THE LANGUAGE OF MULTIPLYING FRACTIONS

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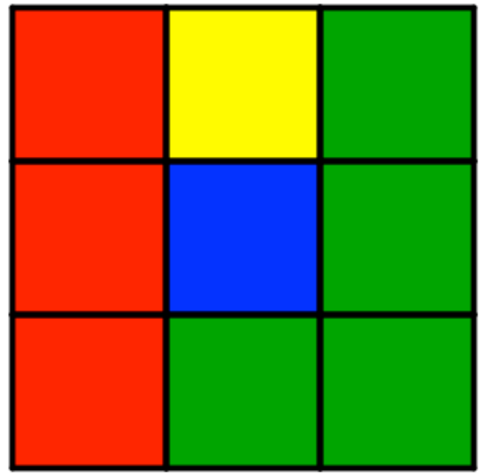
WHO AM I?

Currently teach for Boston Public Schools, at a K-8 school

- Have taught or served as math coach from
  Grades 2 through Grades 12
- At a K-8 school with considerable population of ELLs
- Previously taught for 3 years in a small Japanese town

Have received (and believe in) lots of strong content-based PD

- *Math for America* through Boston University
- Park City Math Institute (PCMI)



Where can you find each of the following fractions in the diagram above?

$\frac{1}{9}$ $\frac{4}{9}$ 0.25 $\frac{1}{3}$

8 MATH TEACHING PRACTICES

• Establish mathematics goals to focus learning	• Pose purposeful questions
• Implement tasks that promote reasoning and problem solving	• Build procedural fluency from conceptual understanding
• Use and connect mathematical representations	• Support productive struggle in learning mathematics
• Facilitate meaningful mathematical discourse	• Elicit and use evidence of student thinking

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DEFINITION OF MATHEMATICAL UNDERSTANDING

"A mathematical idea is understood if it is part of an internal network."

"A mathematical idea, procedure, or fact is understood thoroughly if it is linked to existing networks with stronger or more numerous connections." (p. 67)

Hiebert, J., & Carpenter, T.P. (1992). Learning and teaching with understanding. *Handbook of Research Mathematics Teaching and Learning:* 65-92

REPRESENTING MATHEMATICAL IDEAS

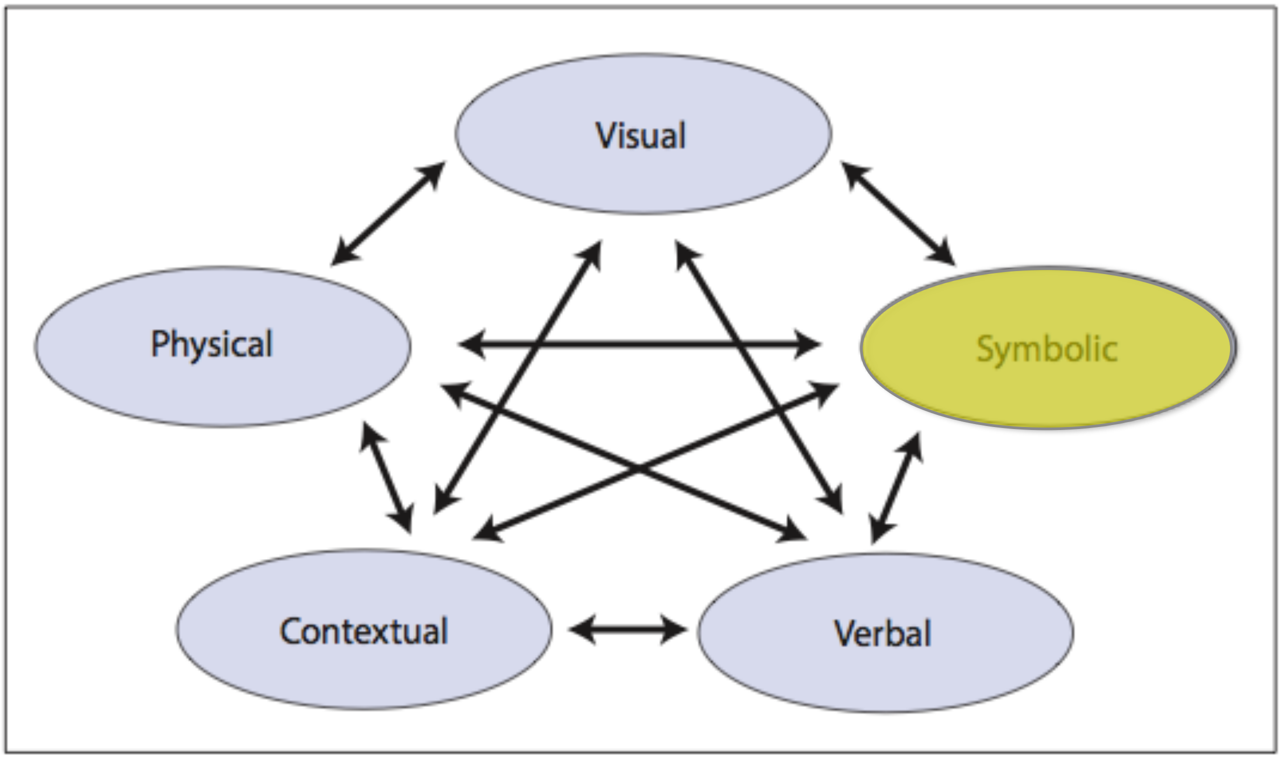
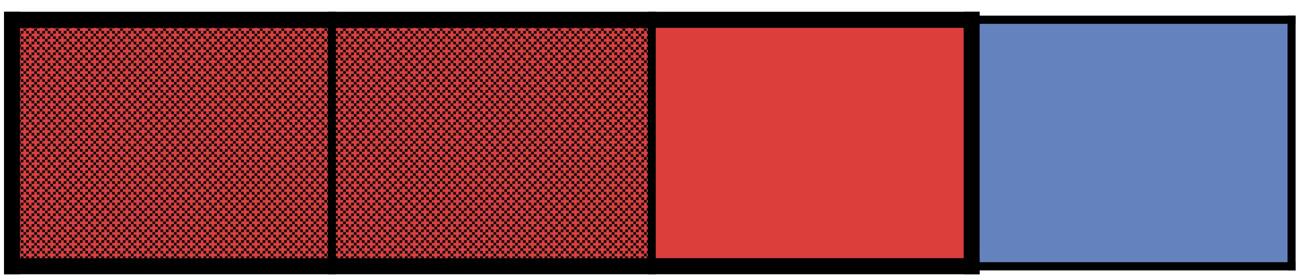


