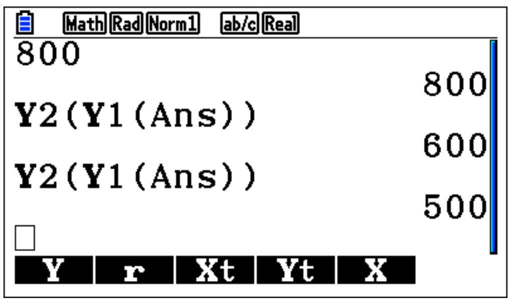
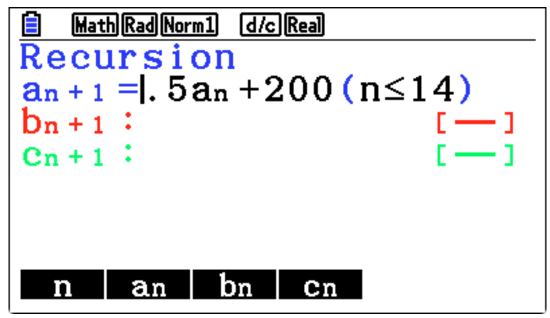
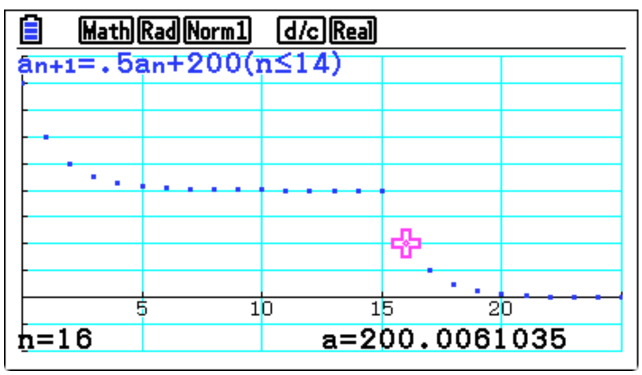
Exploring the Connections Between Recursive Sequences and Composition of Functions

NCTM Annual Conference
San Francisco, CA
April 14, 2016







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Common Core State Standards Addressed

Standards for Mathematical Practice

CCSS.Math.Practice.MP1 Make sense of problems and persevere in solving them.
Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends.

CCSS.Math.Practice.MP2 Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.

CCSS.Math.Practice.MP4 Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. They are able to identify important quantities in a practical situation.

CCSS.Math.Practice.MP7 Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. They notice if calculations are repeated, and look both for general methods and for shortcuts. Mathematically proficient students continually evaluate the reasonableness of their intermediate results.

Interpreting Functions

Understand the concept of a function and use function notation.

CCSS.Math.Content.HSF-IF.A.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*.

CCSS.Math.Content.HSF-IF.A.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Building Functions

Build a function that models a relationship between two quantities.

CCSS.Math.Content.HSF-BF.A.1: Write a function that describes a relationship between two quantities.

CCSS.Math.Content.HSF-BF.A.1a: Determine an explicit expression, a recursive process, or steps for calculation from a context.

CCSS.Math.Content.HSF-BF.A.1b: Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model*.

CCSS.Math.Content.HSF-BF.A.1c: Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time*.

Composition Functions and Recursion Sequence Questions

1. The Tree-Hugger Nursery has 7000 white pine trees. Each year, the nursery plans to sell 12% of its trees and plant 600 new ones.

   A. Write a function f(x) and store in Y1 that represents the number of trees at the nursery that are **not** sold.

   B. Write a function g(x) and store in Y2 that represents the number of trees at the nursery after 600 are planted.

   C. Using the functions from Parts A and B, write a composition of functions that represents the number of trees at the nursery each year.

   D. What is the seed number?

   E. Find the number of trees at the nursery after one year, three years, and five years.

   F. What will be the number of trees in the long run (over a long period of time)?

2. Cici purchased $1500 worth of merchandise with her credit card this past month. Then she was unexpectedly laid off from her job. She decided to make no more purchases with the card and to make only the minimum payment of $50 each month. Her **monthly** interest rate is 1.5%.

   A. Write a function f(x) that represents the credit card balance after the payment is made each month. Store it in Y1.

   B. Write a function g(x) that represents the credit card balance after the finance charge is added to the balance each month. Store it in Y2.

   C. What does the function f(g(x)) represent?

   D. What is the seed number?

   E. Find the balance on the credit card after the first month.

   F. How many months will it take Cici to get the balance under $1300?

   G. When will Cici pay off the total balance on her credit card?

   H. What is the total amount paid for the $1500 worth of merchandise?

3. Often an oral surgeon prescribes a mega dose of antibiotics prior to a procedure to prevent infection. Following the procedure, a lesser amount of antibiotics is taken for a period of time, also to prevent infection. The amount and percent of medicine remaining in the body is of concern to both dentist and patient to eliminate the chance of infection.

