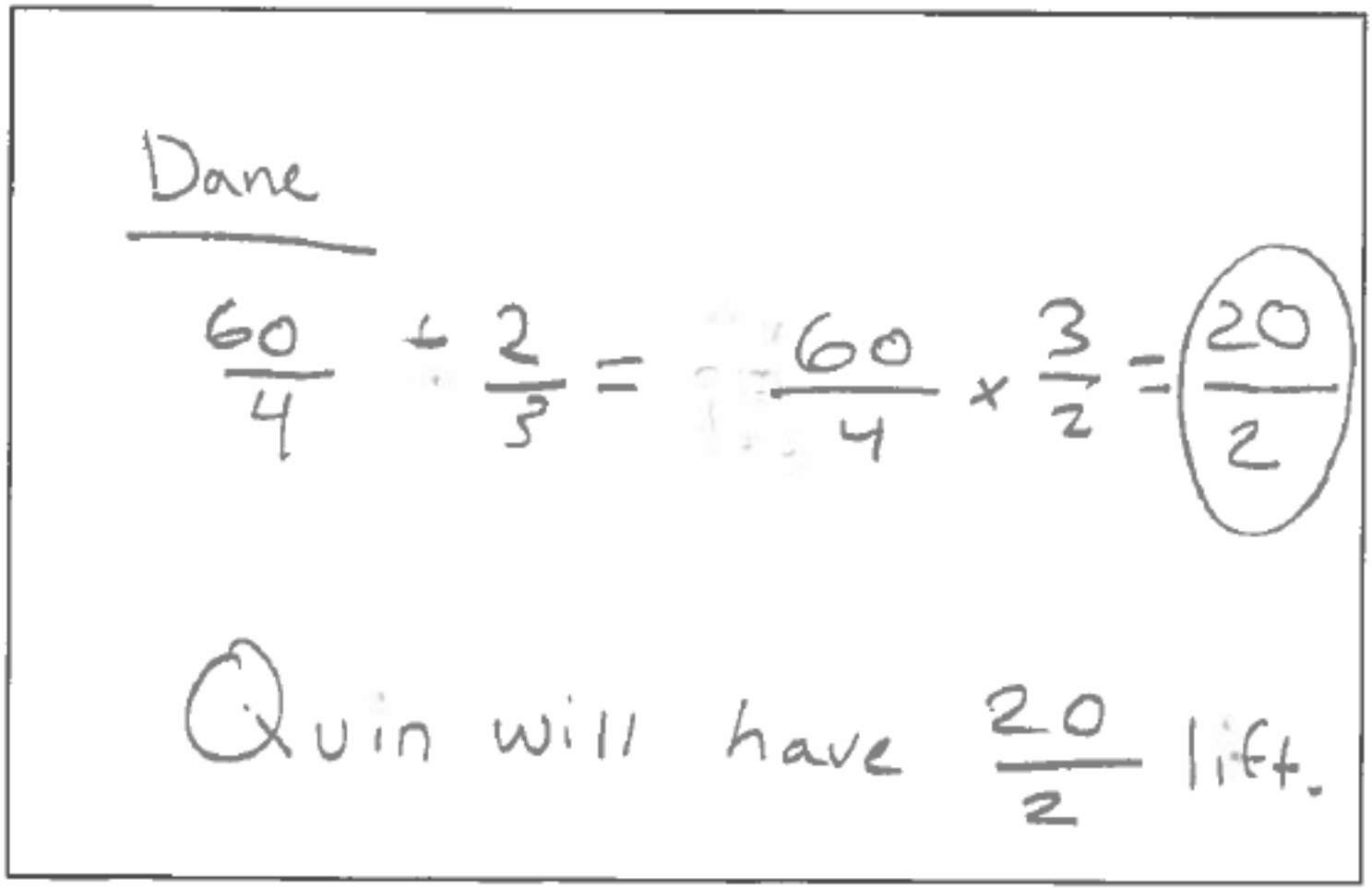
Student Work

Procedural Errors



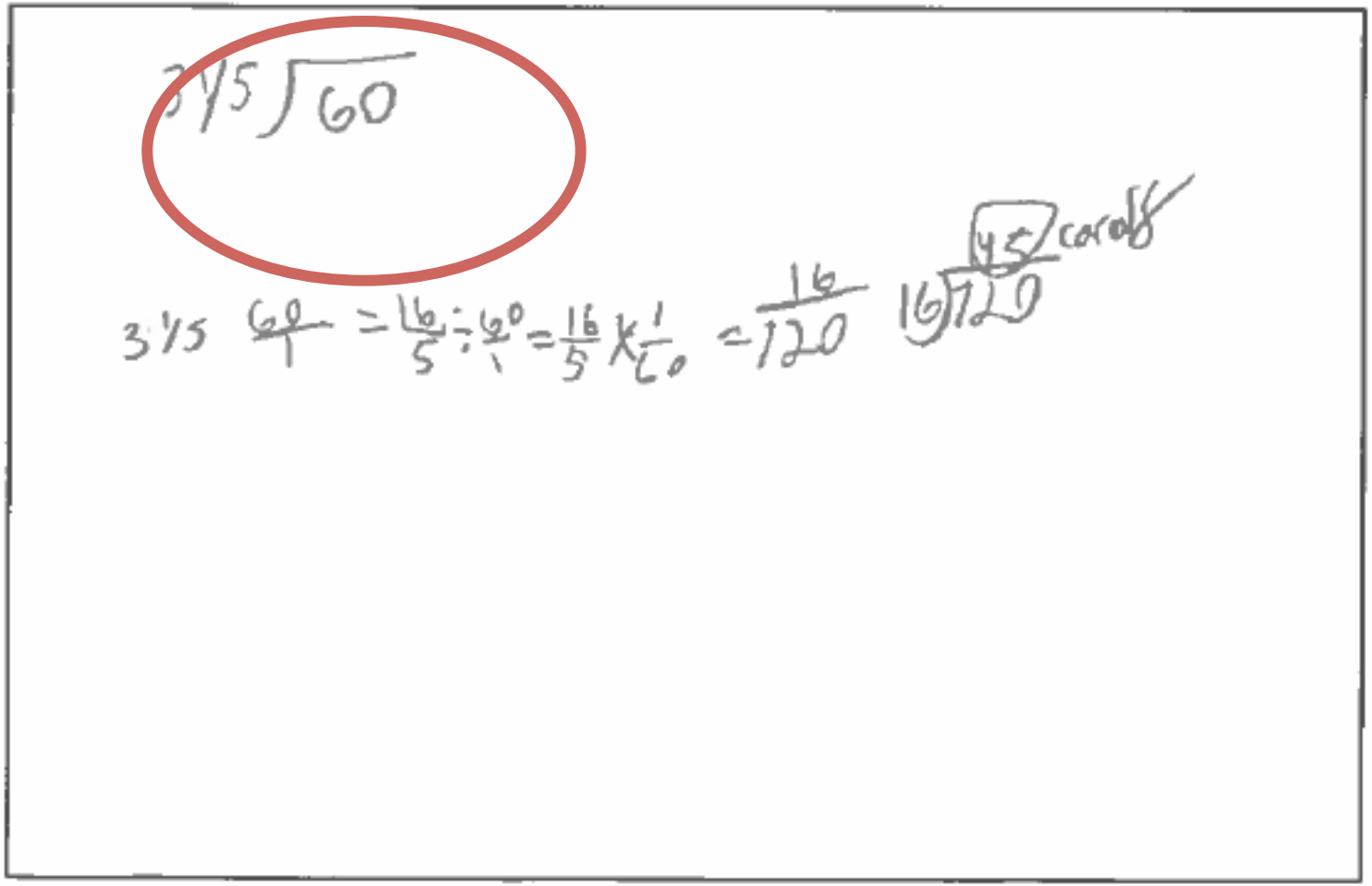
Student Work

Translation Trouble



Student Work

Conceptual Errors and more



Solution using a Tape Diagram

Before

Dane	15	15	15	15
Quinn	15	15	15	

4 : 3

After

Dane	15	15			
Quinn	15	15	15	15	15

2 : 5

What if Dane originally had 60 cards? How many does Quinn have after the switch?

Hinge Question

A check for understanding at a 'hinge-point' in a lesson

1. It is the point where you move from one key idea/activity/point on to another.
