See It! Touch It!
Concrete to Conceptual NCTM Regional Conference 2016

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## See It! Touch It!

Concrete to Conceptual

## Why Concrete?

- I hear and I forget.
- I see and I remember.
- I do and I understand.

Confucius

## Why Concrete?

- Students of ALL ages need to do math to understand math.
- Manipulatives provide a way for students to do mathematics in a concrete manner.
- Manipulatives help students conceptually learn and understand mathematics.


## Why Concrete?

- Learning is elevated when a math concept is $\qquad$ presented in a variety of ways.
- Student engagement is heightened when
$\qquad$ students construct their own knowledge while using manipulatives.
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## Why Concrete?

- Middle school students are in transition $\qquad$ between Piaget's concrete operational and formal operational stages of cognitive development. Many middle school students do not completely understand math concepts when presented in an abstract way.


## Why Concrete?

"Without manipulatives, children are too often lost in a world of abstract symbols for which they have no concrete connection or comprehension.'

Marilyn Burns

## USING CONCRETE MATERIALS

- Give students time to explore and play
- Best in a small group/teacher led station
- Concrete first before abstract
- Quick and move on for the majority of the students

PURPOSE FOR TODAY'S LEARNING

- Use concrete materials to represent ratio problems.
- Use concrete materials to model proportional and nonproportional relationships.
- Use concrete materials to model and construct $\qquad$ linear functions.


## RATIO \& PROPORTIONAL REASONING

"Proportional reasoning is one of the most fundamental topics in middle grades mathematics. Students' ability to reason proportionally affects their understanding of fractions and measurement in elementary school, and it supports their understanding of functions and algebra in middle school and beyond. Given the importance of ratio and proportion, it is typical to see extensive class time devoted to the topic in upper elementary and middle school grades.

What Does It Mean to Understand Ratios?
"Reasoning with ratios begins with learning to attend to two quantities simultaneously.:

Teaching Ratio \& Proportion in the Middle Grades, NCTM Research Brief by Amy Ellis

## 6.RP.A. 3 RATIO AND RATE REASONING

Jackie and Brian each have a bag of candy containing 20 gumballs that are either purple or white. The ratio of purple to white gumballs in Jackie's bag of candy is $1: 3$ and the ratio of purple to white gumballs in Brian's bag of candy is $1: 4$. How many more purple gumballs does Jackie have than Brian?

## 6.RP.A. 3 RATIO AND RATE REASONING

The ratio of gumballs to lollipops in a vending machine is $2: 5$. If there are 16 gumballs in the machine, how many lollipops are there?
6.RP.A. 3 RATIO AND RATE REASONING

The ratio of nitrogen to potassium in a sample of soil is 12:9. The sample has 36 units of nitrogen. How much potassium does the sample have?

NC RELEASED TEST ITEM

## 8.EE.C. 8 EXPRESSIONS \& EQUATIONS

Kim made soup which contains 75 total ounces of beans.

- The soup has two kinds of beans, black and red.
- There are 4 times as many ounces of black beans as red beans.
How many ounces of red beans are in the soup?
NC RELEASED TEST ITEM


## USING MODELS TO BUILD AN UNDERSTANDING OF FUNCTION

"Middle school students' experiences with function should be based on problems that incorporate manipulative materials and involve students in collecting data. The problems should lead students to build connections between the concrete model and the numerical patterns or functional relationships observed in the data.'

Cramer, Kathleen. "Using Models to Build an Understanding of Functions." Mathematics Teaching in the Middle School 6 ( (anuary 2001): 310-318. Print.

## TRIANGLE DESIGNS (7.RP.A.2)

Build a polygon design with the green triangles following the pattern shown.

Stage 1
Stage 2
Stage 3


Predict the perimeter of the design with 100 triangles?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## HEXAGONAL DESIGNS (7.RP.A.2)

$\qquad$
Build a polygon design with the yellow hexagons following the pattern shown.


Predict the perimeter of the design with 100
hexagons?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## TRIANGLE DESIGNS (7.RP.A.2)

Compare and contrast the two patterns. Use multiple representations.


ADDITIONAL RESOURCES


POLYGON PATTERNS: WHAT COMES NEXT?

- Using the pattern blocks, create a proportional relationship and a non-proportional relationship.
- Create a table of values, a graph and an equation for each pattern that you create.
- Be prepared to justify proportionality.


## STACKING STAIRCASES (7.RP.A.2)

Build each stage of a 3 unit staircase using snap cubes following the pattern shown.


## STACKING STAIRCASES (7.RP.A.2)

Build stage 4 and 5 of a 3 unit staircase.

Predict the volume and surface area for a staircase with 300 snap cubes.
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$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

STACKING STAIRCASES (7.RP.A.2) $\qquad$
What patterns do you notice?

| Stage \# | Volume | Surface Are |
| :---: | :---: | :---: |
| 1 | 3 | 14 |
| 2 | 6 | 24 |
| 3 | 9 | 34 |
| 4 | 12 | 44 |
| 5 | 15 | 55 |
|  |  |  |
| 10 | 30 | 104 |
| 25 | 75 | 254 |
| $n$ | $3 n$ | $10 n+4$ |

## STACKING STAIRCASES (7.RP.A.2)

Is the relationship between stage number and volume proportional? Explain.

Is the relationship between stage number and surface area proportional? Explain.

## STACKING STAIRCASES (7.RP.A.2)

In groups, create the following staircase patterns:

- Group A: 4 unit staircase
- Group B: 5 unit staircase
- Group C: 6 unit staircase
- Etc.

Create multiple representations for the volume and surface area of your staircase pattern.

## STACKING STAIRCASES

| Units | Volume | Surface Area |
| :--- | :--- | :--- |
| 3 | $3 x$ | $10 x+4$ |
| 4 | $4 x$ | $12 x+6$ |
| 5 | $5 x$ | $14 x+8$ |
| 6 | $6 x$ | $16 x+10$ |
| 7 | $7 x$ | $18 x+12$ |
| 8 | $8 x$ | $20 x+14$ |

What patterns do you notice?

## MODELING LINEAR FUNCTIONS USING MULTIPLE REPRESENTATIONS

"Key understandings are built upon facility with concrete materials that form the foundation for working with pictorial, tabular, graphic and eventually symbolic representations. Students need many varied experiences in meaningful contexts with each of these representations before they can truly understand the symbolic expressions and rules of formal algebra."
Lawrence, Ann, and Charlie Hennessy. Lessons for algebraic thinking. Sausalito, CA: Math
Solutions Publications, 2002 Print. Solutions Publications, 2002. Print.

## See It! Touch It! Concrete to Conceptual

- What is the value of using concrete materials for teaching mathematics?
- How might these concrete models help students gain a better understanding of math concepts?
- How might these activities help teachers identify student strengths, gaps and/or weaknesses?


## Manipulatives provided by

## ETA hand <br>  <br> , mana

Thank you for attending this session. If you would like to learn more about any of the manipulatives you used, please stop at the
ETA hand2mind booth in the Exhibitors Hall.

