Presentation: 238
Creating Puzzlement and Perseverance: A Productive Failure Problem-Solving Model
NCTM 2016 Regional Conference and Exposition: Philadelphia, Pennsylvania
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The Task:

The Game Place

You were just hired as the store manager of "The Game Place". The Game Place sells used video games. As the store manager, you are responsible for purchasing games from other used video game distributors. You will need to know some basic math skills to solve problems involving the budget and purchasing. For example, two new types of games are available to offer in our store. The first type of game costs $30 dollars and the second one is $20. You want to stock at least $600 worth of games to be competitive with the surrounding stores, but your store's purchasing budget cannot exceed $1200 worth of games.

Question: How many possible combinations of orders can be made that will satisfy the minimum and maximum requirements?

(adapted from a word problem from the Jordan and Granite online textbook, p. 82)

Pre-Test

1. What do you know about graphing linear functions?
2. What do you know about graphing linear inequalities?
3. Can you give an example of a problem that could be solved by graphing a line?
4. What do you know about systems of linear equations?
5. What do you know about systems of linear inequalities?
6. What are the possibilities when solving a system of linear inequalities?
7. Can you make up a problem that could be solved by graphing a linear inequality?
8. Can you make up a problem that could be solved using a system of linear equations?

Questionnaire 1:

1. After working through the task, what have you learned about finding possible solution combinations?
2. While working through this task, what did you find easiest? What was most useful?
3. What was most puzzling about this task? What helped you to keep trying?
4. In general, what did you learn?
5. How do you feel about this task?

Instructional Plan 1
Unit 2: Reasoning with Equations and Inequalities
Lesson: System of Linear Inequalities
Length: 6 days

The Purpose:

The purpose of this instructional unit plan is to study a problem solving task that was designed using Kapur's (2010) productive failure model for problem solving to use for my case study, *A Case Study of How Ninth-Grade Mathematics Students Construct Knowledge during a Productive Failure Model*. The instructional plan will provide a framework to ensure the standards, performance task, essential questions, assessments, and time frame are adequately implemented. The productive failure model for problem solving will allow for students to work through a problem and make their own assumptions before the teacher aids in helping the students find an answer.

Overview of a Productive Failure Model:

Kapur (2010) suggested that teachers should incorporate "productive failure" as a problem-solving method as opposed to allowing for supports during problem solving. Productive failure is a method that capitalizes on the understanding that students need to be able to experience opportunities of unsettlement in order to expand their learning (Kapur, 2010, 2011). Kapur (2011) stated that:

> it is important to note that invariant across the Piagetian notions and conceptual change is the idea that a disequilibrium (e.g., between internal schemas and environment) or a discrepancy (e.g. between learners' and canonical conceptions) – both forms of temporary failures – are a necessary condition for learning. (p. 562)

Therefore when students are problem solving and are challenged to the point of bewilderment, it can be a positive symptom of solving an authentic problem. Cotic and Zuljan (2009) stated that problem solving should be an "intellectual search" that "provoke[s] cognitive tension" (p. 299).

Kapur (2010) stated that when designing a complex problem, the problem should have "multiple solution paths leading to multiple solutions" (p. 527) and the problem should enable

students "to make some inroads into exploring the problem and solution spaces without necessarily solving the problem successfully" (p. 528). Cotic and Zuljan (2009) gave five types of complex problems that should be given to students: 1) "problems that contain insufficient amount of data", 2) "problems containing more data than needed", 3) "multiple-solution problems", 5) "problems that are solvable in various ways", and 4) "problems with contradictory data or no solution" (p. 300).

Kapur's (2008) productive failure model for problem solving provides students with opportunities to hypothesize multiple solutions to the problem. He found "that students from the productive failure condition produced a diversity of linked problem representations for solving problems that were ultimately unsuccessful in their efforts" (p. 523). Although the students were not able to solve the problem during the period that they worked without supports, Kapur (2008) found that "despite seemingly failing…students from the productive failure condition outperformed their counterparts from the lecture and practice condition on well-structured and higher order application problems on post-tests" (p. 523).

The Lesson:

I will use Kapur's (2012) productive failure design for problem solving in order to determine how students construct knowledge during a productive failure task. Table 1, shows the modified version of Kapur's (2012) productive failure model that will be used for each problem solving cycle in this case study. Each cycle will last six days. Table 1 shows a day-by-day plan of the activities conducted during a productive failure cycle, the duration of each activity, and the qualitative data that will be collected for each day of the cycle.