2. Understanding the content before the hinge is a prerequisite for the next chunk of learning.
3. Immediate feedback.

Hinge Question

Rational vs Irrational

Molly entered $7 \div 31$ on her calculator.

This is what appeared: 0.24137931034482758620 6

Is $\frac{7}{31}$ rational or irrational? Explain your answer

1 student – I'm not sure

9 students – Irrational because it's a decimal and doesn't repeat

6 students – Rational

7 students – Rational (All fractions are rational)

Hinge Question

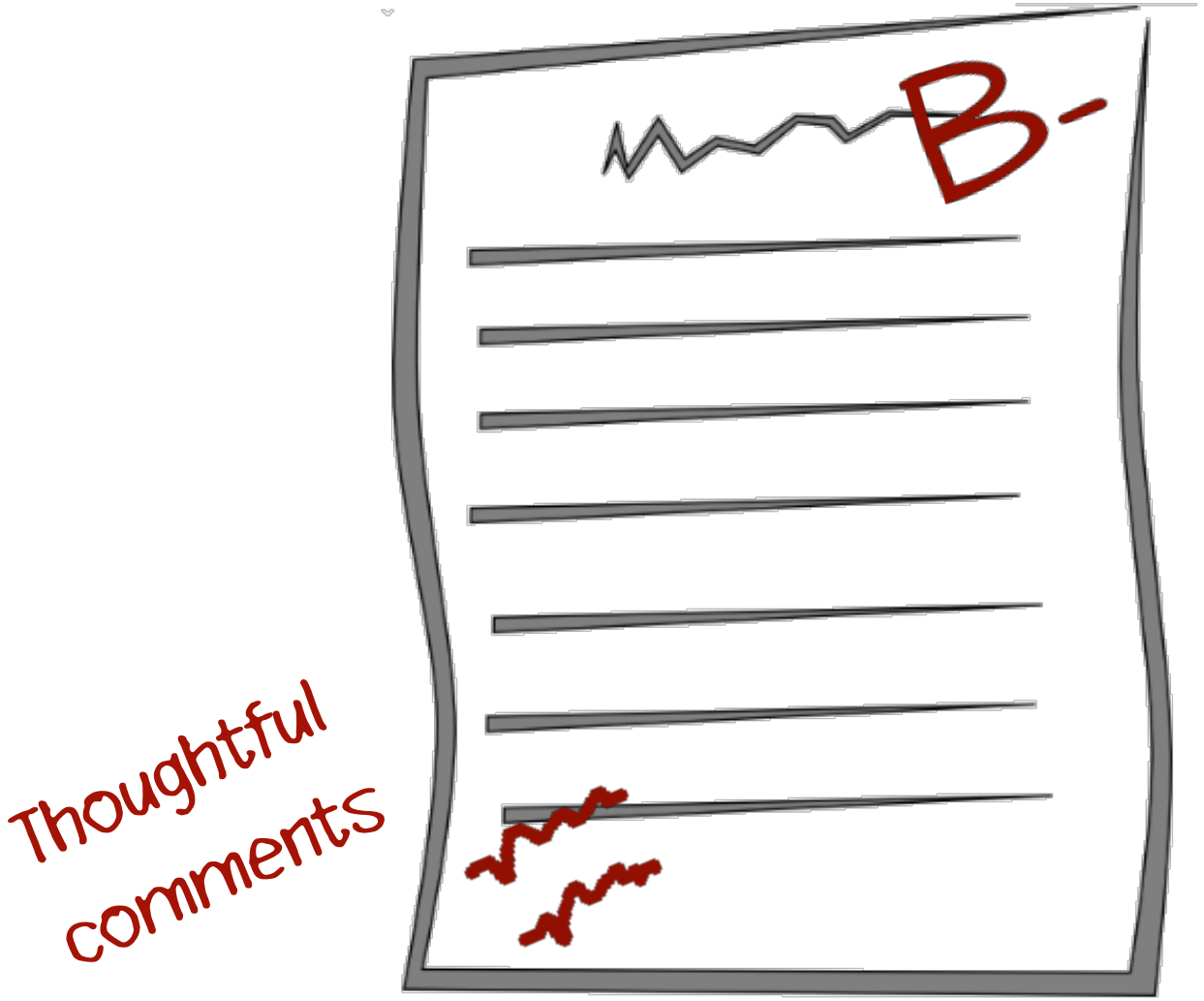
Molly entered $7 \div 31$ on her calculator.

