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Investigating Learning and Success: Innovating in college remediation

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Objective: Remedial mathematics is widely considered a barrier to student success in community college. Yet there is little research that explicitly examines whether increasing student learning will improve grades or completion rates. This study examines the relationship between different types of student learning in developmental math and measures of progress toward a degree, such as grades.

Method: A mathematical skills assessment was given to all intermediate algebra students at a large, urban community college, and to students in the following college level class at the beginning of the next term. Assessment scores were compared with student characteristics, grades in intermediate algebra, grades in college-level math, and whether the student earned a credential.

Results: Self reported grades in previous math classes were the largest predictor of grades in college math. Procedural algebra skills did not predict grades in college-level math. Conceptual skills predicted grades in general education math, but not in precalculus. Math skills did not show any clear relationship between either grades in intermediate algebra or earning a credential. Scores on the procedural scale were significantly lower for students who had taken at least one term off from mathematics courses. In contrast, conceptual skills remained reasonably constant, regardless of the time since the student had taken their previous math class.

Contributions: The findings challenge the assumption that increased student learning in remedial mathematics will improve student outcomes. They suggest that the type of mathematics students learn matters, and support calls for further examination of what happens in a community college classroom, what students learn and how it benefits them.

Investigating Learning and Success: Innovating in college remediation

Conducting research in mathematics education generally requires paying close attention to many technical details simultaneously. Focusing narrowly and intensely makes it possible to lose site of the bigger picture. This introduction attempts to connect the research presented here to the context of student experience and current policy issues. After this intro a more traditional and academic paper follows. (This intro is only endorsed by one author—Davis). For those wishing to skip ahead begin toward the bottom of page 10.

Each year millions of students attend college with the hope that doing so will improve their lives. They believe that earning a degree will create access to careers with better earnings and job satisfaction. Sadly, many students have these dreams and aspirations blocked because of mathematics requirements.

The standard across community colleges and state university systems is to require students to demonstrate mastery of high school mathematics by achieving a certain score on an exam[1]. Those that do reach specified scores are required to enroll in remediation. These course sequences vary in length from two to six quarters. The content of remedial programs varies to some degree, but generally remediation course sequences contain the same mathematics that is taught in k-12. The courses are quicker paced, more rigorous and offer less student support than high school versions.

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[1] Many institutions are now also using transcripts to support placement. Unsurprisingly, performance in courses over many years is a more accurate predictor of future success than a brief exam. However, in many cases the decision rules for placement require GPAs and courses completed that leaves only a small minority of students affected by transcript analysis. It is also important to note that this approach still requires students to demonstrate mastery of the material without questioning the assumption that the knowledge is valuable for them or the requirement sensible.

Those students that are stymied by mathematics requirements are not necessarily those unable to master the technical proficiencies required of those joining a STEM field. Many students thwarted by math requirements are trying to earn degrees in the humanities, social sciences and professional degrees such as in health fields. For these students, there is little connection between the mathematics content of the remedial course sequences and the knowledge they need for success in school and work. [2]

In those colleges that allow students to take college-level courses before completing remediation it is common for students to meet all of the requirements for a degree except for their mathematics requirements. Given this evidence it is difficult to understand how remedial course requirements can honestly by viewed as preparing students for college.[3] Remediation plays a different role for many students.

There was perhaps a time when society needed colleges to help select students by weeding out the less able. Mathematics is the most effective content area for this purpose because the difficulty of tests can always be increased to ensure that only the desired number succeeds and it is straightforward to design assessments that can be graded cheaply and objectively.

The times have changed. We have many spaces for enrollment in college and increasingly fewer jobs for those that have no post secondary education. We do not need colleges to weed out the weak students (whatever remaining need there is to select students, there is little basis for continuing to use mathematics performance as the measuring stick). We need colleges

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[2] Pathways approaches are promising emerging trends, although they generally do not go far enough toward teaching the concepts and tool use students would find valuable. Moreover, they are fierce policy battles regarding pathways that do not teach intermediate algebra.

[3] Obviously this does not describe the needs of students in STEM fields.

to give students access to the many fields that have worker shortages. In such a context, it becomes critical to consider what students need to know for the degrees they are seeking and the fields they plan to enter.

There are, of course, reasonable arguments against focusing solely on the mathematics students need to know for success in specific college majors and particular careers. Mathematics is important and has value for all students. Reasoning about quantity and shape is a part of our human biological and cultural inheritance. Mathematics is powerful. Yet, the current standard practice does not provide consistent access to the power or utility of mathematics. There is a need for more evidence regarding the types of mathematics that is sensible to require of all college students. It is difficult to see how the current remediation regime of preparing students to study calculus by having them take algebra through the intermediate level makes sense.