Suppose that 50% of the medicine remains in the body after 24 hours, measured each morning. A dental patient takes 800 mg. of Trimox (an antibiotic) on the morning of a dental procedure and 200 mg. daily after breakfast for two weeks.

   A. Write a function Y1(x) that represents the amount of medicine in the body after waking each morning.

   B. Write a function Y2(x) that represents the amount of medicine in the body shortly ***after taking each dose***.

   C. Using the functions from Parts A and B, write a composition of functions that represents the amount of medication in the body each day.

   D. What is the level of medication in the patient's body after one day? Five days? Two weeks?

   E. After two weeks, the patient stops taking the drug. How many days after the procedure will there be less than 1mg. in the bloodstream?

F. If the patient began the regimen with a mega dose of 1000 mg., how much Trimox is in the bloodstream after two weeks?

G. If the patient did not take a mega dose, and took 200 mg. on a daily basis, how much Trimox is in the bloodstream after two weeks?

4. Tony and Tania are working at the town pool for the summer. They need to provide a "shock" treatment of 450 grams (g) of dry chlorine to prevent the growth of algae in the pool. They add 45 g each evening at closing after the initial treatment. Each day, the sun burns off 15% of the chlorine.

A. Write a function Y1(x) that represents the amount of chlorine in the pool after as a result of the sun burning it off each day.

B. Write a function Y2(x) that represents the amount of chlorine added to the pool each day.

C. Using the functions from Parts A and B, write a composition of functions that represents the amount of chlorine in the pool each day.

D. What is the chlorine level after 1 day? 3 days? 10 days?

E. What will the chlorine level be in the long run (over a long period of time)?

5. Your rich uncle, Uncle Scrooge, opens up a special bank account as a gift for you the day you begin classes as an undergraduate at an esteemed four year college. He makes a single deposit of $12,000 into this account. The account earns 0.3% per month. You are allowed to withdraw $300 per month for expenses at college.

A. Explain how you can calculate the monthly balance in the account.

B. What is the balance after one year?

C. Assuming that you earn an undergraduate degree in four years, will the money in the account be sufficient if you made the withdrawal every month? Explain!

D. Assuming that you had a paid summer job for three months and did not make the withdrawal while you were working, will the money in the account be sufficient? If so, what would be the balance in the account when you earned your degree after four years? Explain!

6. Jen and Priya decide to go out to the Hamburger Shack for lunch. They each have a 50-cent coupon from the Sunday newspaper for the Super-Duper-Deluxe $5.49 Value Meal. In addition, if they show their I.D. cards, they'll get a 10% discount. Jen's server rang up the order as a Value Meal, coupon, and then I.D. discount. Priya's server rang it up as Value Meal, I.D. discount, and then coupon.

A. How much did each girl pay?

B. Write a function, C(x) that will deduct 50 cents from a price, x.

C. Write a function, D(x) that will take 10% off a price, x.

D. Find C(D(x)).

E. Which server used C(D(x)) to calculate the price of the meal?

7. Nicky purchased $1500 worth of merchandise with her credit card this past month. Then she was unexpectedly laid off from her job. She decided to make no more purchases with the card and to make only the minimum payment of $50 each month. Her annual interest rate is 1.5 per cent per month. Since Nicky is born to shop, her rich aunt promises her that if she does not use her credit card, she will give Nicky financial aid. The plan is that every third month, the aunt will make a $200 payment for Nicky.

A. Write a function that represents Nicky's monthly balance. Store this function in Y1.

B. Write a function that represents Nicky's balance for the months when her aunt gives her $200. Keep in mind that Nicky does not have to make her own payment. Store this function in Y2.

C. Using the functions that are stored in Y1 and Y2, what a function could be written, using a composition of functions that represents the balance every three months? Store this function in Y3. **Do not use numbers! Express it in terms of Y1 and Y2!**

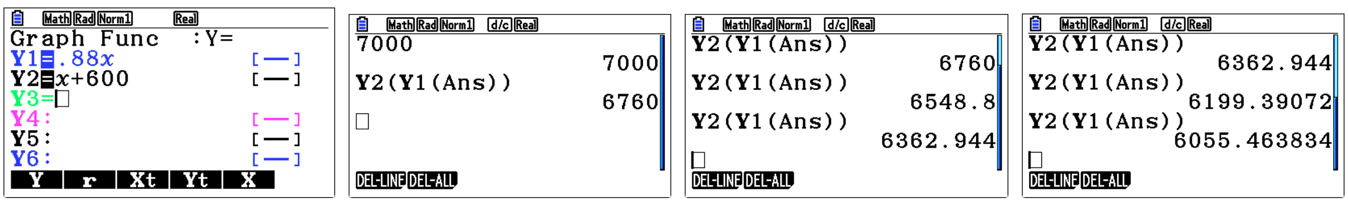
D. Find the balance on the credit card for each of the first six months.