This is what appeared: 0.24137931034482758620 6

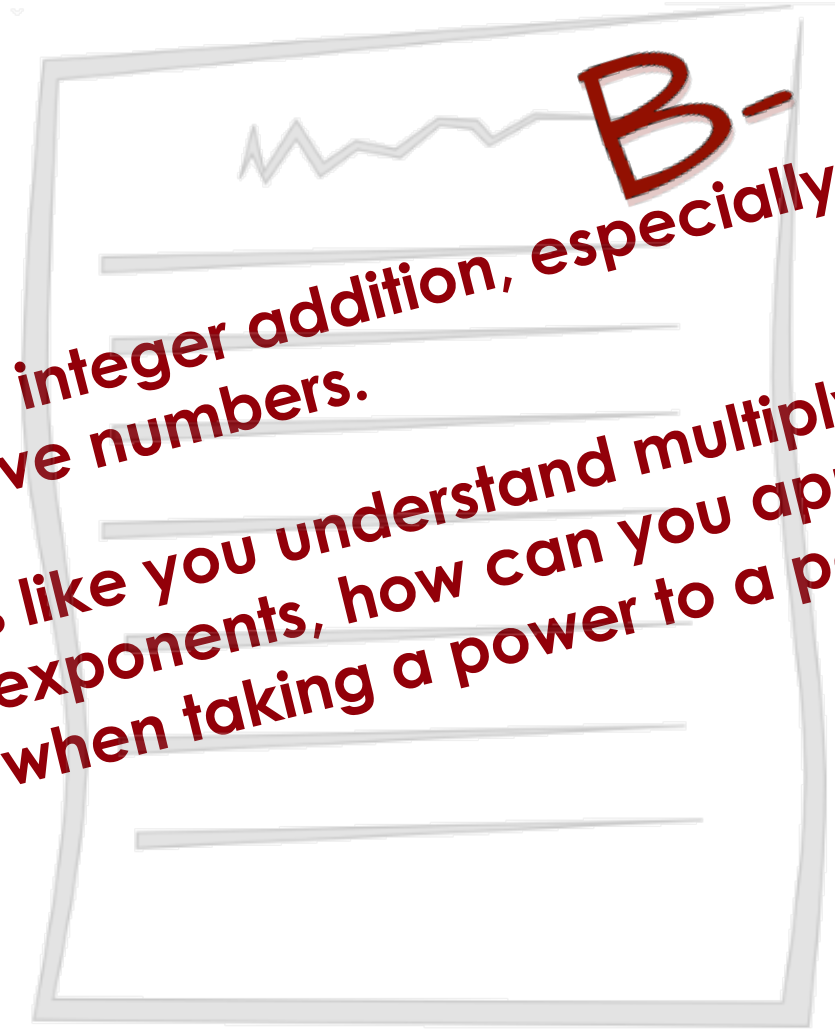
Is $\frac{7}{31}$ rational or irrational? Explain your answer

Mistakes or misconceptions are the most important thing that happens in any classroom, because they tell you, the teacher, where to focus on the specific concepts.

Student Perception



Written feedback



- Review integer addition, especially with negative numbers.
- Looks like you understand multiplying with exponents, how can you apply that when taking a power to a power?

Hopeful student thinking

What are my mistakes?

What was I thinking?

Why did I make these mistakes?

Students view

So why don't students view their mistakes as a valuable asset?

~~Rationally~~

EMOTIONAL!!!

SHAME

Put words into their mouths

Help students be specific about their misconceptions.

Self-analyzing

I made a mistake here because _____

Something to remember is _____

A strategy I tried is _____

In a previous problem, I _____

Put words into their mouths

Help students be specific about their misconceptions.

Struggling

I am confused about _____ because _____

When I checked my work, I noticed _____

Put words into their mouths

Help students be specific about their misconceptions.

Simple mistakes

I wrote the wrong _____

I didn't follow directions correctly, I should have _____

My work was really messy and I need to ___

We would like our students to…

- See mistakes as a source of understanding
- Improve motivation and self-esteem by responding to and overcoming mistakes
- Encourage independent mistake correction as a matter of habit