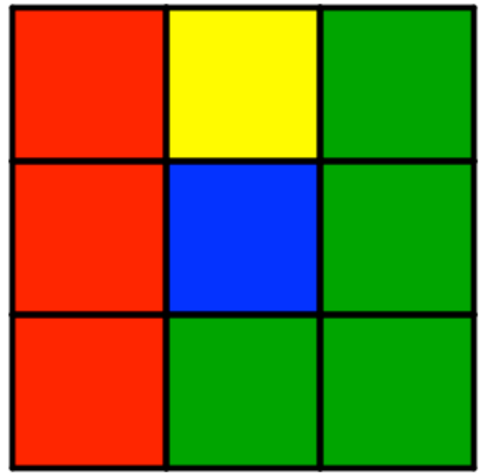
Fig. 9. Important connections among mathematical representations

p. 25, citing Lesh, Post, and Behr (1987)

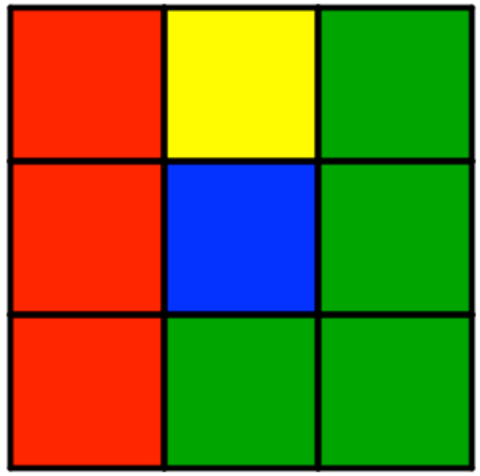
FOR EXAMPLE…

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$$

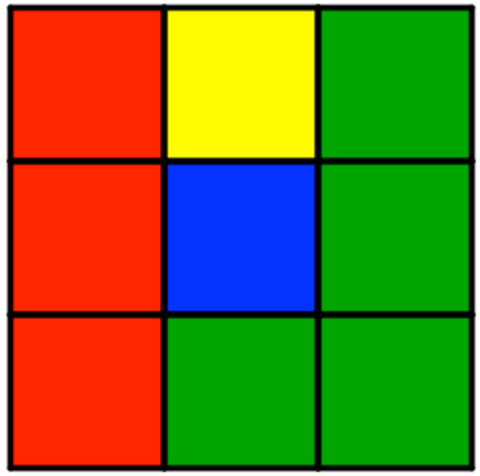




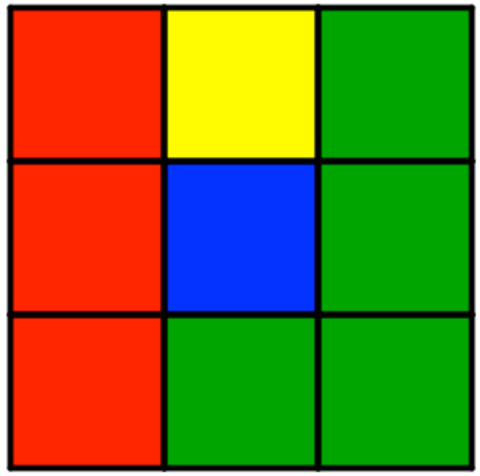
1. What are we pretty sure the student understands?
2. What are we pretty sure the student doesn’t understand?
3. What does the student’s statement not reveal?