If social scientists organized to influence the chancellors of their college systems to mandate that all students take a course with a reading list of turn of the century foundational theorists like Durkeim, Weber, and James as part of general education requirements they would have some solid arguments. These texts contain fundamental ideas essential for the study of humanity across several academic disciplines. Studying the individual, the group and the relationships between them is essential for understanding civilization and participating in the modern intellectual project. The social scientists might concede that the language of these texts is a bit challenging, but students need not be forced to read in German. If many students failed these courses, the professors could potentially argue "college students should be able to make sense of essays in English and write about them coherently".

Thankfully, social scientist faculty members have not been able to mount a movement for such a requirement. One challenge faced in countering arguments like the above is that there is little that is rhetorically effectively against statements such as, "college students should be able to do X". These statements appeal to common sense and stand on firm ground when those expectations are widely shared. It is difficult to engage in meaningful discussion about why students should be able to do X or what would it matter if graduates were not able to do X, when it seems *prima facie* that they should be able to do so.

This common-sense appeal supports the current state of mathematics requirements. To many it seems entirely reasonable that college students should be able to demonstrate mastery of intermediate algebra on the entrance exam and if not they should be able to learn to do so quickly in remedial courses (regardless of the quality of instruction). Professors of mathematics can and do argue that the placement exams are easy. (They are certainly easy for them). Yet at community colleges the majority of students are placed into remediation (Bettinger & Long 2005; Attewell, Lavin, Domina & Levey, 2006). Attempts to alter the requirements to reduce the number of students forced to take remedial courses and to provide students with mathematics learning experiences that would be useful for their work and lives are commonly resisted. Reformers are accused of the awful specter of "lowering standards'.

These debates have been going on for a long time. Perhaps a historical perspective can shed some light on our current situation. There was a time in this country that classical languages were a required part of a college education. Reformers tried to remove these requirements to make room for other options (e.g., so students could choose to study modern languages and read Voltaire and not Tacitus). The arguments in defense of the classics were similar to those arguments used to support the status quo in mathematics--classical languages sharpen the mind and make people better thinkers. A college-educated person should know classical languages. A Yale university report (1828) in support of the classical curriculum opined:

> *iIs this a time for the college to lower its standard? Shall we fall back, and abandon the ground which, for thirty years past, we have been striving so hard to gain? Are those who are seeking only a partial education to be admitted into the college, merely for the purpose of associating its name with theirs? Of carrying away with them a collegiate diploma, without incurring the fearful hazard of being over-educated? Why is a degree from a college more highly prized, than a certificate from an academy, if the former is not a voucher of a superior education?*

There is little evidence that there are generalized problem-solving skills that can be taught in one domain of human activity that are transferred to other domains (Vygotsky, 2014). While there are certainly benefits to learning mathematics, Latin, and or Greek, success does not appear to sharpen the mind or make people more practical as was claimed in the Yale Report. If this were so, we would expect to find that the faculties of colleges, particularly those departments in the STEM fields and the classics were among the most well-reasoned and practical groups of our society. Advance study does not necessarily appear to develop these generalized skills. One study of remedial college students found that as students took more remedial courses they performed worse on mathematics problems that required reasoning (Stigler et al., 2010.). It seems that the benefits of mathematics education may not even transfer to other types of mathematics problems.

The above is an effort to position this paper as moving the discourse in mathematics education policy beyond appeals to ideas about what a college student should be able to do. This work is attempting to investigate the value of mathematical knowledge. We call for more studies that examine the benefits of different mathematical learning experiences for varying student pathways.

The aim of our project was to investigate the value of remediation and we considered a few different approaches to remediation. The innovations explored were not radical departures from the norm. They included an accelerated sequence that taught a three-quarter remediation sequence in two quarters and an intermediate algebra course that focused on teaching concepts through contextualized problems.

The more innovative aspect of the research was asking the questions about how these different approaches related to success in college-level math. What wanted to understand what is the value of all this effort at remediation when students actually do succeed and enroll in college level math?

We are seeking to stimulate more dialogue and research about what remediation sequences are offering college students. Many are focused on improving success rates through redesigning course sequences, improving assessment and placement, and this work is valuable. We also want to know how does the learning that is taking place help students. A deeper understanding of the answers might help guide redesigns and placement work.

The methodology of our work could have been better and the measures we used could be improved. Given what is at stake for students, we hope that the field of mathematics education becomes more rigorous in investigating how the courses that are offered, especially those that are required, benefit students.