Error Analysis: Why it's a good idea

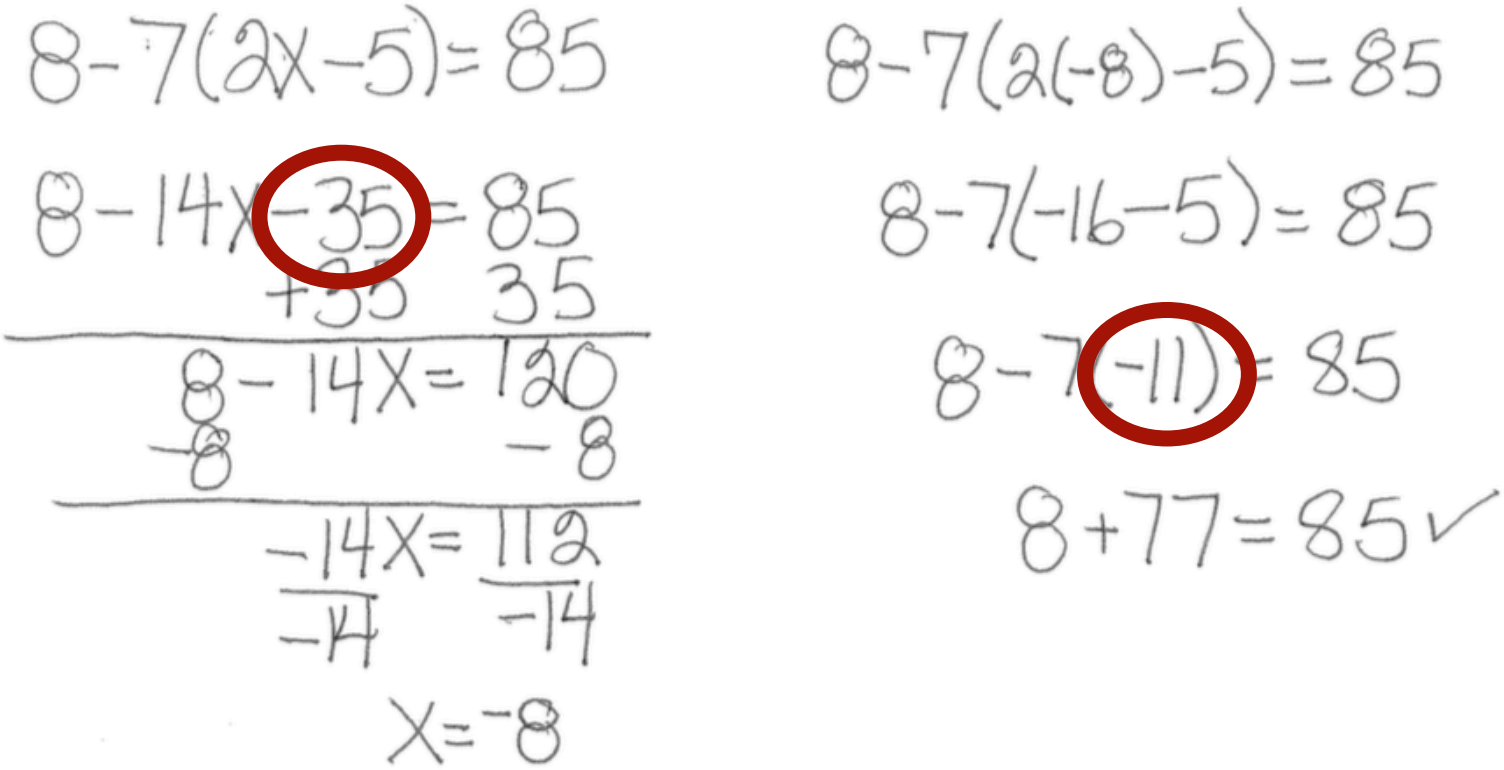
- It promotes higher level thinking.
- It aids in conceptual understanding.
- It is a great strategy for to applying previous concepts to new material.

Using Incorrect responses

Incorrect responses can be a wonderful starting place for discussion and analysis of important and challenging mathematical ideas.

Error Analysis

x = -3 ✓ or x = -8



Ask students:
What advice would you give to this student?

Idea from: mathmistakes.org

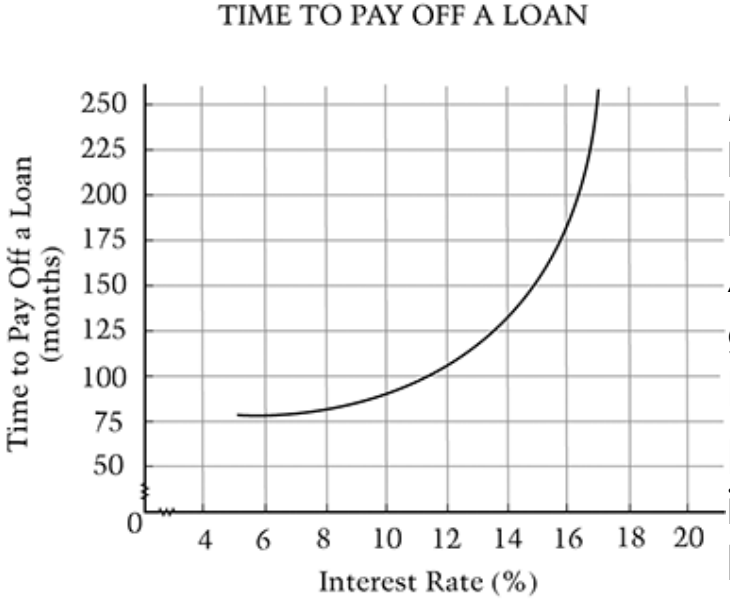
Dane and Quinn collect sports cards. Dane has 4 cards for every 3 cards that Quinn has. If Dane gives Quinn ½ his cards, what will be the new ratio of Dane's cards to Quinn's?

2 for 5

How can you use this method to answer the question:

What if Dane originally had 60 cards? How many does Quinn have after the switch?

Idea from: mathmistakes.org



Mrs. Brown would like to pay off a loan in 180 months. According to the graph, what should be the approximate percent of the interest rate on her loan?

NAEP 2013

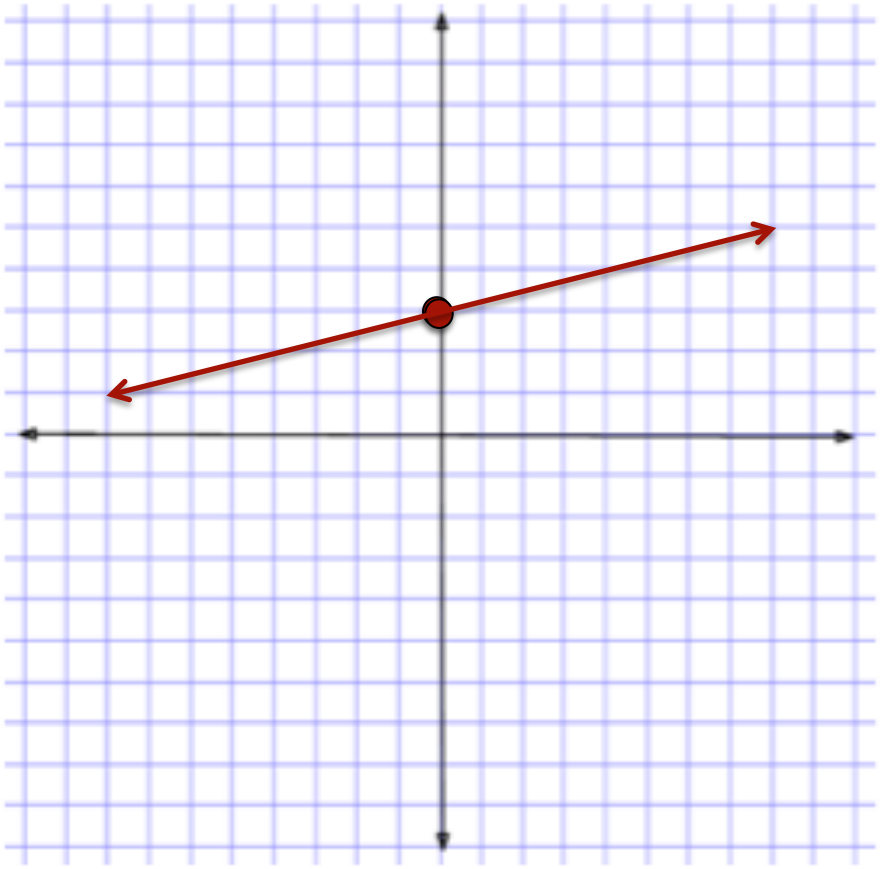
Answer: 16.40/0

Show this answer on the graph

How many months would she have to pay if the interest rate is 8% ?