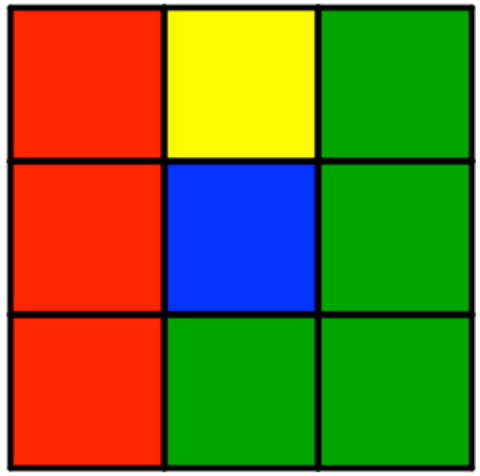
Student A: $\frac{1}{9}$ is yellow.



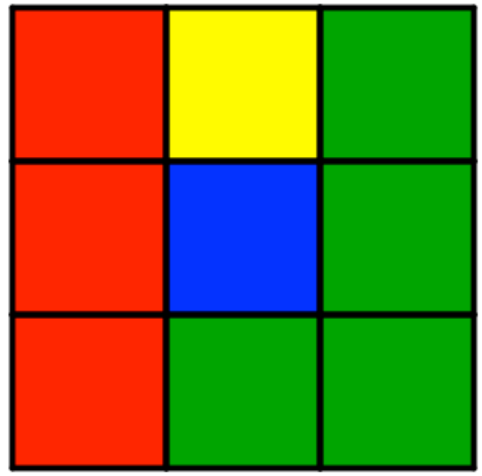
Student B: There’s $\frac{4}{9}$ of green.



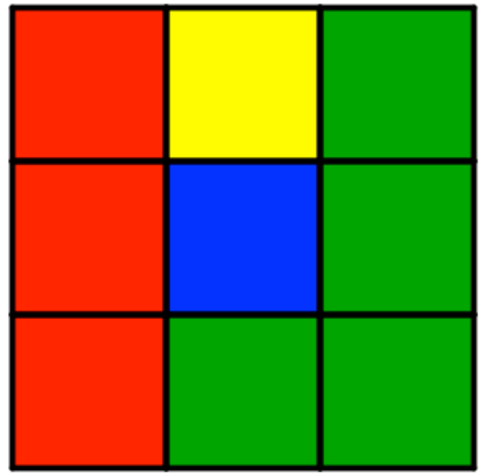
Student C: $\frac{4}{9}$ of the tiles are green.



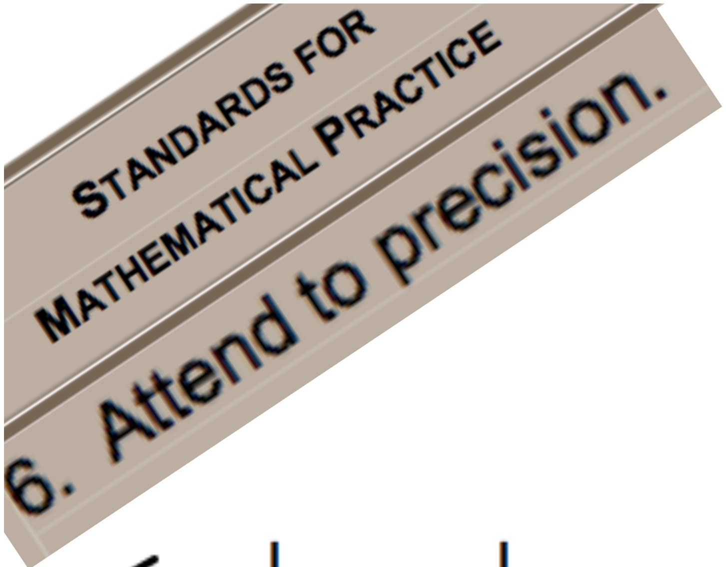
Student D: I don’t think it has 0.25, because that’s ‘hundredths’ and there aren’t 100; there are only 9.



