Development of Teacher Candidates Mathematical Knowledge for Teaching (MKT) in a Preservice Setting

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1. Purpose

One of the principal challenges facing candidates is shifting from "student" to "teacher." This transition requires candidates to reorient from thinking about how they solve mathematics problems to engaging with students and their work, understanding student representations, and asking questions to encourage student thinking.

To scaffold this transition, we developed a five-step Mathematics as Teacher Heuristic (MATH) that requires candidates complete the following:

1. solve a rich task as a "doer";
2. assess student work samples associated with the same task;
3. consider questions to ask struggling students;
4. develop scaffolded instructional materials addressing student needs (gleaned from earlier analyses); and
5. reflect on the process.

In earlier implementations of the heuristic (Meagher, M., Edwards M.T. and A. Ozgun-Koca, 2013), the use of student solutions encouraged candidates to consider alternate approaches and was successful in the development of candidates' Mathematical Knowledge for Teaching (MKT) (Bass, 2005) and their Pedagogical Knowledge.

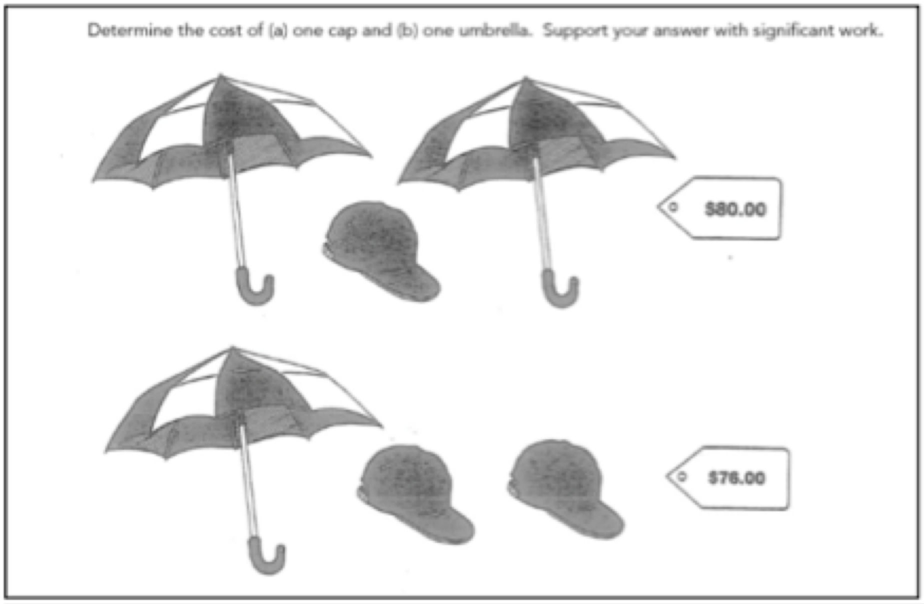
2 Theoretical framework

Research led by Bass (2005); Ball, Hill and Bass (2005); and Hill, Rowan & Ball (2005) has provided “an emerging theory of what we have named *mathematical knowledge for teaching* (MKT)” (Bass, 2005, p. 423). Bass characterizes this knowledge as “the mathematical knowledge, skills, habits of mind, and sensibilities that are entailed” (p. 429) in teaching mathematics. There is evidence that development of MKT is directly correlated to student achievement (Hill, Rowan & Ball, 2005; Hill & Lubienski, 2007). In another effort to describe the shift undertaken by candidates, Hill, Ball & Schilling (2008) developed the construct of “knowledge of content and students (KCS)” (p. 373) - a combination of subject content knowledge and knowledge of how students think about and engage with content knowledge. Such knowledge directly involves candidates making a shift from “student” to “teacher.”

“Unpacking” (Adler & Davis, 2006) i.e., examining student solutions to understand “the relationship between a mathematical result or answer and the process of its production” (p. 274) is one aspect of “professional noticing of children’s mathematical thinking” (Jacobs, Lamb & Philipp, 2010). Building on van Es and Sherin (2008), Jacobs et al. (2010) provide a framework for analysing the way candidates engage with student work. They develop the construct of “professional noticing of children’s mathematical thinking,” characterizing this noticing in three ways: (a) attending to strategies, (b) interpreting understandings and (c) deciding how to respond to understandings. Their work shows that the ability to “notice” in this professional way is not something candidates typically have but can be developed with sustained professional development.

3. Methods and data sources

The twenty-two participants in this study were undergraduates enrolled in the second course of a yearlong methods sequence. This course built upon candidates' first semester experiences with planning and assessment. Candidates worked through the MATH heuristic with the following set of tasks with the data consisting of their responses to each task.