Idea from: mathmistakes.org

Graph the function $y = -\frac{1}{4}x + 3$



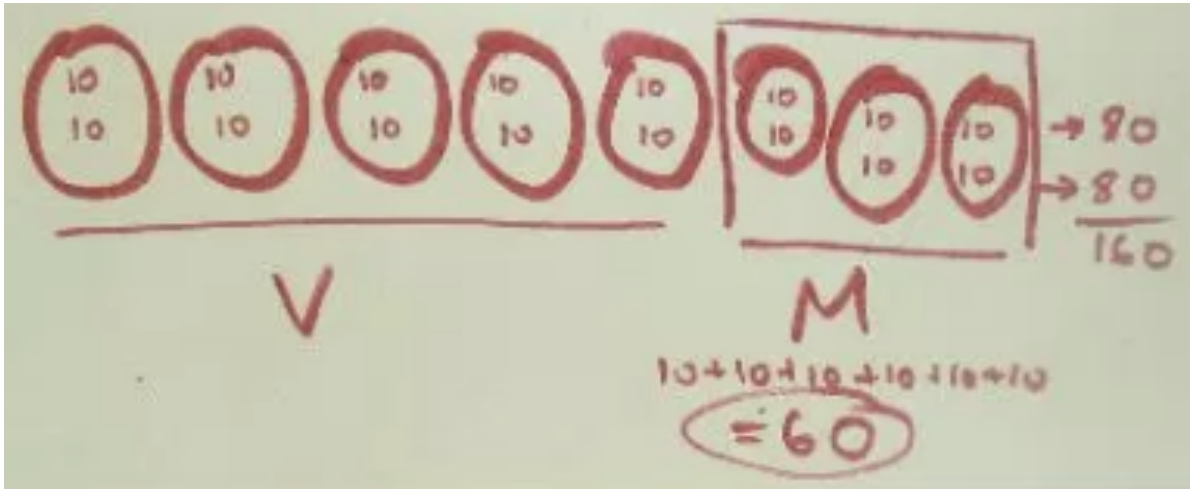
What was this student thinking?

What advice can you give to this student?

From: mathmistakes.org

Lani surveyed 160 people. 5 out of 8 of the people she surveyed prefer vanilla to mint. How many preferred mint?

Julia's work

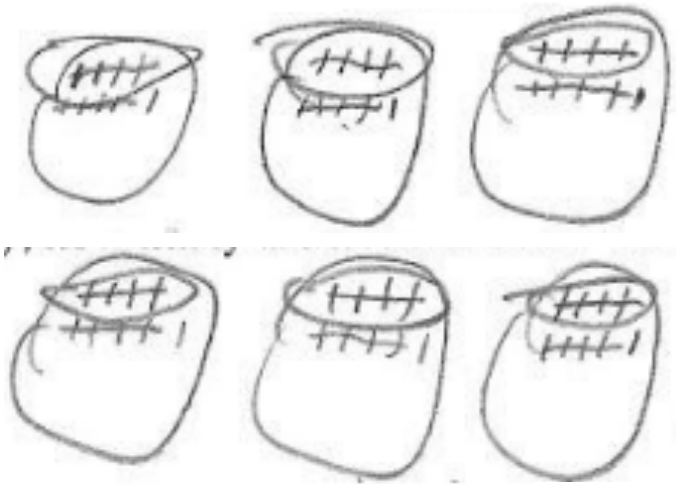


Why did Julia put 10 and 10 in each circle?

2 out of 3 of my cards are basketball. I have 96 cards. How many are basketball?

From: mathmistakes.org

In a class library, 5 out of 6 books are non-fiction. If there are 66 books in the library, how many are fiction? Show all work.