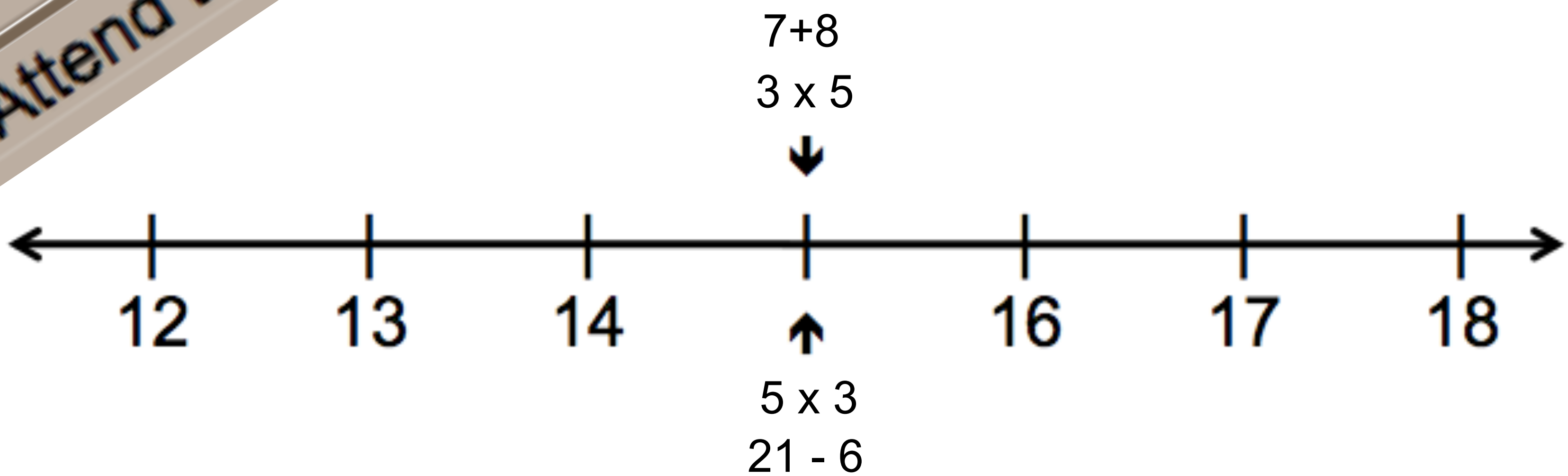
Student E: The blue is $\frac{1}{3}$ of the red.



Student F: I can see $\frac{1}{3}$ in two ways. First, $\frac{1}{3}$ of all the tiles are red. Also, the area of the blue is $\frac{1}{3}$ the area of the red.



IS 3 X 5 THE SAME AS 5 X 3?