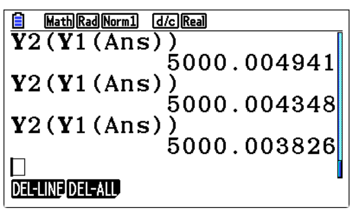
E. How many months will it take Nicky to get the balance under $1000?

F. When will Nicky pay off the total balance on her credit card?

8. If the $200 that Nicky received from her aunt was paid every third month *in addition to* Nicky's $50 payment, when would Nicky pay off the total balance on her credit card?

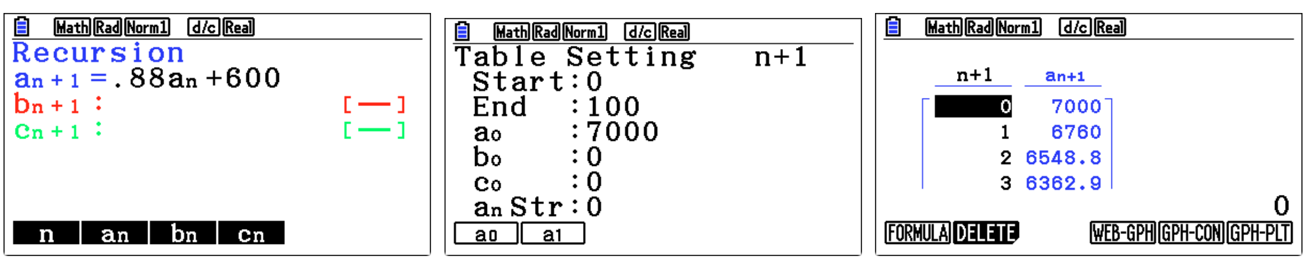
1. Composition Solution:

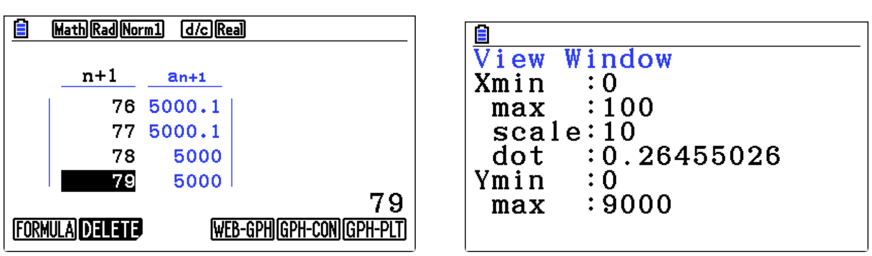


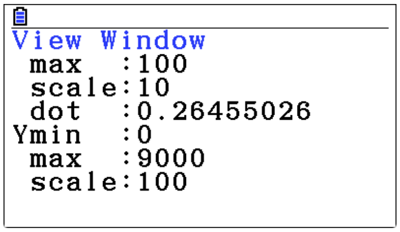


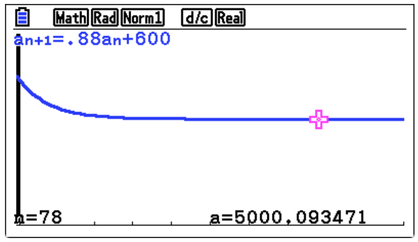
D) 6760.00 trees; 6362.90 trees; 6055.50 trees **E)** 5000 trees

1. Recursive Solution:

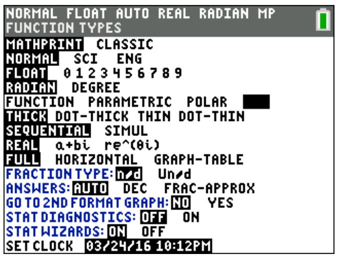


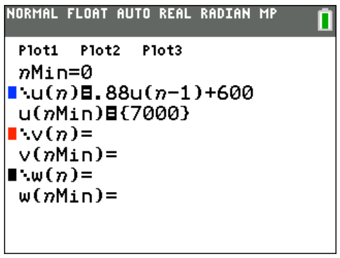


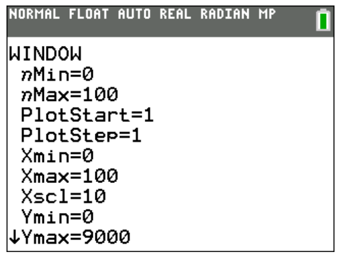


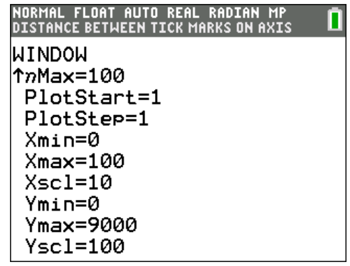


TI Screen Shots for Recursion of Problem 1:









Screen Shots and Answers

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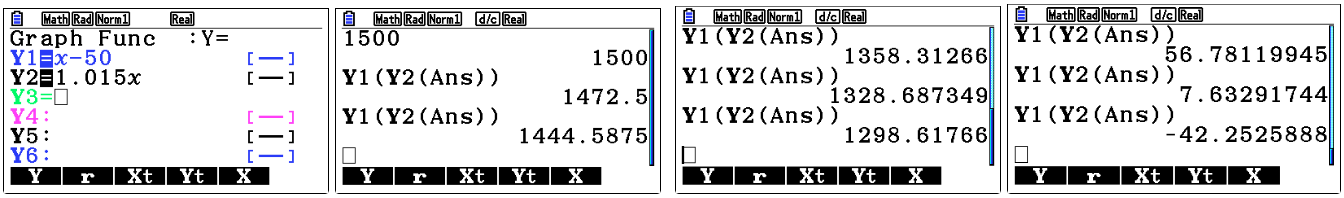
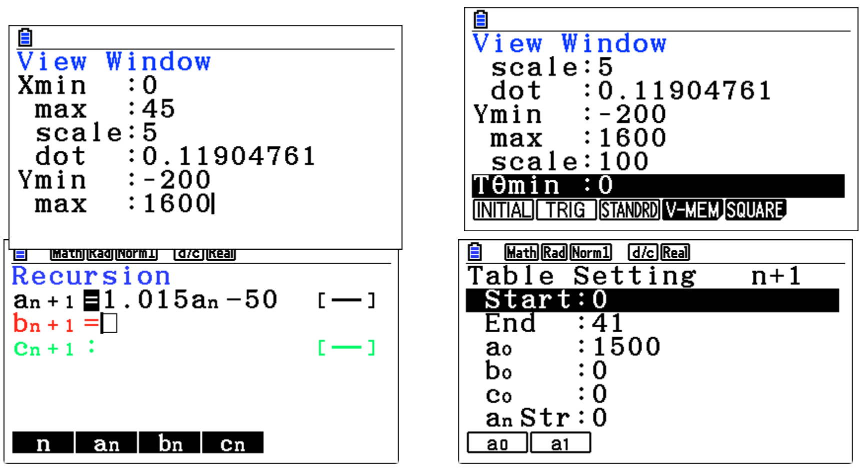
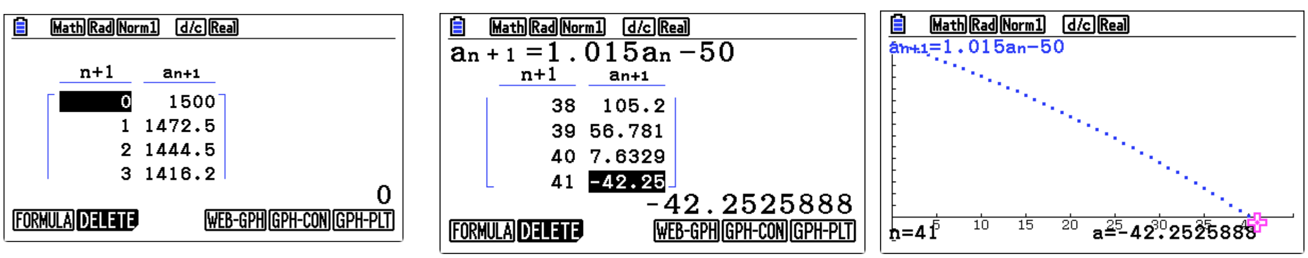


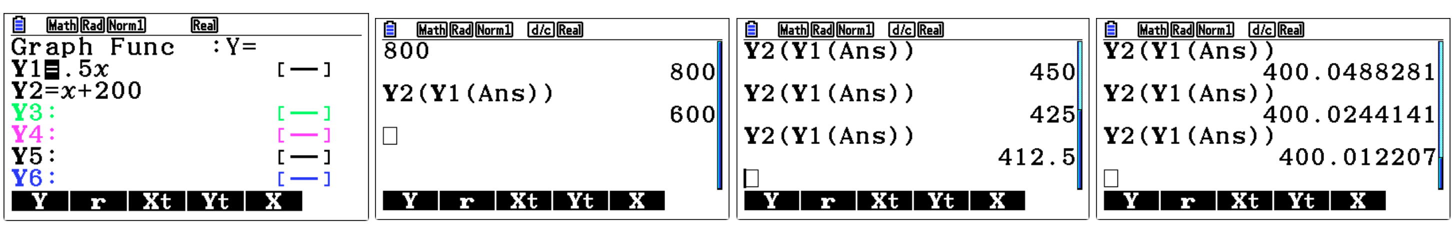
F) 7 months \$1298.62 **G)** 41 months -\$42.25 **H)** 40* \$50 + \$7.63 = \$2007.64