Table 1 Productive Failure Model Cycle: Data Collection and Methods

Day	Activity	Duration	Data Collection
Day 1	Pre-Assessment-open ended questions	25 minutes	Qualitative data – pre-assessments are recorded in learning log
Days 2 and 3	Problem Solving Task Cycle 1: Systems of Linear Equations and Inequalities	45 minutes	Qualitative data 1. Student work is recorded each day in the learning log. 2. Students will be video-taped while they work in groups on the task. 3. At the end of the task each day, each student records in the learning log a questionnaire page (Student Questionnaire 1 and Student Questionnaire 2). 4. Researcher Log–records general observations that were noticed during the task before the video-tape is viewed.
Day 4	Consolidation Lesson – students make a poster	90 minutes	Qualitative data 1. Students will be video-taped during their presentation. 2. Teacher will collect the poster used during the presentation.

	and present their findings to the class		3. At the end of the presentation, students answer a questionnaire page in the learning log (Student Questionnaire 3). 4. Researcher Log- records general observations after the consolidation lesson before the video-taping is viewed.
Day 5	Teacher Directed – teacher models way to solve the problem solving task	45 minutes	Qualitative data 1. Students are asked to take notes in the learning log. 2. At the end of the lesson, students answer a questionnaire page in the learning log (Student Questionnaire 4). 3. Researcher Log – records general observations after the lesson occurred.
Day 6	Assessment- new open-ended isomorphic questions – individually -teacher goes over the answers-student makes any revisions in a different colored pencil	90 minutes	Qualitative data 1. Students answer the isomorphic questions in their learning-logs with color coded corrections/revisions after the teacher goes over the answers. 2. Final questionnaire – answered by each person in the group (Student Questionnaire 5). 3. Students are interviewed as a group while watching the clips of the video-taping from days 1 and 2 – this interview is video-taped. 4. Researcher Log- records general observations after the group interview.

Commented [JSH1]: Are these new or the same open-ended questions from Day 1? On page 6, you wrote "each student will be asked the same 10 open ended questions from the pre-assessment."

Essential Questions:

- How do I graph a linear equations and linear inequality in two variables?
- How do I find possible solutions when two variables are given?

Commented [JSH2]: Linear equations and linear inequalities

Instructional Technology:

1. Graphing Calculators – at least one per group
2. Computer – optional – as a resource if students need further information on a topic
3. LCD projector/computer – optional - used to present any work student has made
4. Internet – students may need to research a question further, define a word, used as an available resource (the classroom is a BYOT (Bring Your Own Technology) so the students will have access to internet through their own devices

Materials:

1. Copy of Performance Task, "The Game Place" , one per person
2. Learning Log
3. Graph Paper and straightedges
4. Graphing Calculators – as listed under instructional technology
5. Chart Paper and markers
6. Textbook

7. Colored pencils
8. Notebook paper and pencils
9. Scientific calculator
10. Whiteboard and markers

Instructional Objectives:

Instructional objectives are defined as "a statement of performance to be demonstrated by each student in the class, derived from an instructional goal and phrased in measurable and observable terms" (Oliva, 2009, p. 310).

Cognitive Domain

1. The student will develop a plan for solving the task in order to gain a deeper understanding of systems of linear equations.
2. The student will develop a plan for solving the task in order to gain a deeper understanding of systems of linear inequalities.

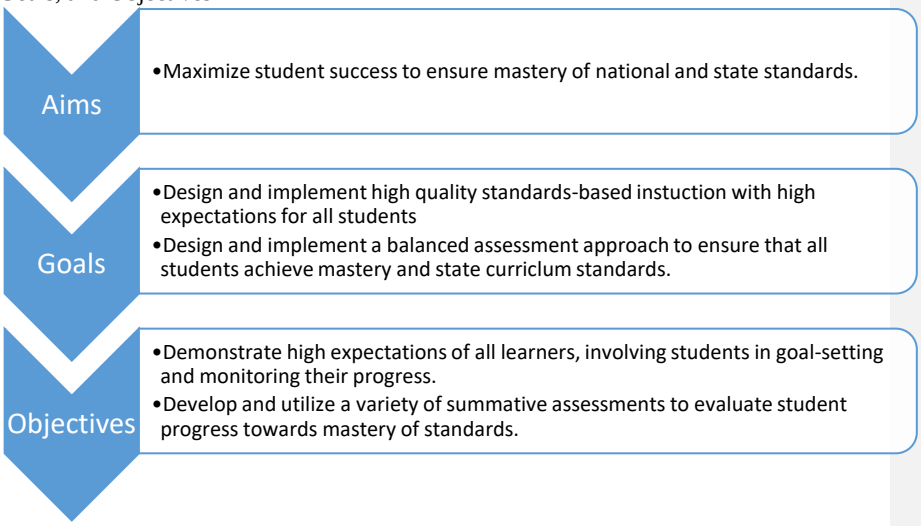
Affective Domain

1. The student will grapple even when the task may become frustrating and show persistence when attempting to solve the problem in order to gain a deeper understanding of systems of linear equations.