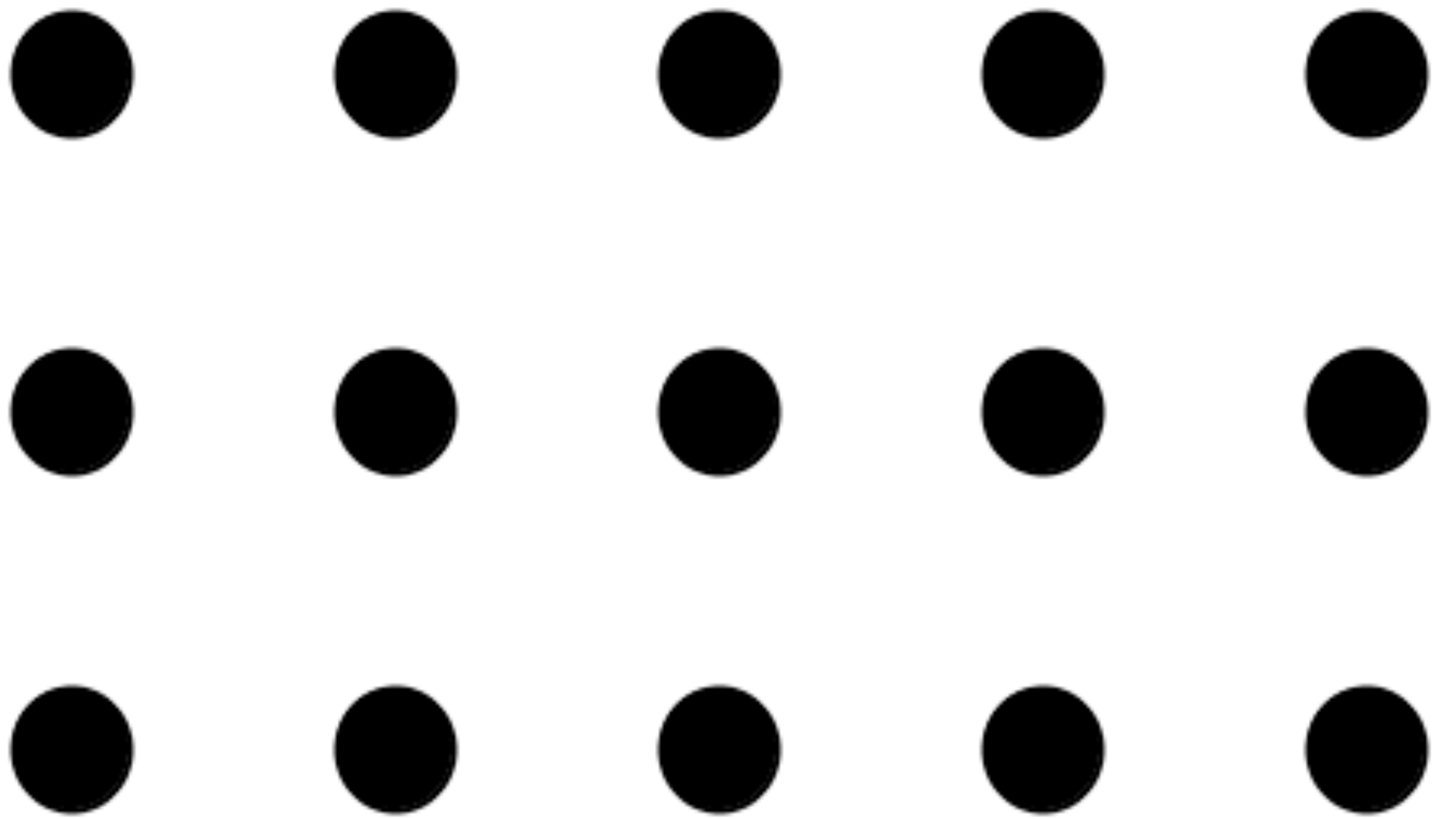


$$5 \times 3 = 3 \times 5 = 7 + 8 = 21 - 6$$

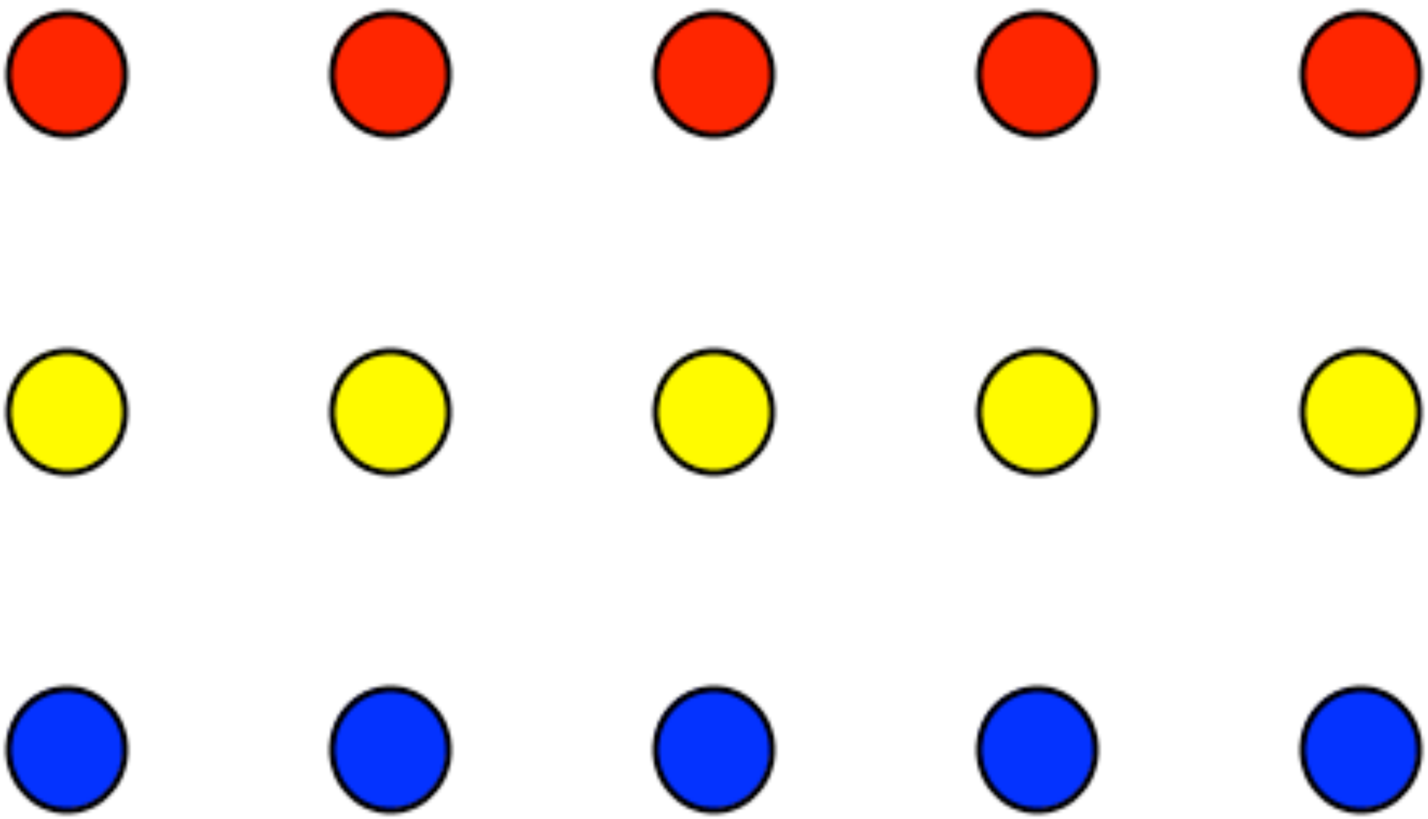
1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*

