2016 NCTM RESEARCH CONFERENCE
April 11-13 • San Francisco

Paper Title: A Field Experience to Support Facilitating Mathematics Discussions: A Case of Two Preservice Elementary Teachers

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Session Title: Preservice Elementary Teachers' Development in Facilitating Mathematics Discussions

Session Type: Interactive Paper Session

Presentation Date: Tuesday, April 12, 2016

Presentation Location: San Francisco, California

A FIELD EXPERIENCE TO SUPPORT FACILITATING MATHEMATICS DISUCSSIONS: A CASE OF TWO PRESERVICE ELEMENTARY TEACHERS

Facilitating a mathematics discussion is a "high leverage practice," one essential for novices to know and be able to carry out on their first day of teaching and that has the biggest payoff for student learning (Ball, Sleep, Boerst, & Bass, 2009). Blanton, Berenson, and Norwood, (2001) claimed that "mathematics teachers' ability to cultivate serious mathematical thinking in students rests on the nature of classroom discourse" (p. 241). The Mathematical Practices of the Common Core State Standards (National Governors' Association & Council of Chief State School Officers [NGA & CCSSO], 2010) include "construct viable arguments and critique the reasoning of others" (p. 6) and the National Council of Teachers of Mathematics (NCTM, 2000) has described how this should look: classrooms should provide opportunities for students to discuss, represent, analyze, evaluate, and justify their and others' thinking about mathematical ideas. Research on teachers facilitating discussion provides only snapshots of practice, often of experts (e.g., Lampert 1990). Few examples (e.g., Connor, 2007; Ghousseini, 2008) describe changes in novice teachers' practice over time in facilitating discussions, or what features of teacher education promote teachers' development. This paper contributes to this body of research by analyzing preservice elementary teachers' (PSTs) growth in facilitating discussions over time and one feature of a field experience that supported that growth. I present some of the results of a study that focused on developing PSTs' ability to facilitate mathematics discussions that align with the vision described in NCTM (2000) standards documents and the Mathematical Practices of the Common Core State Standards (NGA & CCSSO, 2010). In particular I address the research question how did PSTs' abilities to facilitate discussion change over a 6-week field experience, and what elements of the field experience design might account for these changes.

Background

One prevalent mode of communication in mathematics classrooms is the initiation-response-evaluation (IRE) model, where the teacher *initiates* interaction with a question (often focused on procedures) that is followed by short student *responses* that are quickly *evaluated* by the teacher for correctness (Mehan, 1979). While a rapid-fire question and answer session like IRE may be useful for keeping students on-task during whole class direct instruction, it fails to engage students in rich mathematical discourse where they gain practice construing and refuting arguments. Contrast this with dialogic discourse (Knuth and Peressini, 2001), where the speaker and listener generate meaning through shared dialogue. In this situation, a teacher uses questioning to elicit student thinking and *press for meaning* in order to hold students accountable for explaining and justifying their ideas, not merely stating a solution or describing a procedure (Kazemi & Stipek, 2001). In lieu of evaluating student responses to questions, teachers continue to question to explore student thinking and attune student s set up students to judge the validity of their own solutions (Boaler & Brodie, 2004; Crespo, 2000). By orchestrating discussion around incorrect solutions, rather than merely correcting them, the teacher gives students the opportunity to understand for themselves why solutions are invalid (Staples & Colonis, 2007). Brendefur and Frykholm (2005) call teachers supporting students in explaining and justifying their thinking and helping students connect their thinking to important mathematics concepts *reflective communication* and *instructive communication.* And this is the mode of mathematical communication that I sought to help PSTs develop.

However, several factors impede teachers' abilities to facilitate mathematics discussions. First, the fundamental basis of instructional practice, student and teacher interaction, is wholly different from teacher-led direct instruction, where teacher is the sole arbiter of mathematical truth, doing school mathematics means memorizing rules and performing procedures modeled by the teacher, and students can be confident that correctly applying rules and procedures indicates mastery of a concept (Lampert, 1990). The work of the teacher in orchestrating discussion, while not readily apparent as it would be in a teacher-centered classroom, remains significant and daunting: choosing appropriately challenging problems that are both accessible to students and address important mathematics (Smith & Stein, 2011); determining which student ideas to follow to move the mathematics forward (Wood et al., 1993); posing questions that press for complete and clear explanations (Kazemi & Stipek, 2001); connecting among different student ideas (Lampert, 1990; Smith & Stein, 2011); establishing and modeling norms for discussions and helping students attend and respond to one another's thinking (Lampert, 1990; Staples, 2007; Yackel & Cobb, 1996).