Measures of Learning and Progress

The majority of community college students never get a degree or certificate, and those with low socioeconomic status have the worst outcomes (Washington State Board for Community and Technical Colleges, 2013). The majority of those assigned to remediation do not complete the sequence (Bailey, Jeong & Cho 2010). Annual dollars spent for these programs have been estimated between 1 and 2.3 billion dollars (Strong American Schools 2008; Breneman, Abraham & Hoxby, 1998).

Our study is focused on the relationship between two different elements of student success: *learning* and *progress toward a degree*. Both are important components of success (Kuh, et al., 2006, Committee on Measures of Student Success, 2011). Measurements of progress toward a degree include grades, term-to-term retention, the number of credits earned, and whether the student earned a degree or certificate. These data are often readily available, and, at least in name, the measures are universally understandable. As a result, they are commonly used for research, state performance funding, and in calls-to-action by policy makers & funders, such as the Lumina Foundation's Goal 2025 and President Obama's American Graduation Initiative.

In contrast, student learning is the set of skills and understanding that a student gains, regardless of whether they receive a degree or pass a given class. Unfortunately, there is little research which directly assesses the relationship between measurements of progress and learning in college.

Different Types of Mathematical Learning

One challenge in studying this relationship between learning and progress lies in the measurement of learning itself. Accurately measuring student learning can be challenging for researchers, and is more difficult to do consistently at scale by large numbers of faculty who typically have little formal training in educational assessment. Furthermore, there are different types of mathematical knowledge, which are learned and assessed differently.

One well-studied divide is between conceptual understanding, which Hiebert & Lefevre (Hiebert, 1986) describe as "knowledge that is rich in relationships", and procedural skills which are algorithms or sequences of steps "tied to particular problem types" (Rittle-Johnson & Schneider, 2015). Research reviews examining this pair of ideas have concluded that instruction focused on conceptual understanding tends to also improve students' procedural skills, but that the converse is not necessarily true (Hiebert & Grouws, 2007; Rittle-Johnson & Schneider, 2015).

Another framework comes from a National Research Council (NRC) report which focuses on pre-kindergarten through eighth grade. This report identifies a more expansive definition of mathematical proficiency which includes five "interwoven and interdependent" strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Connecting these different representations of an idea facilitates retrieval and improves retention (National Research Council, 2001).

Unfortunately, measures of student learning too often focus on the most easily measured and taught component of mathematical learning: procedural fluency. Also, it is clear that the type of teaching and learning happening in classrooms is not aligned with the vision of teaching and learning expounded by professional teaching organizations (National Council of Teachers of Mathematics, 2000) and current standards documents (National Governors Association Center for Best Practices, 2012) that emphasize sense-making, reasoning, and conceptual understanding (Richland, et al., 2012; Cox, 2015; Jacobs, et al., 2006). This lack of alignment is reinforced by widely used placement tests which test predominantly procedural fluency.

This paper does not aim to settle these issues. We assume a framework similar to that of the NRC. Namely, mathematical proficiency in college encompasses a variety of strands that, when combined, allow individuals to effectively use mathematics in a variety of contexts. Procedural skills are one piece of a puzzle that also includes making connections between mathematical ideas and with genuine contexts, as well as productive dispositions. Because of the predominance of procedural skills in remedial math education, we single out two types of algebraic problems, which we will call "procedural" and "conceptual." Procedural problems involve manipulating one type of algebraic expression or equation to achieve a solution. Conceptual problems are those that connect multiple strands, for example by asking students to reason about a real-world situation by creating and manipulating an algebraic formula, or by asking students to write about the interpretation of a graph.

Local Context

The study was done at a large, urban community college in Washington state. At the time of the study, the college had just finished a series of reforms of their developmental math program. Because of this, there were three classes at the intermediate algebra (IA) level, the final level of remediation: Traditional IA, Accelerated Algebra 2, and Conceptual IA. Traditional IA was the final class of a three-quarter algebra sequence. This content of this sequence was primarily procedural algebra. Problem sets generally asked students to solve equations or simplify expressions.

The college also had an accelerated, two-quarter sequence which had the same outcomes as the traditional sequence. Topics were arranged in the accelerated sequence to remove repetition and decrease the number of required courses. Acceleration in mathematics is a common type of reform which has been shown to improve the percentage of students who complete college math (Hern, 2012; Hayward & Willett, 2014), but has shown little success in improving graduation rates (Hodara & Jaggars, 2014). The final course in the accelerated sequence was Accelerated Algebra 2, which had different but overlapping outcomes when compared with Traditional IA.