X

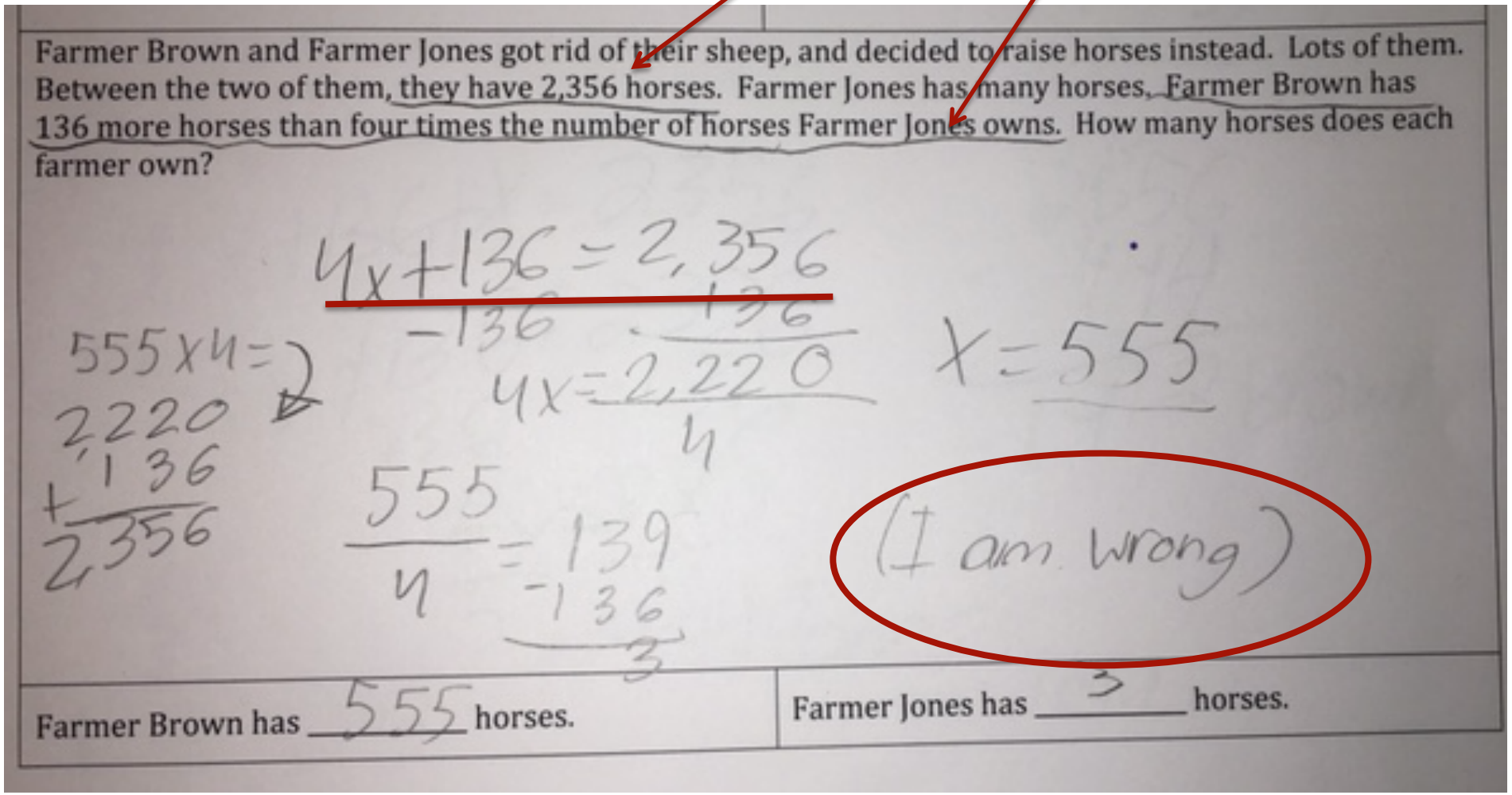
Is 36 out of 66 the same as 5 out of 6? Show how you know.

3 out of 4 of my cards are baseball. I have 60 cards. How many baseball cards do I have?

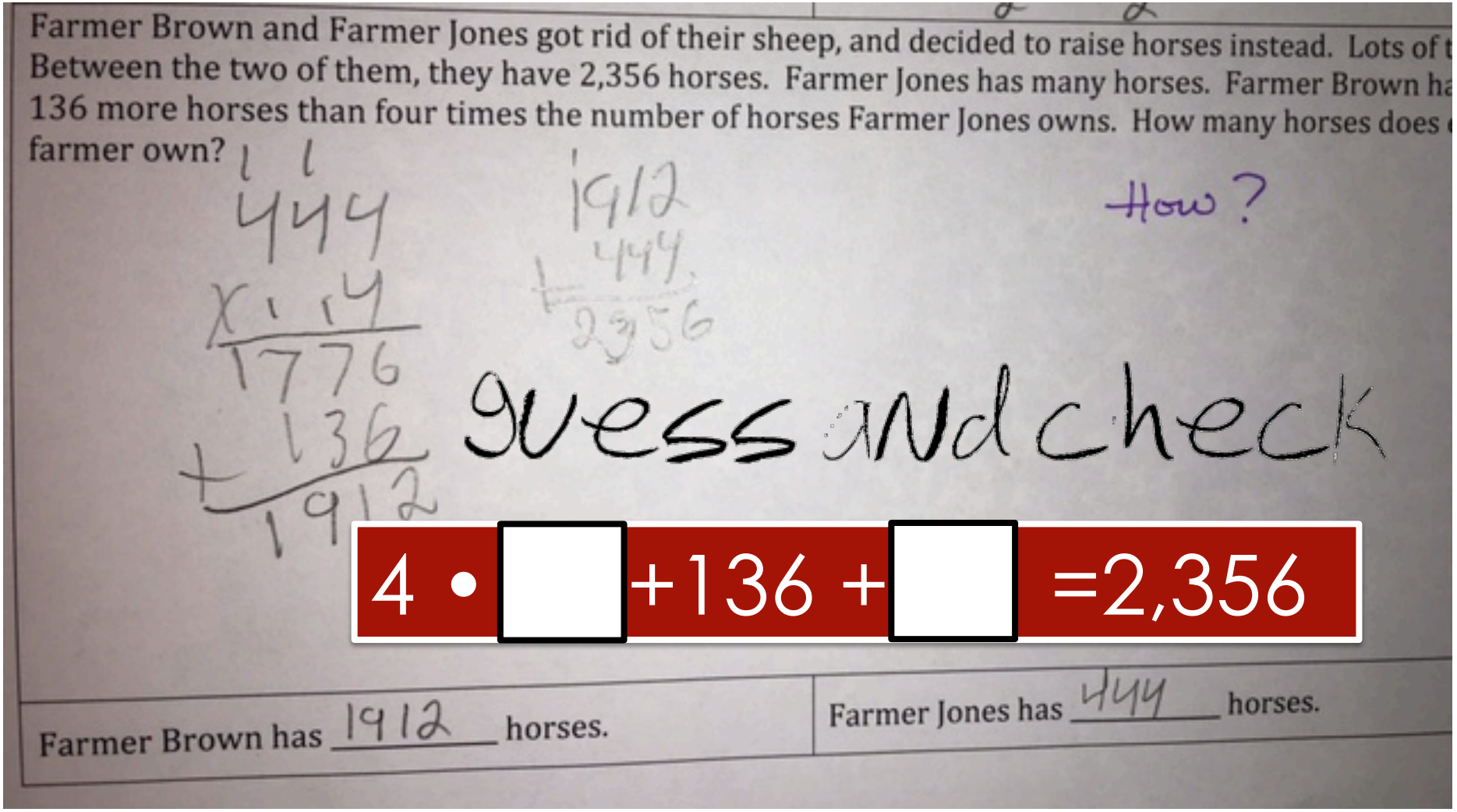
My Favorite NO

Farmer Brown and Farmer Jones got rid of their sheep, and decided to raise horses instead. Lots of them. Between the two of them, they have 2,356 horses. Farmer Jones has many horses. Farmer Brown has 136 more horses than four times the number of horses Farmer Jones owns. How many horses does each farmer own?