Task 1: Constructing a Solution: First, you will solve the problem as a student likely would - with your primary goal being to "find a correct answer" (although we ask that you do this using at least two different methods). In all cases, you are to support your answers with significant work.

Task 2: Assessing Student Work: Next, you will use a pre-fabricated rubric to assess the mathematics understanding / performance of high school students evidenced in their written work. This task mirrors more closely the work of a beginning secondary school teacher.

Task 3: Considering Good Student Questions: Prior to revising the task, you will analyze student work carefully, paying particular attention to student stumbling blocks with the task. Brainstorm questions that you might ask a student who is "stuck" while trying to solve the problem. This task also mirrors more closely the work of a beginning secondary school teacher.

Task 4: Revising the Task: After assessing student work samples, paying particular attention to student misconceptions associated with the task, you will construct teaching materials that better support students as they explore the problem. You will construct materials that illustrate your understanding of the interconnectedness of mathematical ideas across multiple courses, constructing one worksheet tailored for pre-algebra students and another for first and second year algebra students. The worksheets should be designed to foster student connection making, generalization, and creativity, while providing students with significant support for solving the tasks you create.

Task 5: Reflection: Lastly, you will consider what you have gained as a teacher of mathematics through your work on this assignment. You will be asked to compare your initial solution strategy to those employed by high school students as well as ways in which your mathematical understanding of the task has grown through the engagement with the 5 basic mathematical practices (problem solving, reasoning, communicating, connecting, representing).

Figure 1 MATH heuristic task

The research question is: what does the MATH heuristic show about teacher candidates' MKT?

4. Results

In this paper we focus on the first three components of the MATH heuristic, using the first component (i.e., solve a rich task as a "doer") to discuss issues with candidates' MKT and using the third component (i.e., consider good questions to ask struggling students) to demonstrate that the development of KCS and MKT remain areas of concern.

Teacher Candidate as a Doer

Candidates first solved the mathematics task. Seventeen of 21 solutions were algebraic, centering on solving a linear system of equations: elimination method (4), substitution method (8), graphing (2), and matrices (3). When asked to provide the second solution, most candidates simply changed how they solved the linear system of equations.

Many acknowledged that it was difficult to come up with more than one solution, indicating the limits of the candidates' KCS and MKT: they can solve the problem themselves but struggle to imagine multiple solution paths.

Noticing students' problem solving strategies and mathematical thinking

Using Jacobs et al.'s framework, we saw that candidates were successful in *noticing* the correct strategies that most closely aligned with their own work. They had varied success in *noticing* and *interpreting* incorrect student work with misconceptions. There were two main categories of misconceptions in unsuccessful student solutions: (i) the hats in the top picture could have a different price from the hat in the bottom picture and (ii) all items have the same price. In the second case, there were two variations: (ii) (a) incorporated misconception (i) whereby the students divided $80 by 3 and then divided $76 by 3 to get the same price for each of the items in the top and for each of the items at the bottom but each picture had different prices and (ii) (b) whereby they added $80 to $76 and then divided by 6. No candidate made explicit reference to either of these categories of misconception as a trend in the student work samples. In fact, their analyses did not deploy KCS or MKT, rather they tended to focus on more general pedagogical concerns.

Candidates considered negative trends to be qualities such as not showing work and not applying the meta skill of checking their work "A negative trend I saw was a lack of asking, "Does this make sense?" I feel after one completes a math problem, this question is essential to ask this of the final answer." In terms of positive trends the candidates again valued broader pedagogical issues such as "correct use of mathematical language. Even if students never even got close to a correct answer" and willingness to try "Despite most of the students showing little to no algebraic skill, most students attempted the problem. This was inspiring because it showed me that the students were at least willing to try the problem."

Focusing on general strategies, rather than analyzing the mathematical content of student work, shows that engagement with student work often activates the pedagogical skills but not their KCS and MKT, especially on the level of *noticing* general trends.

Close analysis of student work

When it came to close analysis of student work in response to the third prompt, candidates were able to get "inside the student mind" even if they were not *noticing* larger trends. However, in many cases, focus on general pedagogical issues persisted.

Our full analysis of the data examines candidate responses to student work on each of the three major misconceptions but owing to space in this paper we focus on misconception (ii) (a).