2. Recursive Solution:



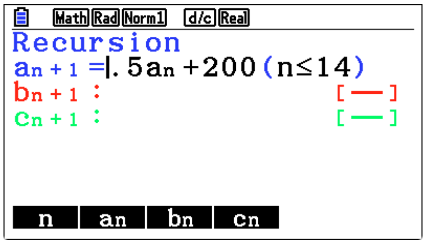


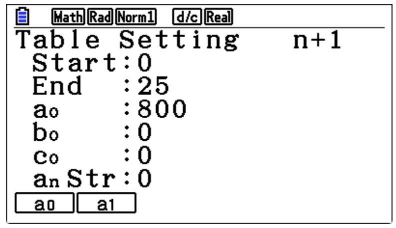
3. Composition Solution:

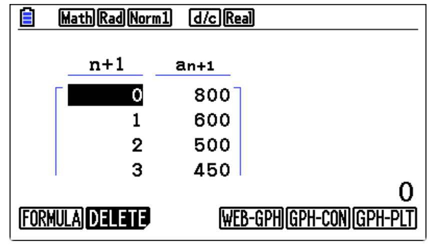


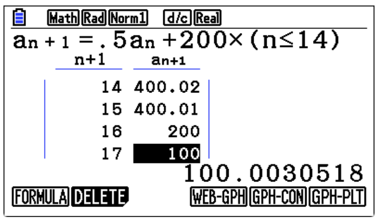
D. 600 mg ; 412.5 mg ; 400.02 mg **E.** after 23 days there will be .7812 mg **F.** 400 mg **G.** 400 mg

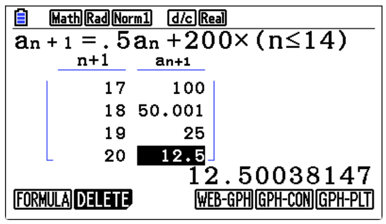
3. Recursive Solution:

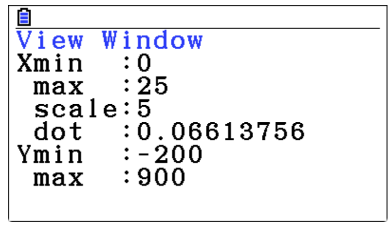


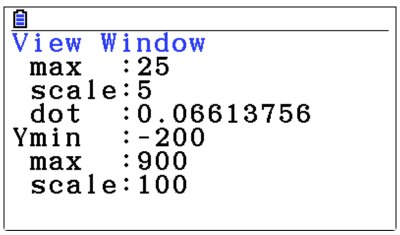


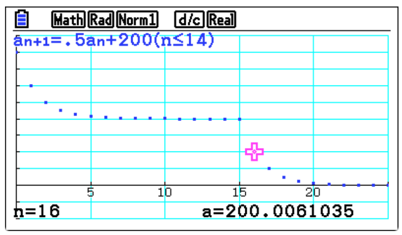




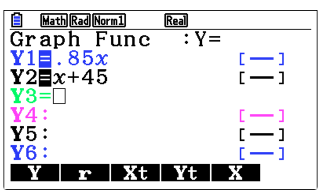


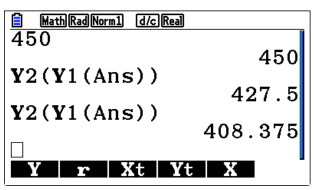


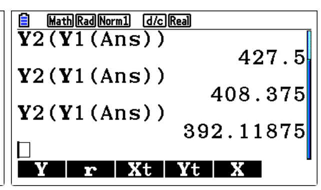


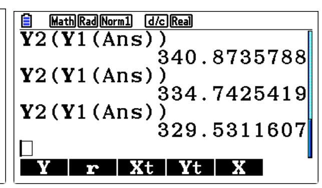


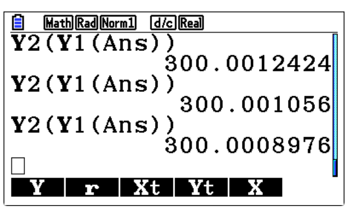
4. Composition Solution:





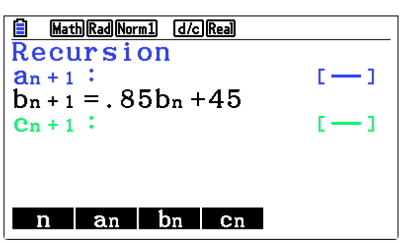


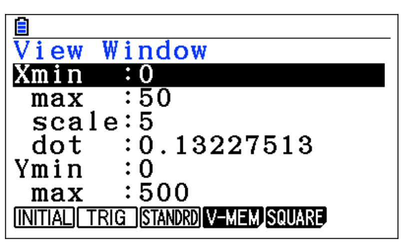


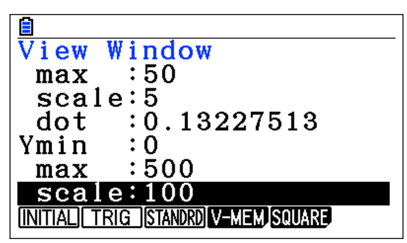


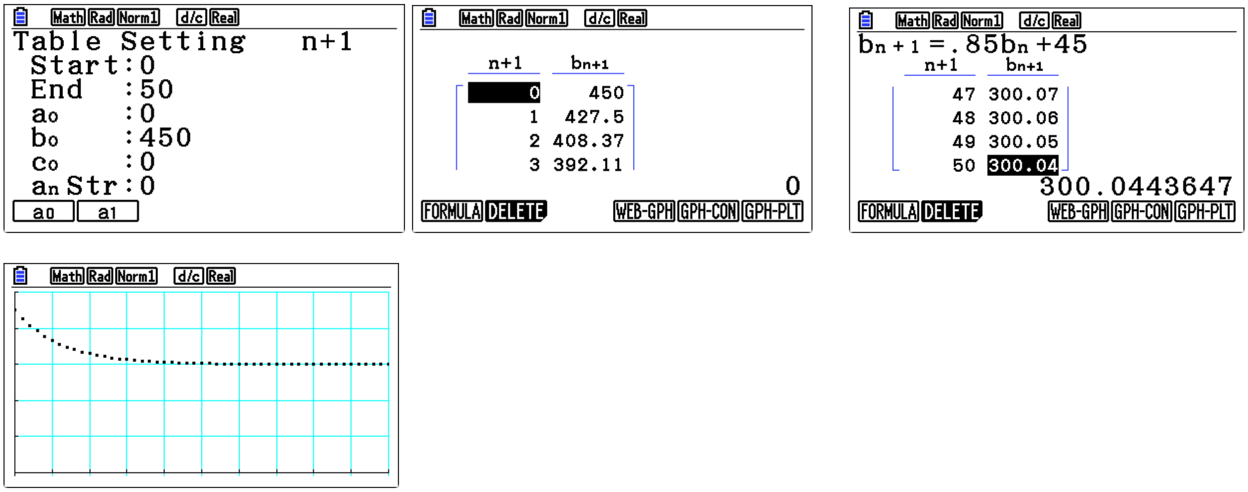
D) 427.50 mg.; 392.11 mg. ; 329.53 mg. **E)** 300 mg.