Second, one of the underlying assumptions of facilitating mathematical discussions is that doing so in ways that engage students in authentic mathematical discourse, means that the teacher does not explicitly model procedures and solutions. Productive struggle is needed, yet learning mathematics in this way may contradict teacher or student beliefs about how mathematics is learned. The teacher performs a balancing act between maintaining the challenge of a task while also supporting and attending to student thinking. The uncertainty inherent in discussions can be difficult to manage for those with weak content knowledge (Ball, 1993) and novices who have limited experienced with children's mathematical thinking (Yackel, 2002).

Without support to overcome challenges teachers may diminish reliance on discussions and revert to teacher-centered instruction (Baxter & Williams, 1996; Nathan & Knuth, 2003). To make this complex practice accessible to novices, researchers suggest decomposing teaching into smaller grain-sizes that can be "articulated, studied, and rehearsed" (Sleep & Boerst, 2012, p.

1039). This allows teacher educators to scaffold novice's learning through *approximations of practice*, "the opportunities for beginning teachers to engage in the practice in ways that approach its enactment in the profession" (Boerst et al., 2011, pp. 2845–46). One strategy for scaffolding novice's learning is to incorporate a practice-based approach to teacher education (cf., Ball & Forzani, 2009). More than merely integrating theory and practice via field work, practice-based education should "emphasize repeated opportunities for novices to practice carrying out the interactive work of teaching and not just to talk about that work" (Ball & Bass, 2000, p. 503). This study, situated within a practice-based mathematics teaching methods course, provides opportunities for PSTs to repeatedly carry out a task of teaching, receive and reflect on feedback, revise their efforts, and try again, thereby allowing them to hone in on improvements at a particular skill.

Framework

While practice-based teacher education is gaining traction, there are few examples of research that describe the changes in prospective teachers' practice over time, particularly in regard to learning to facilitate discussions, or that provide insight about what features of practice-based programs support developing teachers' practice. Connor (2007) studied how secondary student teachers made use of argumentation in the context of student teaching. Following preservice elementary teachers over several semesters of their program, Ghousseini (2008) analyzed tools that supported their learning to lead mathematics discussions. However, both failed to offer a systematics way of tracking teacher change. I chose two frameworks to analyze data that allowed for a careful comparison of PSTs changes over time: Stein, Smith, Henningsen, and Silver's (2000) mathematical task analysis guide and Hufferd-Ackles, Fuson, and Sherin's (2004) math-talk framework.

Cognitive demand refers to the "cognitive processes in which students actually engage as they go about working on the task" (Stein, Grover, & Henningsen, 1996, p. 461). Stein et al. found the intended demand of a task may change in a teacher's task set-up and implementation (e.g., requiring students explain their ideas versus performing routine procedures). They described categories of cognitive demand: *memorization* (recall of memorized fact); *procedures* without connections (execution of known procedures without attention to underlying concepts); procedures with *connections* (connect procedures to underlying mathematical concepts); and *doing mathematics* (synthesis of knowledge to develop new procedures, generalizations, or justifications). In this study, these categories were assigned a 0–3 value and used to assess PSTs' implementation of *doing mathematics* tasks.

Hufferd-Ackles et al.'s framework originated from a year-long study of one teacher's work to build a classroom math-talk community. The goal was to develop a classroom in which both teacher and students worked together to develop shared understanding. The framework addresses four components of discussions, *questioning*, *explaining* mathematical thinking, source of *ideas*, and *responsibility* for learning, and describes four levels (0–3) for each component ranging from a teacher-directed lecture to a classroom where student thinking drives mathematical work. The four components make it useful for comparing teacher moves to cognitive demand and multi-levels make it appropriate for studying teacher development. In practice, using the math-talk framework required some modification. Originally developed in a whole class setting with an experienced teacher, it did not translate smoothly to the context of two teachers working with only two students. Nor did it capture the small changes typical of beginning teachers. Thus, I modified the descriptions of the levels to fit the context of two teachers working with two pupils (e.g., "Teacher is physically at the board, usually chalk in hand, telling and showing students how to do math," was modified to, "Teacher shows how to solve or tells correct answers or appropriate strategies."). Using the same process the original creators described, I added mid-levels (0.5, 1.5, 2.5) to describe PST actions that did not fit cleanly into only one level (for more detail on this process, see Hallman-Thrasher, 2011).