Grade 3, CCSS-M

3 X 5 OR 5 X 3



3 X 5 OR 5 X 3



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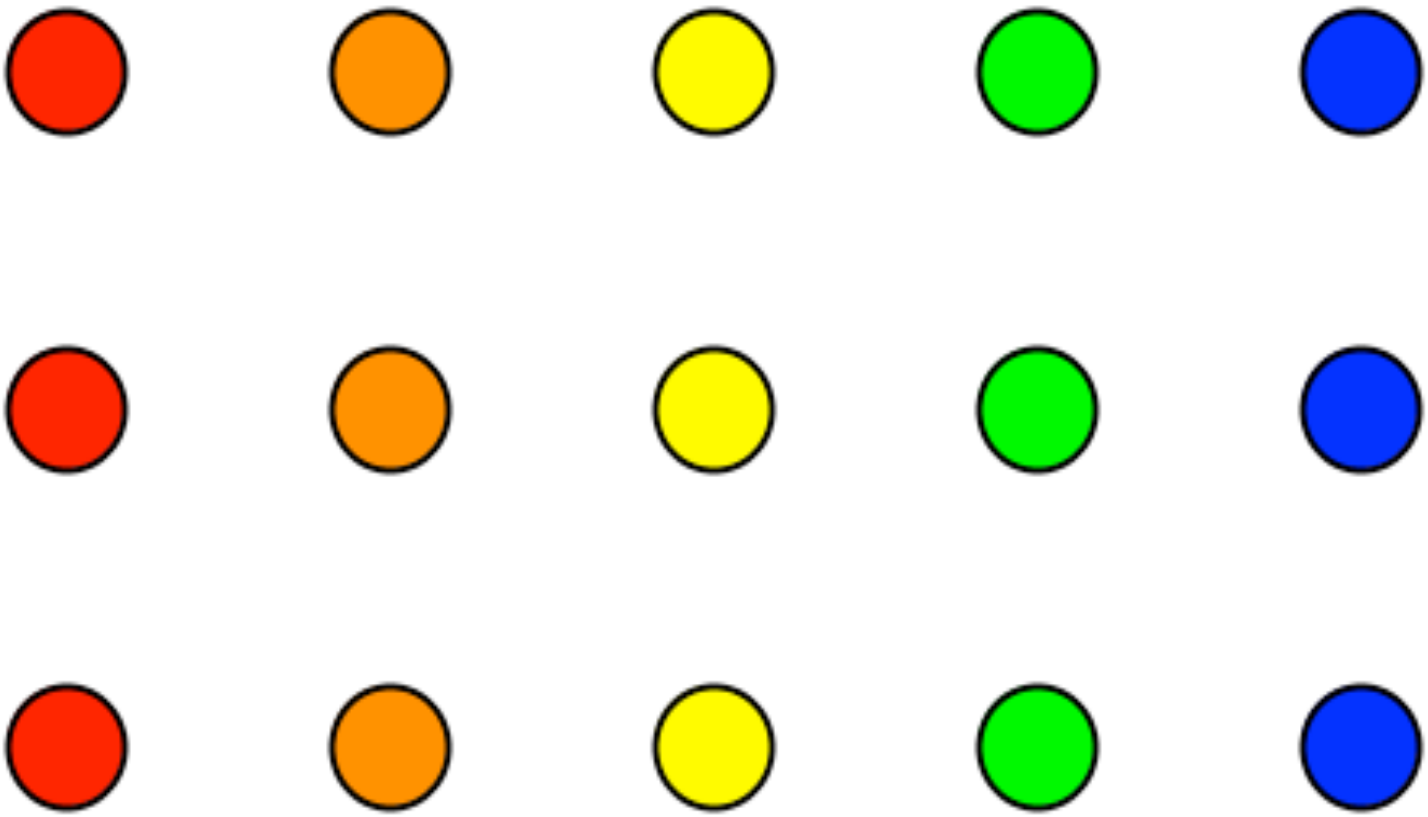