4. Recursive Solution:

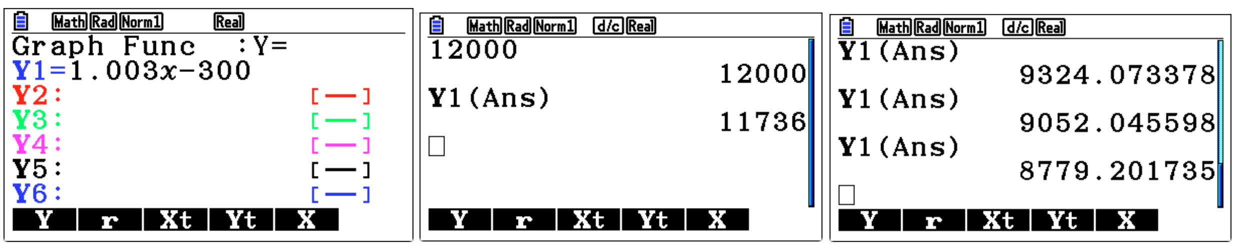




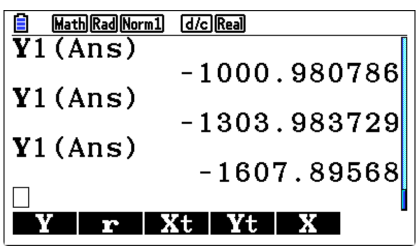




5. **Composition Solution:**

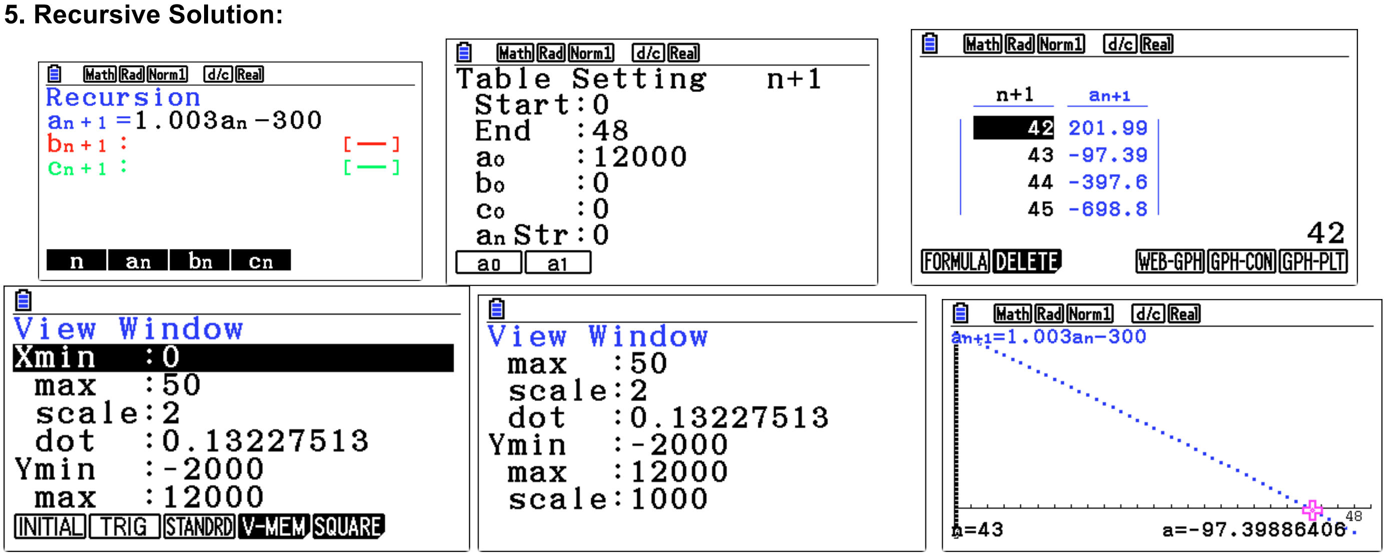


12 Month balance

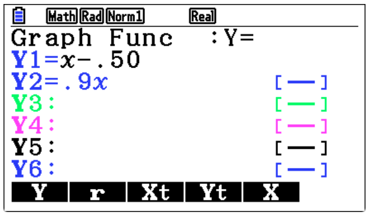


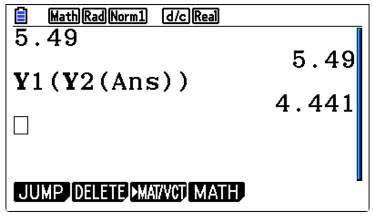
4 year balance (not enough money)

5. Recursive Solution:

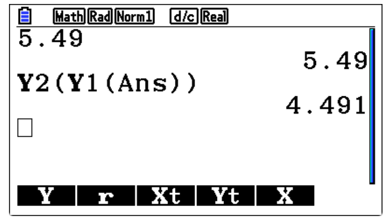


6. **Composition Solution:**





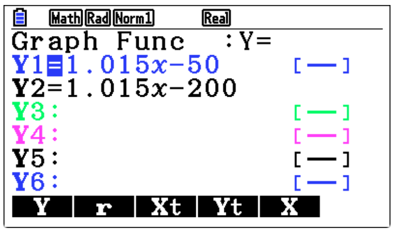
Priya's Server

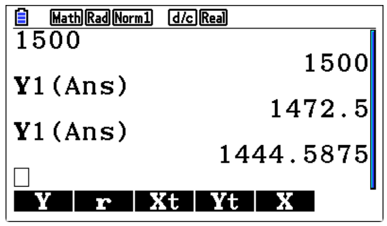


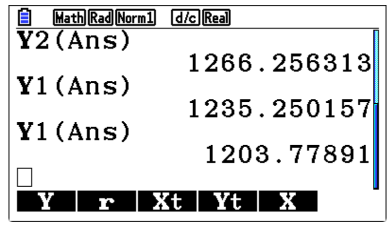
Jen's Server

6. Non-Recursive

7. Composition Solution:





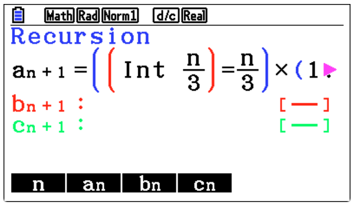


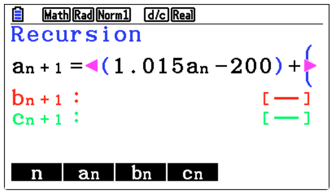
Month	Pay Amt	Formula	Balance
0	-	-	1500
1	50	Y1(ans)	1472.5
2	50	Y1(ans)	1444.59
3	200	Y2(ans)	1266.26
4	50	Y1(ans)	1235.25
5	50	Y1(ans)	1203.78
6	200	Y2(ans)	1021.84
7	50	Y1(ans)	987.17
8	50	Y1(ans)	951.98
9	200	Y2(ans)	766.26
10	50	Y1(ans)	727.74
11	50	Y1(ans)	688.66
12	200	Y2(ans)	498.99
13	50	Y1(ans)	456.47
14	50	Y1(ans)	413.32
15	200	Y2(ans)	219.52
16	50	Y1(ans)	172.81
17	50	Y1(ans)	125.41
18	200	Y2(ans)	-72.71

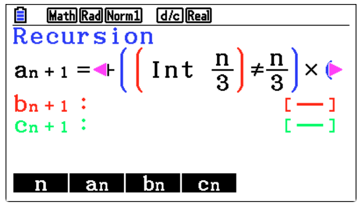
E. At 7 months balance is $987.17 **F.** Balance paid at 18 months.