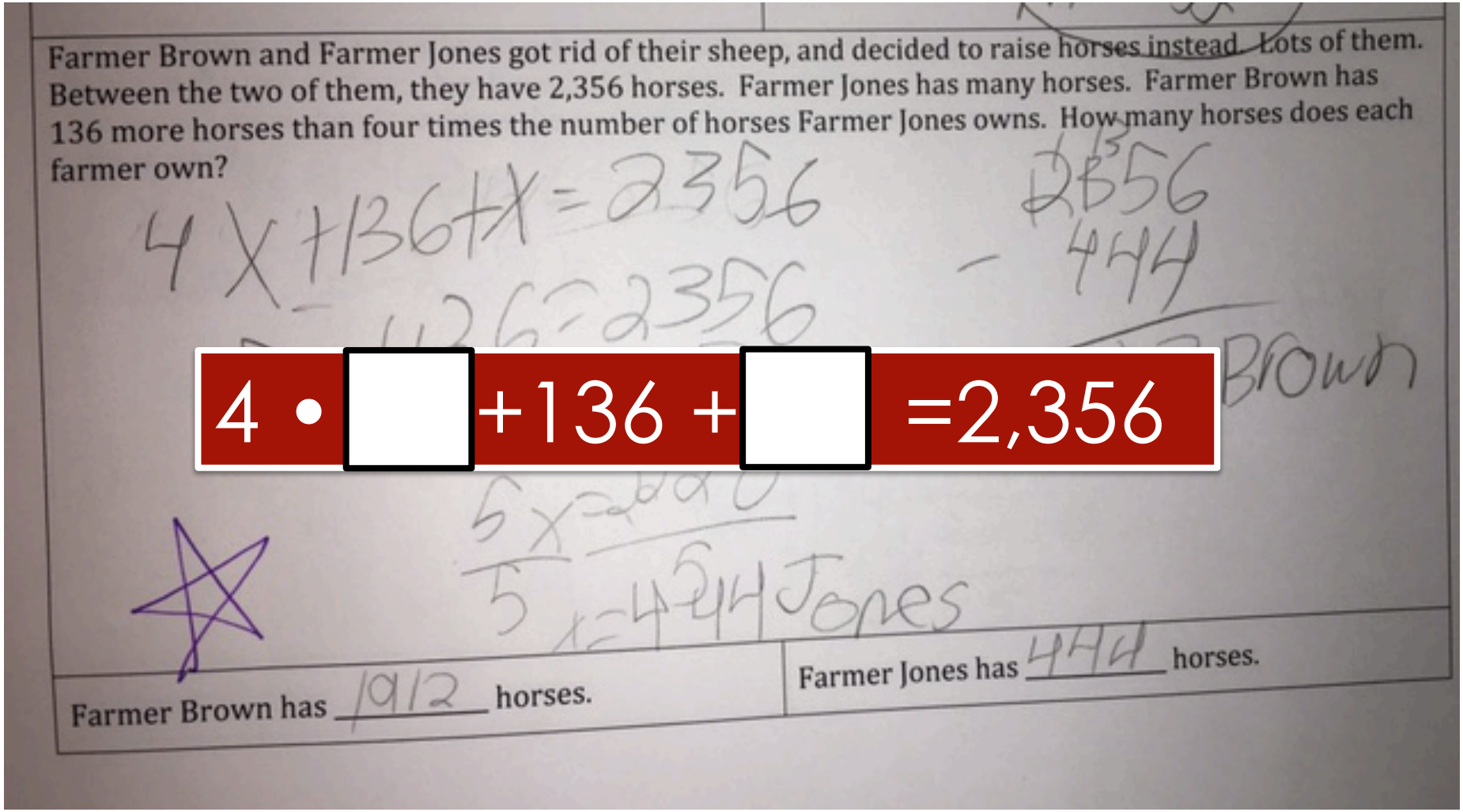
My Favorite NO



My Favorite NO

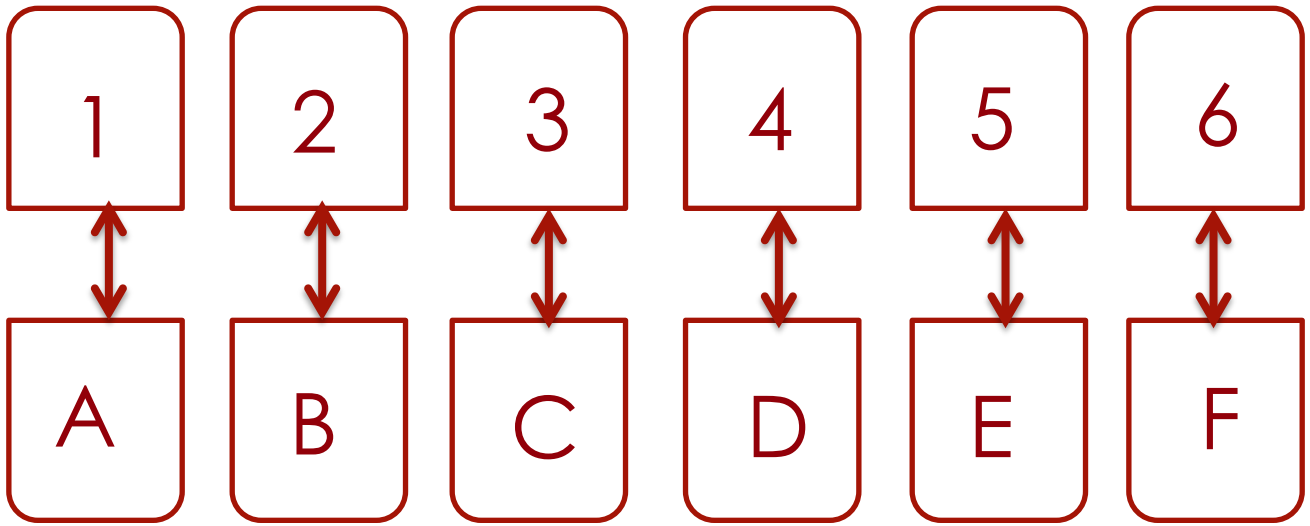


My Favorite NO



Classroom Sign

Peer Grading

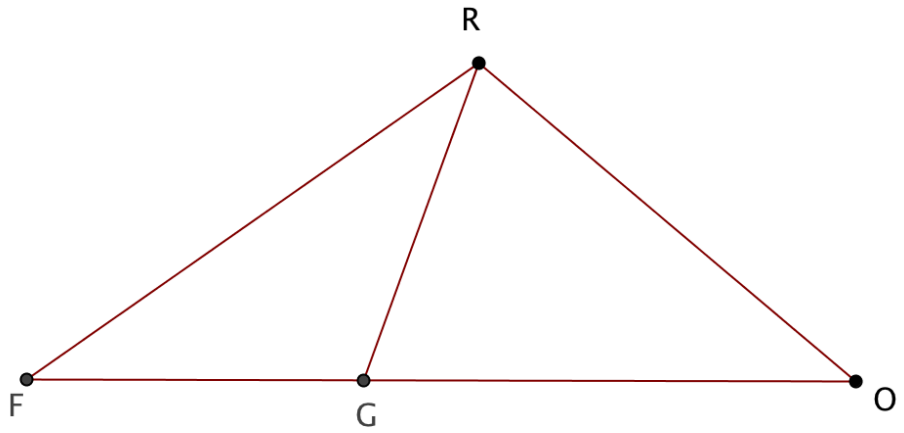


Isosceles Triangle Problem

$\overline{FR} \cong \overline{OR} \cong \overline{OG}$

$\overline{FG} \cong \overline{GR}$

Find $m\angle GOR$



Student: How are we suppose to do this if there are no angle measures?

Student: Aren't the angle measures arbitrary?

Student: Should I prove the triangles are congruent?

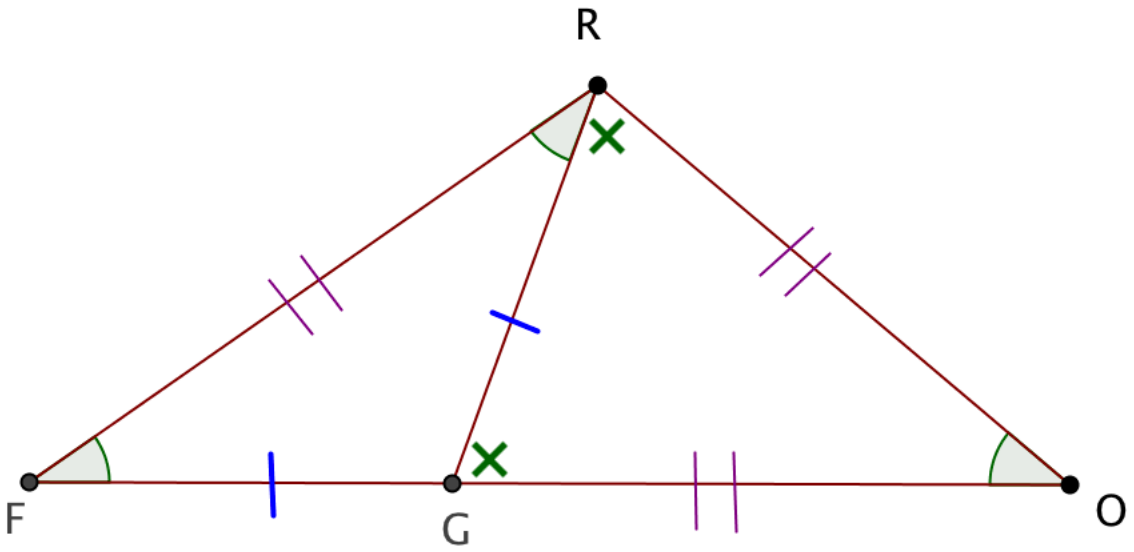
Student: I have no idea where to start.

Isosceles Triangle Problem

$\overline{FR} \cong \overline{OR} \cong \overline{OG}$

$\overline{FG} \cong \overline{GR}$

Find $m\angle GOR$



Student: The first thing I noticed are the 3 isosceles triangles.

Student: Mark the figure to show what's given . . .

Student: I'm sure there is something else ….

Student: Aren't some of the angles congruent?

We would like our students to…

- See mistakes as a source of understanding
- Improve motivation and self-esteem by responding to and overcoming mistakes

- Encourage independent mistake correction as a matter of habit

Famed Cosmologist on Cosmetology

Cosmetology school was robbed. The students
A - beat him up with curling irons
B - the old hairspray and cigarette lighter trick, chasing him with bursts of flame
C - they put a cops hat on a wig stand and scared him into leaving.

New spa treatment over in Bali is which of these?
A - you are covered with house paint, which is allowed to harden, and then peeled off, taking toxins with it;
B - the python massage, in which snakes crawl all over you. It's exfoliating.
C - instead of hot rocks, hot lava is dripped onto your back.

Thailand beauty treatment
A - a one hour foot massage from an inmate at the Chiang Mai Correctional Institution
B - a haircut from a skilled artisan wielding a power saw
C - a manicure from a monitor lizard trained to love the taste of fingernails.

Neil DeGrasse Tyson

I love being wrong 'cause that means in that instant, I learned two things today.

NPR Wait Don't Tell Me
(3 questions on cosmetology, October 23, 2015)

Neil deGrasse Tyson
@neiltyson

If you never make mistakes then you are not on the frontier of discovery, for there is where mistakes are made all the time.

Summary

Eliciting evidence of student thinking by creating a **classroom atmosphere** where mistakes are expected, inspected and respected.

Use methods like error analysis (created questions or student work), hinge questions and peer grading to assess student thinking and learning.

Of course you should!

If we learn from our mistakes, shouldn't I make as many mistakes as possible?

Classroom Poster

First
Attempt
In
Learning

Thank You

I appreciate your feedback

Barbara Lynch
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