Conceptual IA was an alternative to Traditional IA in which some of the more symbolic, procedural algebra was replaced with data analysis, communication, and contextualization. Projects and active learning were integral to the course. This type of reform is consistent with current knowledge about what makes mathematics useful and meaningful for students (Bransford, Brown & Cocking, 2000; Freeman, et al., 2014)

Methods

Research Questions

This study examined the relationship between learning in intermediate algebra (IA) and progress toward a degree, by looking at how pre & post scores on an assessment of algebra skills related to three measures of progress: grades in IA, grades in the following college level class, and earning a degree or certificate. In order to get a more nuanced picture of the relationships we found, we also explored whether students retained conceptual learning longer than procedural learning.

Participants

Students were given an assessment at two time points: (1) At the beginning of each of the college's three IA courses (Traditional IA, Accelerated Algebra 2, Conceptual IA), and then (2) the following quarter in all of the college's four introductory college-level math courses (STEM

Precalculus, Business Precalculus, Statistics, and Math in Society). Because this was a community college, there was a high level of diversity in student ages, backgrounds, and personal characteristics. The significant attrition that is common in community colleges was apparent in this cohort as well. For example, the majority of people who took IA in Winter did not take college-level math the following quarter. (Tables 1 & 2 in Appendix B give information about the progression of students who took the pre-test & post-test, respectively).

Discussion of Research Questions

How well does a grade in intermediate algebra reflect the amount students learned in the class?

The noteworthy part of this analysis was not that the results suggested one answer to the research question. Rather, the results pointed at a variety of answers, depending on the course. In Traditional IA, there was a moderate positive relationship between student grades and their post-test scores. This suggests that the method of assigning grades in these classes was aligned with high stakes, procedural tests. In Accelerated Algebra 2, we did not see a significant relationship. In Conceptual IA, there was strong relationship between learning *gains* and grades, though the sample size was small in this case.

The results should not be interpreted directly as a statement about the effects of remedial reform, since the effects in this case were dependent on local conditions. Instead, they point out that students in different math classes with different instructors are subject to different performance expectations. The college in the study had no uniform assessments, standard syllabus, or grading standards, even between classes with the same learning outcomes. Conversations with instructors suggested large differences in instructional approaches consistent with those Cox (2015) found in remedial math classrooms. In such a situation, the relationship between grades and different types of algebraic knowledge is highly dependent on the instructor (Armstrong, 2000). This variation in expectations and assessment practices may, in itself, be a barrier for student success. Rather than grades being a true measurement of learning, grades may be a de facto measurement of how well a student can "read the teacher."

Do stronger intermediate algebra skills help students get better grades in college-level math?

After controlling for grades in previous courses, stronger procedural algebra skills gave little to no help to students' grades in college math. Perhaps most surprising is that procedural skills did not help precalculus students get better grades. This is reminiscent of previous research which has shown a disconnect between placement test scores and grades (Armstrong, 2000; Belfield & Crosta, 2012; Scott-Clayton, 2012). However, the items in our assessment were taken from final exams used at the college, which were themselves patterned on the content of traditional algebra textbooks. These are exactly the skills that many faculty consider necessary for passing precalculus.

One possible explanation for this result again comes from variability in instructor expectations. Pedagogical and assessment norms at the college were the same for college-level classes as for intermediate algebra. It is likely that the relationship between learning and grades at the college level varies like it does in intermediate algebra. If so, it would be no surprise that starting levels of mathematical knowledge did not help students get better grades - even though they might give students the foundation they needed to *learn* more.

Alternatively, it may be that a one-time snapshot of student knowledge as measured by a test is not a good measurement of the mathematical skills that a student is able to call on throughout the academic term. Even if people cannot immediately recall information, it often takes less time to relearn it than it did originally. This is a concept that psychologists refer to as "savings in relearning" (Ebbinghaus, 1885/1913; Nelson, 1985).

How does learning relate to completion?

We found that greater knowledge of intermediate algebra was not at all predictive of whether students earned a degree or certificate. To the extent that "college readiness" is defined as having the skills necessary to complete college, this finding suggests that perhaps intermediate algebra content does not help students become college ready. Given the high failure rates in these courses, this adds to the body of evidence which challenges the notion that intermediate algebra is a necessary component of remediation.

Is conceptual learning retained longer than procedural learning?

We found a significant difference in how well students remember purely procedural mathematics and math that is "rich in connections". Procedural algebra skills decreased significantly after only one academic term away from math. On problems of this type, for example ones in which students are given an equation and asked to solve it, students often use a memory-intensive, step-by-step process (Stigler, Givvin & Thompson, 2010). Those students must (a) recognize the type of problem, (b) remember which technique to use, and (c) remember how to correctly execute the procedure. This process is fragile. If students are unable to remember any of these steps, or if they apply techniques inappropriately, the problem is wrong.