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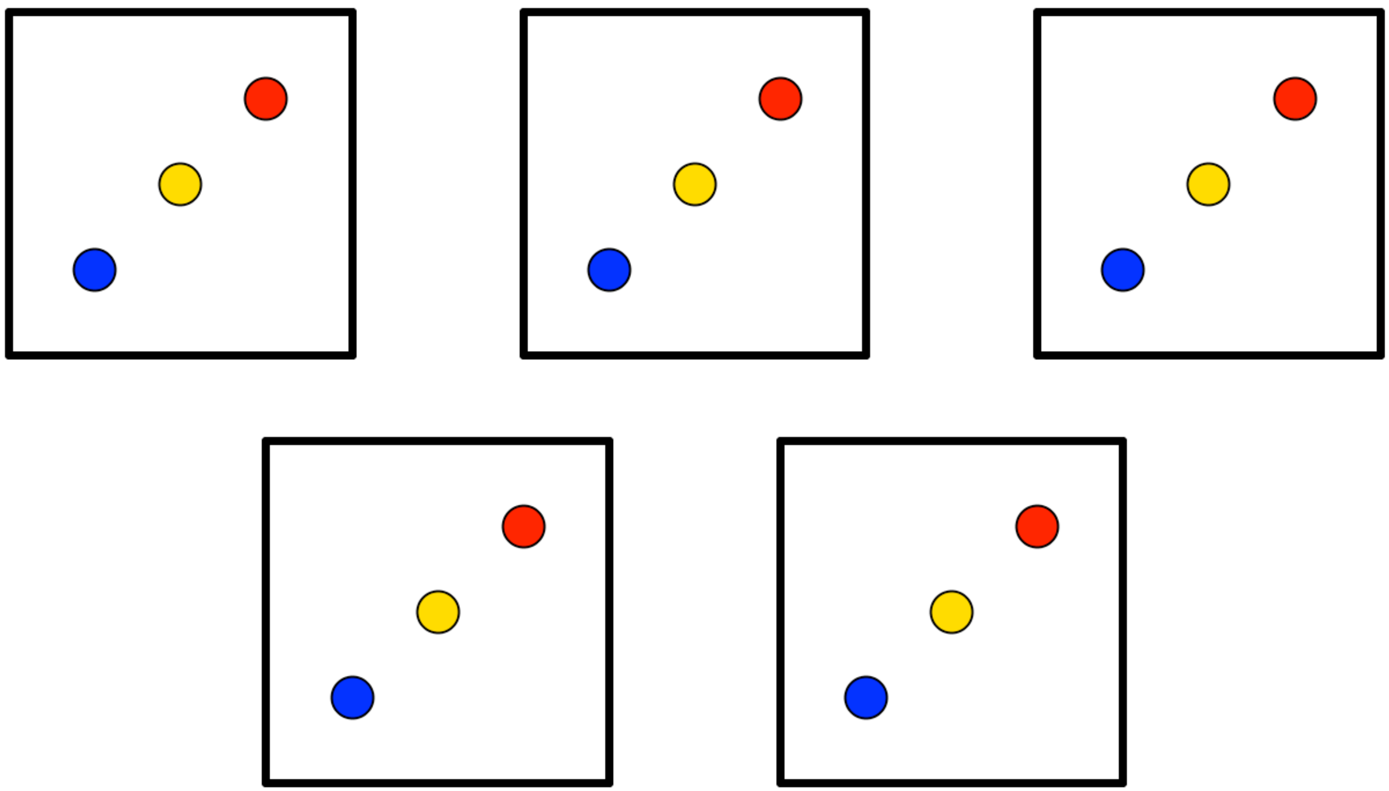
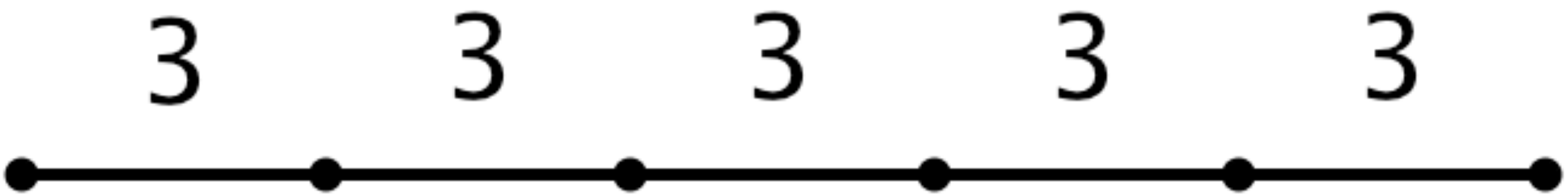
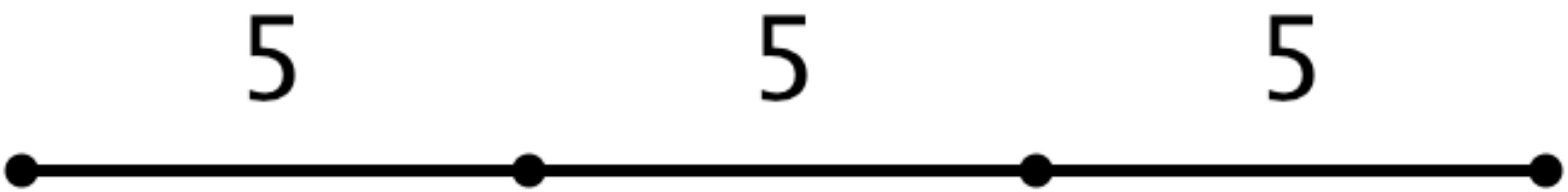


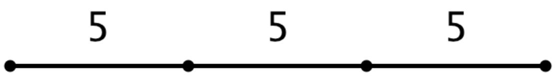
3 X 5 OR 5 X 3



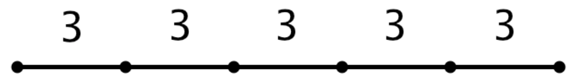
Unambiguously 5 x 3.



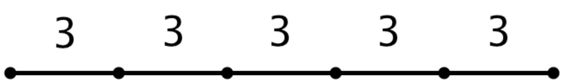
Unambiguously 3 x 5



3 X 5 **OR** **5 X 3**

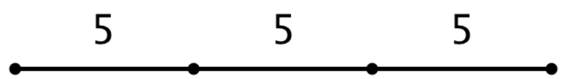


There are 3 T of vanilla used in each recipe. After making the recipe 5 times, how much vanilla did I use?



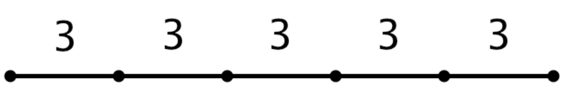
5 x 3

There are 3 groups of students. Each group has 5 students. How many students are there in total?