Methods

Eight study participants, chosen for their strong content knowledge, communication skills, and a willingness to attempt student-centered teaching were selected from among 30 PSTs enrolled in their first elementary mathematics teaching methods course. I used written class assignments, contributions to class discussion, and an individual interview in which they completed and explained their work on a problem-solving task similar to what they would later enact with elementary pupils to inform my participant selection. Fifth-grade students with whom the PSTs worked were selected by their teachers for having average mathematics performance.

The methods course was the first of a two-course sequence. The purpose of the course was to develop an awareness of children's mathematical thinking, how children's thinking differs from adult thinking, and how an understanding of children's thinking could inform teaching practices. The course was structured around a "purposeful, integrated field experience" (Feiman-Nemser, 2001) where PSTs facilitated discussion on *doing mathematics* tasks with pairs of elementary children. This study was conducted during 6 weeks of course's embedded field experience. Eight meetings (one per week) of the course were held at a local elementary school where PSTs worked with children one-on-one and in small groups on mathematics tasks. Myself (the course instructor) and teaching assistant were on site to observe, assist, and model how PSTs should engage children in tasks.

PSTs worked in collaborative 3-person teams, 2 acting as teacher each week and the third as a video recorder. I collected data in two 3-week blocks. Each block focused on two non-routine “doing mathematics” problem-solving tasks (see Appendix A). For the first 3-week block of the study, each PST group implemented two assigned tasks with a different pair children each week. In the second 3-week block, they implemented two new tasks, again with different children each week. By repeating the same tasks with new pupils each week, the PSTs could refine their responses. I collected data in weekly cycles of planning, enactment, and reflection (Kazemi et al., 2010).

Planning data included task dialogues (Crepso, Oslund, & Parks, 2011), and task plans. In task dialogues, I posited 3-4 hypothetical student solutions to each of their tasks and they composed how a student teacher dialogue might follow each solution (for examples, see Spangler & Hallman-Thrasher, 2014). For task plans, PSTs listed specific hints, questions, and teacher moves they would use to help a child who 1) did not know how to start, 2) had an incorrect approach, 3) had a nearly correct solution, and 4) had a correct solution and needed to be further challenged. Before the first week of each 3-week block, PSTs completed a task dialogue for each task and, using my feedback on task dialogues, they created a task plan. Each subsequent week of the block they revised their task plans. Enactment data included video of each session of task implementations. Reflection data included each team’s collective written analysis of the children’s and one another’s work.

To analyze video and planning data were parsed into segments, defined as one student solution, idea, question, or strategy and the PSTs’ response to it. Each segment represented a PST (or several PSTs) responding to a child’s solution, strategy, or idea. Often PSTs responded to no solution situations; a child stalled our and either asked for help or a PST decided to intervene. Each time a child introduced a new solution, strategy, idea, or had no solution and a PST intervened, this defined the start of a new segment. I assigned each segment a level for each component of math-talk (Hufferd-Ackles, et al., 2004) and a cognitive demand category (Stein, et al., 2000). In reflections, I noted thoughtful comments and recurring struggles and successes for each task and each PST. Cognitive demand of the enactment data indicated how each PST's ability to implement the task changed and examining changes in levels over the 6 weeks, I determined four trajectories of teacher development. I then looked for patterns within each trajectory to determine what elements of the experience supported it.

Results

I identified four different types of change in their ability to facilitate discussions that maintained high cognitive demand over the 6-week field experience. Two, Erica and Alice, led consistently teacher-directed discussions, but the remaining 6 PSTs improved in different ways. Rene and Kate, who led effective discussions at the beginning of the study, demonstrated the ability to continue elevating the cognitive demand of their discussions over the 6 weeks. Nadia and Megan, who were not initially successful, showed inconsistent improvement over the 6 weeks. Casey and Dana, also not initially successful at facilitating discussions, showed gradual small improvements over the 6 weeks, Of the 8 participants, 6 made improvements in different ways. I focus on the results of Dana and Casey, who represent the trajectory of initially teacher-led discussions that improved in small targeted ways to focus more on student thinking and achieve a *connections* cognitive demand (Figure 1).