In contrast, ability to solve problems on the conceptual scale stayed relatively constant among different groups of students who had taken time off. Each of these problems involved multiple types of knowledge, such as verbal reasoning, graphs, algebraic manipulation, familiar non-mathematical contexts. The context points of these problems provide students entry points to start making sense of the scenario, so that students could use different strands of mathematical knowledge to make sense. Unlike with the purely procedural items, these different strands of mathematical knowledge are mutually reinforcing. As the National Resource Council report (2001) says:

> *The central notion that strands of competence must be interwoven to be useful reflects the finding that having a deep understanding requires that learners connect pieces of knowledge, and that connection in turn is a key factor in whether they can use what they know productively in solving problems.*

Regardless of the mechanism that causes students who have been out of math for a term to have weaker procedural knowledge, these results suggest that teaching students procedural knowledge alone is a short-term prospect. Students who take a traditional developmental math class and then take time off, or those who take time off between high school math and college math, will likely have weaker skills when they continue studying mathematics. However, there may be little consequences of this skill decay if, as our data suggests, procedural skills do not predict success in college-level math.

More troubling is that the results of placement test that measure purely procedural knowledge, such as those commonly used by community colleges, are heavily influenced by whether a student has taken math recently. In addition to being poor predictors of grades in college-level math, these tests may just be poor measurements of mathematical knowledge in general because of this loss of procedural skills.

One limitation of our study was the assessment itself. It was a single snapshot of student ability on the day of the exam. While this type of assessment is commonly used in college intake placement, students in a classroom usually know what will be on an exam and are given the materials to prepare for it. A follow-up study might use deeper measurements of knowledge, such as savings in relearning, to study the same research questions. Such a study might also highlight the differences between performance on a placement test and knowledge in a classroom, and give some measure of the potential impact of placement test preparation.

Another potential limitation lies in the fact that local conditions have the potential to influence educational outcomes in complex ways. For example, a college that used common assessments may find different relationship between grades and student learning.

Implications for Practice

Overall, we found that measures of learning and measures of progress toward a degree were mostly not related. Where there were relationships, the results varied depending on the course in question. In addition, not all types of mathematical learning are created equal. The results suggest two main implications for practice.

Remedial math sequences that focus on procedural algebra (a) do not seem to prepare students for college-level mathematics, and (b) are teaching students skills that they will not be able to recall within a few months.

The vision of mathematics as set of procedures to be followed dominates remedial education, as well as many popular reforms. Procedural learning is just one of many strands of knowledge involved in successfully using mathematics. Rather than encouraging opportunity for all, there is a strong argument that this type of learning decreases equity in our schools (Gutierrez, 2008; Boaler 2002). Reforms which treat math as a set of steps may not give students the skills they need for their classes, career, and life. Even if a procedural-minded reform allows more students to progress through a remedial sequence, it may not support students learning to use mathematics in their courses, work or everyday life.

Shifting to a more conceptual or multi-stranded view of mathematics pedagogy is an enduring challenge, since different types of pedagogy support learning different types of skills. Productive struggle on challenging problems and explicit attention to concepts are necessary for students to learn this type of material (Heibert & Grouws, 2007). In addition, there are cultural challenges. As Stigler (2009) pointed out, "the primary influence on how teachers teach is the way they were taught." Instructors may be uncomfortable with the negative feedback that comes with expecting students to struggle. There is a lot of promising work in this area: locally designed projects as well as large programs such as the New Mathways Project and Statway/Quantway. But faculty need motivation and support for the pedagogical and political challenges they face in implementing these reforms.

Without local evidence saying otherwise, colleges should not assume that student grades are reliable measurements of student learning.

Our research suggests that there is not a clear relationship between standard measures of learning and measures of progress toward a degree. Student success initiatives, such as tutoring, that target student learning, but measure grades or retention, will not be successful unless there is a clear relationship between the two. Unfortunately, research examining the effects of these interventions often suffers from significant selection bias (Rutschow & Schneider, 2011).

One way to better link learning and progress is through common assessments and grading schemes in high-enrollment courses. Ensuring that the majority of a student's grade comes from a common set of assessments ensures that students are learning the same ideas, though there may be some variation by instructor, and that they are expected to have similar levels of proficiency. Strong common assessments don't need to be exams. Projects, group work, and writing prompts make fine summative assessments which probe student understanding in ways that exams cannot.

Conclusion

Despite millions of dollars poured into reform efforts, remediation remains a significant problem in colleges. This research reinforces the calls by a variety of researchers (Givvin, Stigler & Thompson, 2011; Cox, 2015) for further into what goes on in remedial classrooms and how it benefits students. While we push to improve success in remediation, it is also critical that we examine and justify the requirements. Conducting research on what students know when the proceed to the next course and how it helps them in those courses is a beginning to this important area of investigation.