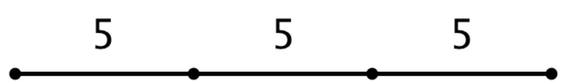
3 x 5

Last week, Jay's sister ran 5 times as much as Jay did. Jay ran 3 miles. How many miles did Jay's sister run?



5 x 3

Yesterday, I spent $5. I spent 3 times that much today. How much did I spend today?



3 x 5

CARD MATCHING ACTIVITIES

Eliciting evidence of student thinking.

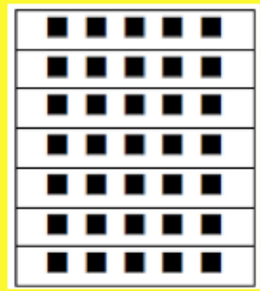
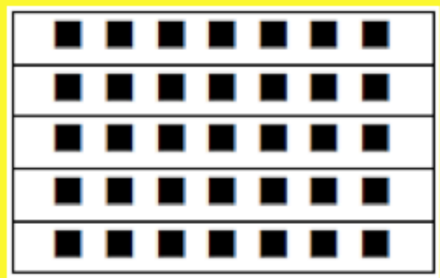
1. Do the task.
2. What would be the experience for students?
3. What could you learn about students?

CARD MATCHING ACTIVITIES

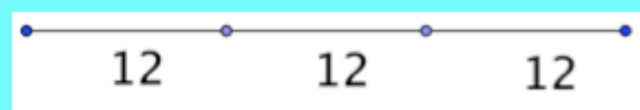
Easy to differentiate.

- Timers
- Concentration
- Smaller decks
- Missing cards

MATCHING ACTIVITIES DEBRIEF

7×5		Mr. Casey bought 7 timers for his classroom. Each timer cost $5. In total, Mr. Casey spent $_____.
5×7		A pack of markers costs $7. Mr. Casey purchased 5 packs of markers. In total, Mr. Casey spent $_____.

MATCHING ACTIVITIES DEBRIEF

3×12	$12 + 12 + 12$
In 3 years, there are _____ months.	

MATCHING ACTIVITIES DEBRIEF

B	$\mapsto$	A
10	$\mapsto$	30
12	$\mapsto$	36
20	$\mapsto$	60
100	$\mapsto$	300

B is $\frac{1}{3}$ of A.

(Car B traveled $\frac{1}{3}$ the distance Car A traveled.)

A is 3 times more than B.

(A ate 3 times more cookies than B ate.)

$\frac{3}{8}$ of the people at the park are adults and the rest are children. $\frac{2}{3}$ of the adults are men, and the rest are women. $\frac{3}{10}$ of the children are boys, and the rest are girls.

How many total people could be in the park?

What fraction of all the people at the park are female?

What fraction of all the people are girls?

$\frac{3}{8}$ of the people at the park are adults and the rest are children. $\frac{2}{3}$ of the adults are men, and the rest are women. $\frac{3}{10}$ of the children are boys, and the rest are girls.

What fraction of all the people are girls?

What are the units of each factor?

$$\frac{7}{10} \times \frac{5}{8} = \frac{7}{16}$$

What are the units of the product?

The class of students ate 3 boxes of doughnuts with 12 doughnuts in each box.

In total, how many doughnuts did they eat?

$$3 \times 12 = 36$$

The class of students ate $2\frac{1}{3}$ boxes of doughnuts with 12 doughnuts in each box.

In total, how many doughnuts did they eat?

$$2\frac{1}{3} \times 12 = 24\frac{1}{3}$$

$$2\frac{1}{3} \times 12 = 28$$

TILE PUZZLE #1

$\frac{3}{7}$ of the tiles are yellow.

TILE PUZZLE #2

$\frac{3}{5}$ of the tiles are yellow. $\frac{1}{2}$ of the remaining tiles are blue.

TILE PUZZLE #3

$\frac{2}{3}$ of the tiles are green. $\frac{1}{6}$ of the tiles are blue.

TILE PUZZLE #4

$\frac{5}{9}$ of the tiles are green. 0.75 of the remaining tiles are yellow.

TILE PUZZLE #5

There are $\frac{3}{5}$ as many green tiles as blue tiles. There are $\frac{1}{5}$ as many yellow tiles as blue tiles.

TILE PUZZLE #6

There are $\frac{3}{5}$ as many green tiles as blue tiles. There are $\frac{1}{5}$ as many yellow tiles as blue tiles.

TILE PUZZLE #7

$\frac{4}{7}$ of the tiles are green. There are $\frac{3}{8}$ as many blue tiles as green tiles.

TWO MAJOR SENTENCE FRAMES FOR FRACTION MULTIPLICATION

[FRACTION] of the [NAME OF WHOLE] (are [NAME OF PART]).

e.g., $\frac{3}{5}$ of the students are from Roxbury.

There are [FRACTION] as many [COMPARISON GROUP] as [REFERENCE GROUP].

e.g., There are $\frac{3}{5}$ as many students from Roxbury as students from Dorchester.