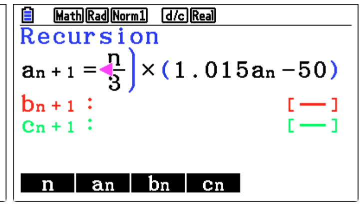
7. Recursive Solution:

$$\left(\left(\text{Int}\frac{n}{3}\right)=\frac{n}{3}\right)x\left(1.015a_n-200\right)+\left(\left(\text{Int}\frac{n}{3}\right)\neq\frac{n}{3}\right)x\left(1.015a_n-50\right)$$

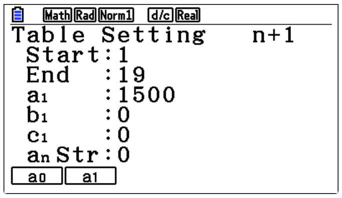


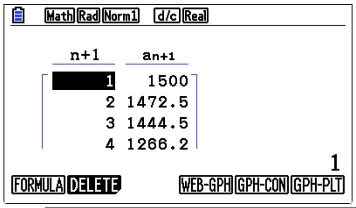


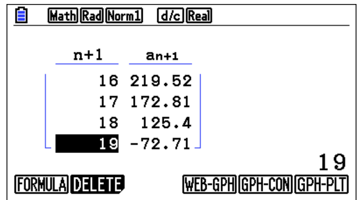


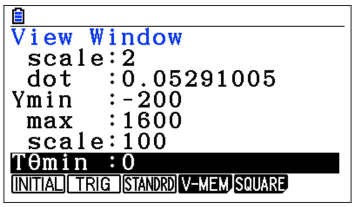


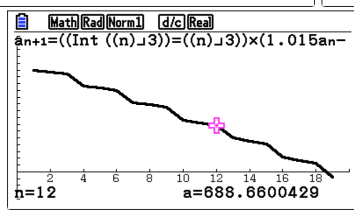
Note that for this problem, we use a_1 and not a_0.

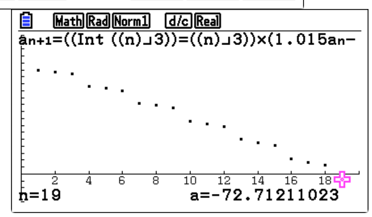




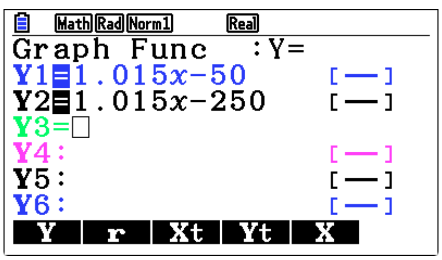


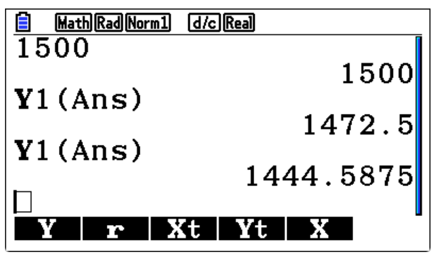


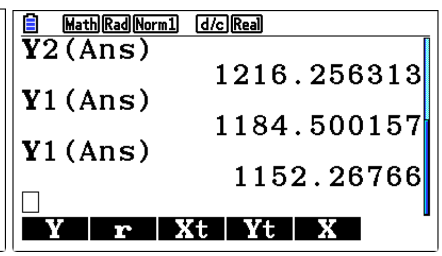




8. **Composition Solution:**







Month	Pay Amt	Formula	Balance
0	-	-	1500
1	50	Y1(ans)	1472.5
2	50	Y1(ans)	1444.59
3	250	Y2(ans)	1216.26
4	50	Y1(ans)	1184.5
5	50	Y1(ans)	1152.27
6	250	Y2(ans)	919.55
7	50	Y1(ans)	883.34
8	50	Y1(ans)	846.6
9	250	Y2(ans)	609.29
10	50	Y1(ans)	568.43
11	50	Y1(ans)	526.96
12	250	Y2(ans)	284.86
13	50	Y1(ans)	239.14
14	50	Y1(ans)	192.72
15	250	Y2(ans)	-54.39

8. Recursive Solution:

Use this function

$$\left(\left(\text{Int}\,\frac{n}{3}\right)=\frac{n}{3}\right)x\left(1.015a_n-250\right)+\left(\left(\text{Int}\,\frac{n}{3}\right)\neq\frac{n}{3}\right)x\left(1.015a_n-50\right)$$

Everything else is the same as problem 7.