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Technical Appendix A: Results and Instruments:

Results

How well does a grade in intermediate algebra reflect the amount students learned in the class?

Since the three intermediate algebra courses in the study had different, but overlapping, learning outcomes, we used different scales to measure the learning outcomes for each course. An OLS regression was run for each class with grades as the outcome variable, and with learning gains and post-test scores as predictors. The cohorts for this analysis included all students who took both the pre-test and post-test. Collinearity was a concern, as post-test scores and gains on each scale had correlations above $r = 0.7$. To tease out interactions, we also ran regressions on each predictor variable. The results are shown in Table 3 in Appendix B. The Conceptual IA sample was small, so it was inspected more closely. The achieved power for the Conceptual IA regressions were above .99. In addition, a scatterplot showed a clear linear trend between learning gains and grades. While we are hesitant to draw any conclusions about the Conceptual IA class from these statistics, we include it as support for the conclusion in the following paragraph.

The most notable thing about the results is the significant variation by course of the relationship between grades and learning/knowledge. In Traditional IA, there was a relationship ($R^2 = .301$) between knowledge (measured the next quarter) and grades. In Accelerated Algebra 2, there was no significant relationship between grades and either learning or knowledge. In Conceptual IA there was a strong relationship ($R^2 = .720$) between grades and learning.

Do stronger intermediate algebra skills help students get better grades in college-level math?

To examine this question, we regressed grades in introductory college math courses on students' conceptual and procedural scores, with students' self-reported average grade in their previous 2-3 math classes and demographic variables as covariates. Because precalculus classes involve more algebraic reasoning than the other college level classes in the study, we ran the analysis on three datasets: (1) students taking STEM Precalculus and Business Precalculus, (2) students taking Statistics and Liberal Arts Math, and (3) groups 1 and 2 combined, which consisted of all students taking an introductory college-level class.

In addition, a stepwise regression was done on groups (1) and (2), with three blocks: The conceptual & procedural scores, then with the addition of previous math grades, then adding in demographic variables. In the initial precalculus model, both conceptual and procedural scores were significant predictors of college grades. However, when a student's previous grades were taken into account, procedural skills became insignificant predictors of precalculus grades. A likelihood ratio test found that the inclusion of procedural skills in the final model did not add any predictive power. ($\chi^2(1) = 2.53,\ p = .11$) The coefficients for the Statistics & Liberal Arts Math model did not vary significantly from those in Table 4 (in Appendix B) as variables were added in.

In the final model, procedural algebra skills did not predict grades in college-level math. Conceptual skills were predictors of higher grades among students who took Statistics & Liberal Arts Math, but not for students who took precalculus. Self-reported grades in previous math classes were the most significant single predictor of grades in Precalculus. A GPA increase of 2.0 in previous math classes was associated with a full letter grade increase in the intro college level course. In addition, age and race played a role in precalculus grades. A 10 year increase in age was associated with a drop in GPA of .32, roughly the difference between a B+ and a B. White students taking precalculus performed approximately a half letter grade below their non-white peers. Pell grant eligibility and gender were originally included in the model, but had no significant effects.

How does learning relate to completion?

To examine this question, we used logistic regression to predict whether a student earned a degree or certificate. The dependent variables were conceptual & procedural assessment scores on the post-test, gender, Pell grant eligibility, previous math grades (self-reported), and whether the student was white. The cohort consisted of all students who took the post-test. There was no significant relationship between completion and either conceptual or procedural knowledge. A likelihood ratio test showed that the model was not a significant improvement over the null model ($\chi^2(6, N = 342) = 4.95, p = .55$).

Is conceptual learning retained longer than procedural learning?

Scores on the conceptual and procedural scales were compared with the amount of time students had taken off of math. The cohort for this analysis was all students taking intro college-level math. Figure 1 shows clearly that students who had taken at least one term off of math had significantly lower procedural learning skills. In contrast, conceptual skills were relatively constant regardless of how long it had been since students had taken math.

<INSERT FIGURE 1 ABOUT HERE>

A regression, run with age as a covariate, confirmed these results. Taking at least one term off of math was associated with a 22 percentage point decrease on the procedural skill scale

$(t(400) = 10.841,\ p < .001)$, but no significant decrease in conceptual skills. For comparison, the standard deviation of the procedural scores was 23 percentage points.