Curriculum Aims, Goals, and Objectives:

As shown in figure 1.1, the instructional plan aligns with the aims, curriculum goals, and curriculum objectives of the school system where the study will be conducted, Coweta County School System. Each of the goals listed in figure 1.1 have multiple objectives, the most relevant was listed for each respectively.

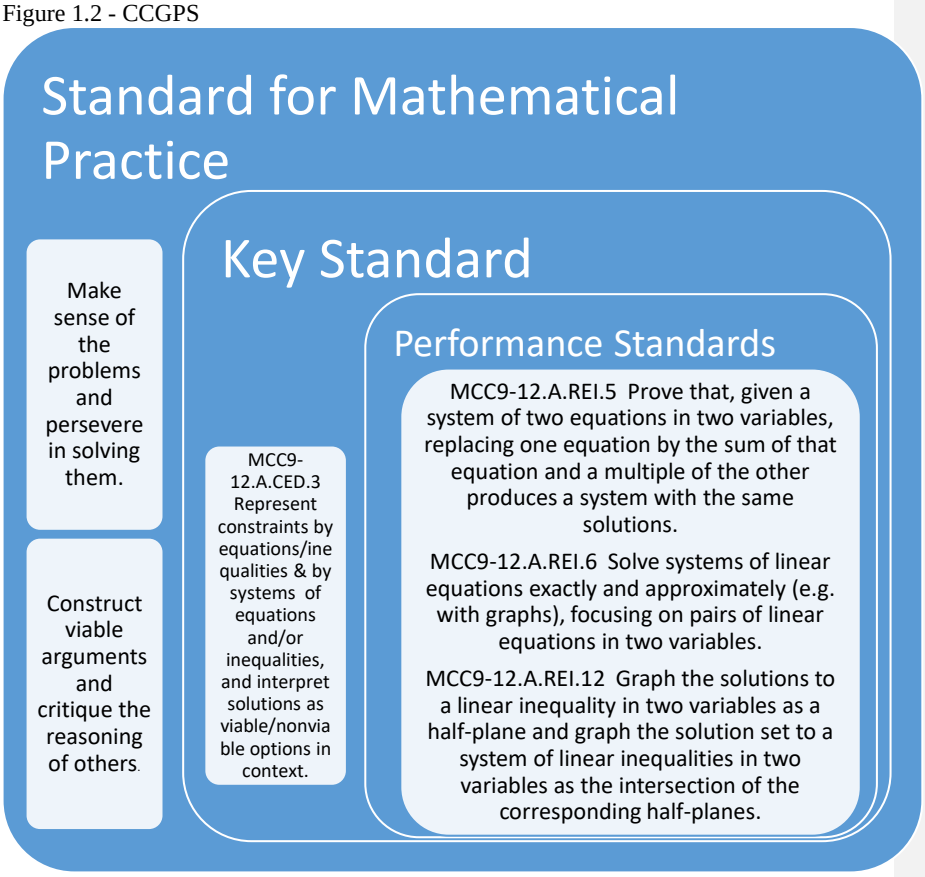
Figure 1.1 – Alignment of Curriculum Guide to Coweta County School System's Aims, Goals, and Objectives



Standards:

As listed in figure 1.2, the Common Core Georgia Performance Standards (CCGPS) that pertain to the problem solving task used in this curriculum guide. This task aligned well with the first five Standards for Mathematical Practice. The two standards for that component which aligned best were chosen for figure 1.2.

1. Figure 1.2 - CCGPS



Note. For a complete list of the Common Core Georgia Performance Standards for Coordinate Algebra, go to: https://www.georgiastandards.org/Common-Core/Common%20Core%20Frameworks/CCGPS_Math_9-12_CoordinateAlgebra_Standards.pdf

Evaluation Techniques:

1. Pre-Assessment: the students will answer 10 open-ended questions
2. Formative Assessment: Data Gathered During the Task

In order to determine how students grapple and persist during the productive failure modeled task, two groups of students will be videotaped during their problem solving sessions. The teacher will be looking for the following:

1. Communication: Students are talking to each other during the task. Since data for my dissertation will be collected in the same manner, I need to make sure that the task's context encourages the students to talk to one another during the task and leads to good discussion.
2. Difficulty Level: The students were challenged during the task to the point of bewilderment, but the students still persisted for the entire allotment of time.
3. Authenticity: The students presented multiple ideas or methods for solving the problems.

To determine whether the students developed a plan for solving the task and showed persistence during the task, the teacher will take detailed notes while the students are working on the task. The teacher will gain further insights and notes from watching back through the videotapes. This qualitative data will be analyzed using the three indicators listed above.

3. Summative Assessment: Data Gathered After the Task

After the students have finished the task and presentations and the teacher has presented the students with the solutions, each student will be asked the same 10 open ended questions from the pre-assessment. This qualitative student feedback will help to determine whether or not the students made learning gains after they completed the task.

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References:

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