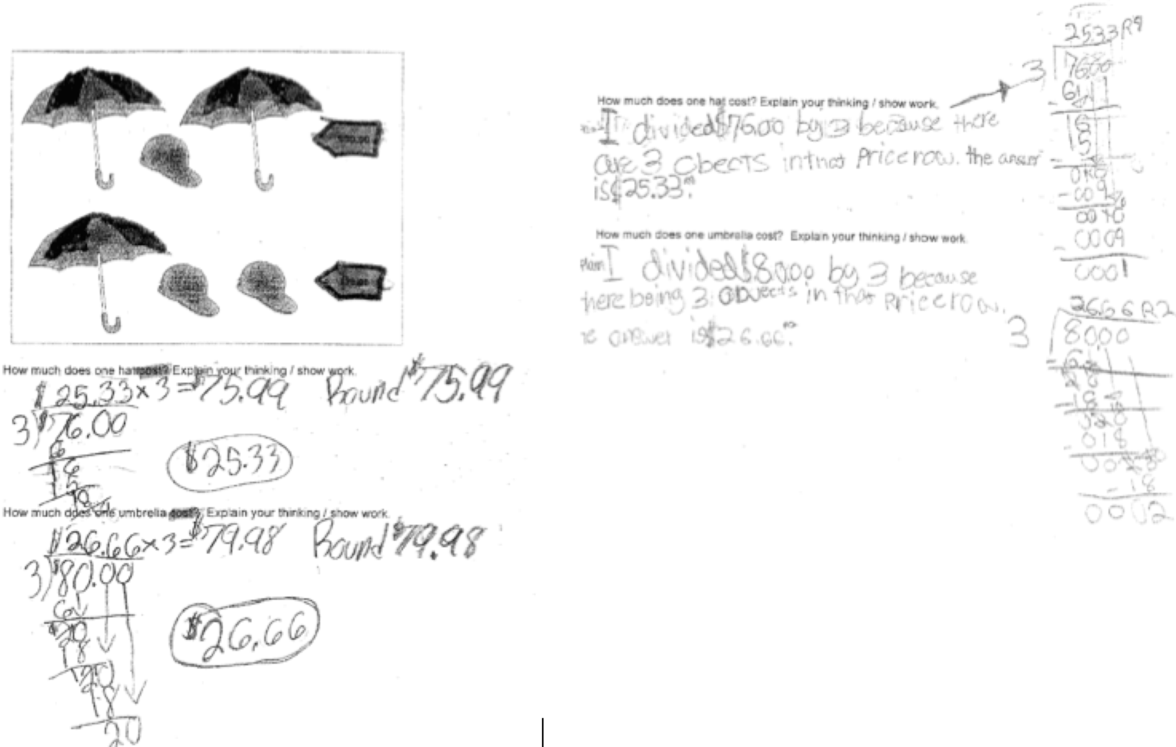


Figure 2. Examples of misconception (ii) (a)

One pair of candidates *identifed* the strategy that manifests misconception (ii) and asked the student, "Does each purchased item have to be the same price? Why or why not?" with the rationale that, "The student found the average cost for the items in each of the two given situations. They seem to lack the understanding that they are trying to find the price of each individual item instead of the average cost." However, again there is no mention of misconception (i) so their *interpret[ing] understanding* is incomplete.

Another pair of students *noticed* the embedded misconception (i) (treating items in the different pictures as different) suggesting the question: "How would your first price compare when plugged into the second equation?" with the rationale "This question will prompt the student to try plugging in the values for the variables. The student should come to the conclusion that the two items are not equal" but make no reference to the "same price" misconception (ii).

A third pair's questions and rationales were not directly concerned with the content of the students' work at all: "What information are you given and what is the problem asking? How can you use the given information to help you solve the problem?" with the rationales that "This would help the student outline what information is given in the diagram and will have the student break down the problem to decide on what the problem is asking" and "This would help the student use the given information to come up with a plan to solve the problem, which could include an equation, table, or other mathematical representations." Once more we see the deployment of Pedagogical Thinking but not of MKT.

5. Significance

In earlier work (Meagher, M., Edwards M.T. and A. Ozgun-Koca, 2013) we found that the MATH heuristic was an effective tool for shifting candidates' thinking. The Noticing Framework of Jacobs et al. (2010) proved a useful tool to analyse the development of candidates' pedagogical knowledge and to identify deficiencies, and successes, in candidates' deep engagement with student mathematical thinking. No candidate discussed explicitly patterns of misconceptions across the more than 100 student samples and, in our analysis, we saw cases where candidates were unable or unwilling to *notice* and *interpret* student thinking. The data shows that the kind of analysis that would show strong MKT does not seem to come naturally to students, but their development in this area is vital if they are to become effective teachers. We plan for an adjustment in the prompts for the MATH heuristic with a requirement to describe trends in student learning.