Instrument

The instrument for this study assessed algebraic skills and consisted of two parts. The "college portion" of the assessment consisted of 13 items adapted from final exams used by IA instructors at the college. This portion was designed to have a high degree of ecological validity by assessing a broad but representative sampling of the learning outcomes in the courses. Ten of the items asked students to solve an equation/inequality or simplify an expression. Three of the items involved modeling an exponential relationship using data, a function, and a graph.
The other portion of the exam consisted of three multi-part items developed by the Mathematics Assessment Resource Service (MARS). These were professionally designed, field-tested, and validated items which were given to tens of thousands of high school students as part of the work of the Silicon Valley Mathematics Initiative (SVMI). MARS items were chosen to correspond to topics that might be taught in an algebra class. These items required students to reason using mathematics, and then explain their reasoning. They variously involved graphs, algebra, data, communication, and real world scenarios. Due to their multi-part structure, MARS items were approximately three times longer than the college items. More information on the MARS assessment and SVMI, can be found in Foster & Noyce (2004).

Because each class had a different set of learning outcomes, all algebra students saw items on the assessment that were not taught in their class. In particular, the MARS items required communication skills not taught in Traditional IA and Advanced Algebra 2.

The assessment was refined during two pilot stages: a focus group of college level students, and a larger pilot involving 127 students. Small adjustments to design, difficulty, and language were made, primarily to the college items, after each stage to ensure that the assessment was understandable and that the range of scores was reasonable for calculations.

In order to minimize self-selection bias, the assessment was given at the beginning of all IA (Traditional IA, Conceptual IA, Accelerated Algebra 2) courses at the college during Winter quarter 2012 and at the beginning of all college-level courses (Statistics, Liberal Arts Math, Business Precalculus, STEM Precalculus) during the following quarter, Spring 2012. All instructors gave the assessment during the first or second day of class. Regardless of when they gave the assessment, instructors were asked not to teach any mathematics before handing out the assessment. Night and online classes were included in the study, and gave the exam during one hour face-to-face blocks. The time point for the post-test was chosen so that we could assess students' algebra skills when they might need them: as the next class started. However, this also meant that students who failed intermediate algebra or didn't take math the next quarter were not included in any analysis of learning gains.

To ensure ecological validity, blinded scoring of the college portion of the assessment was done by college faculty. Care was made to ensure that inter-rater reliability was at least 95%. MARS items were sent to SVMI to be scored. SVMI has a long history of reliably scoring their items, and have achieved a correlation of $r \geq .95$ between first and second scoring on audits.

Variables

Two sets of scales were created from the algebra assessment. The first set of scales divided the assessment into conceptual skills & procedural skills. The procedural skills scale consisted of the 10 items which either asked students to simplify an equation or to solve an equation/inequality. The conceptual skills scale consisted of all MARS items, as well as the three items from the college portion of the assessment which focused on modeling an exponential relationship.

The other set of scales was created to assess learning in each of the three IA courses: Traditional Intermediate Algebra, Accelerated Algebra 2, Conceptual Intermediate Algebra. The three courses had different but overlapping outcomes, most of which were represented by an item on the assessment. To create these scales, learning outcomes were matched up with items on the assessment. A weighted average of the matched assessment items was then generated for each course. Although the MARS items assessed some of the learning outcomes, especially for the Conceptual IA course, the MARS items were purposely excluded from these scales to ensure fidelity with actual course assessments. Scores on each scale ranged from 0 (all items completely incorrect) to 1 (all items completely correct).

Grades used in the study were converted to a numerical scale as if calculating a GPA (A = 4, A- = 3.67, etc.). Students were asked to fill out a brief survey when they took the assessment, which asked them how much time had elapsed since their last math class and average grade in their previous 2-3 math classes. Assessment and survey data were combined with data from the college's database after three years had elapsed. These data included variables such as age, gender, ethnic origin, credentials earned, math classes taken and math grades. Because of low sample sizes in most ethnic groups, ethnic origin was coded as "white" and "non-white" in the analysis. We consider a student to have "completed" if they earned a degree or certificate within three years of taking the post-test.

Data Analysis

A number of techniques were used to analyze the research questions, though most involved multiple regression modeling to assess the size and significance of relationships between variables. Specific techniques used for each research question are explained in the Results section. Data was analyzed using the open source statistical software programs R (R Core Team, 2015) and RStudio (RStudio Team, 2015), including the *lmtest* (Zeileis & Hothorn, 2002) and *QuantPsyc* (Fletcher, 2012) packages. The software program G*Power (Faul, Erdfelder, Lang, & Buchner, 2007) was also used.

Technical Appendix B: Tables

Table 1. Progress toward a degree for students who took the pre-test.

	n	%
Took pre-test	311	100
Earned grade in IA	274	88
Passed IA (C or better)	194	62
Enrolled in intro college-level math next term	125	40
Passed college-level math next term (D or better)	104	33
Ever passed college-level math at the college (D or better)	151	49
Earned degree or certificate	116	37

Table 2. Progress toward a degree for students who took the post-test.

	n	%
Took post-test	426	100
Earned grade in intro college-level math	385	90
Passed college-Level math in Spring 2012	317	74
Ever passed college-level math at the college (D or better)	326	77
Earned degree or certificate	210	49

Table 3. Predictions of intermediate algebra grades by learning gains and post-test scores on scales corresponding to the learning outcomes for each class.
Cohort: All students who took the pre- & post-test from each IA course.

	Traditional IA (n=62)		Accelerated Algebra 2 (n=45)		Conceptual IA (n=12)
Model 1					
TA Gains	.302	AA Gains	.854	CA Gains	4.409 ***
TA Post-test	1.641 **	AA Post-test	.052	CA Post-test	-2.006
R^2	.304	R^2	.043	R^2	.776
Model 2					
TA Gains	1.643 ***	AA Gains	.893	CA Gains	3.409 ***
R^2	.197	R^2	.043	R^2	.720
Model 3					
TA Post-test	1.844 ***	AA Post-test	.813	CA Post-test	2.647
R^2	.301	R^2	.030	R^2	.205

*p < .05, **p < .01, ***p < .001

Table 4. Predictors of grades in introductory college-level math, broken up into Precalculus & non-Precalculus courses. Mean and standard deviation are taken over students in all classes.

	Precalculus (n = 158)			Statistics & Liberal Arts Math (n = 172)			All Classes (n = 330)				
	B	SE	p	B	SE	p	B	SE	p	M	SD
Intercept	.932	.562	.099	.025	.578	.966	.461	.406	.258		
Procedural Skills	.706	.451	.119	.188	.503	.709	.273	.329	.408	.40	.24
Conceptual Skills	.783	.484	.108	1.26**	.464	.007	1.01**	.336	.003	.49	.22
Grades in previous 2-3 math classes	.513***	.137	.000	.490***	.136	.000	.524***	.097	.000	3.0	.75
Age	-.030*	.015	.043	.007	.014	.633	-.009	.010	.364	22	7.6
White	-.500*	.206	.016	.111	.212	.601	-.181	.149	.225	.66	.48
R^2		.216			.142			.150			

*p < .05, **p < .01, ***p < .001

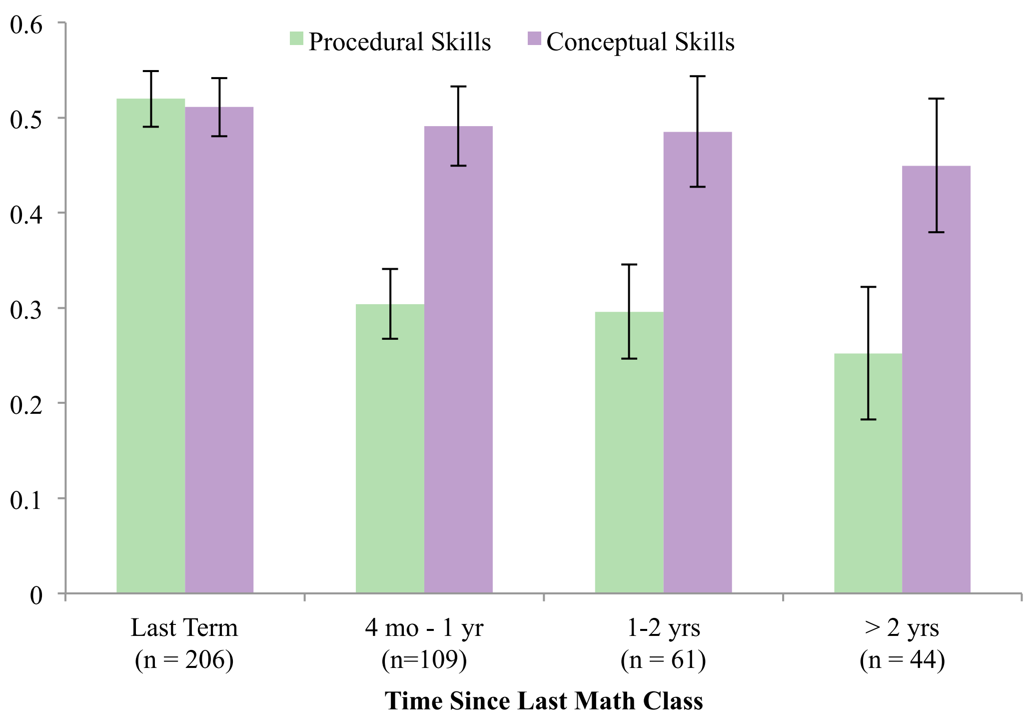

Figure 1. Scores on the algebra assessment by time since last math class. Error bars represent 95% CI. Sample: All students in introductory college-level math.