Initially, Casey and Dana struggled to achieve high cognitive demand in their discussions. In Week 1, their task implementations were mostly teacher-led and low cognitive

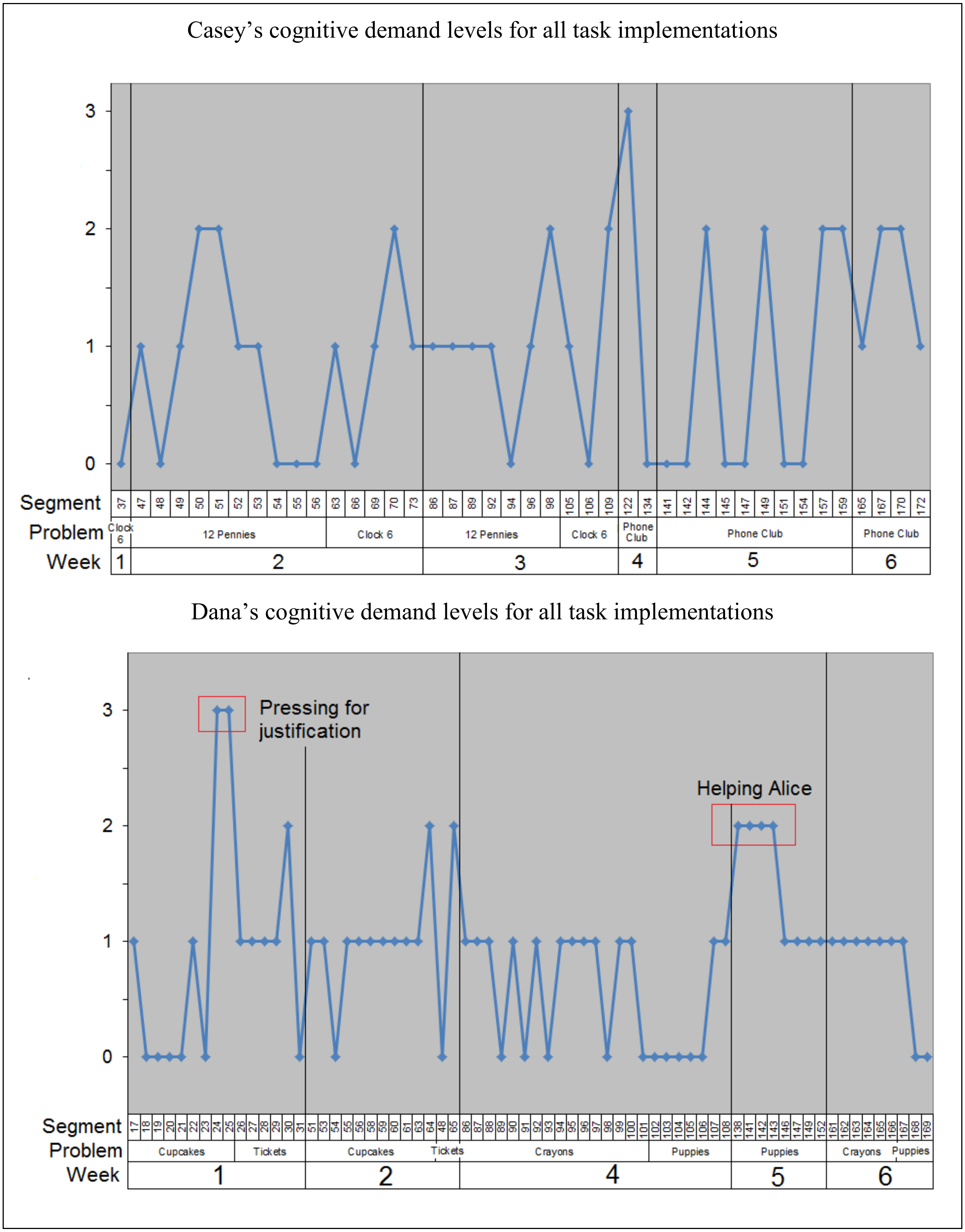

Figure 1. Levels of cognitive demand for PSTs with small, targeted improvements in facilitating discussions.

demand, with only occasional instances of high cognitive demand. However, in subsequent implementations of most of their tasks, they achieved higher levels of cognitive demand, hence their classification as improving. Dana showed consistent improvement across her implementations of the Cupcakes and Puppies Tasks, moving from *memorization* to *procedures with connections*. Casey showed improvements across the 12 Pennies and Phone Club Tasks, moving from sporadic levels of cognitive demand to a more level of *procedures without connections* in the 12 Pennies Task and *procedures with connections* in the Phone Club Task. Each of these two participants had inconsistent performance in one of their problems: the Clock 6 Task for Casey and the Tickets Task for Dana.

Early in the study, Casey frequently asked "How did you get that?" (Group H Video), but did not use the child's response in her follow-up, if she followed up at all. In Week 2, when her child found a correct solution to the 12 Pennies Task she stepped him through verifying that the piles summed to 12. When he suggested a strategy for finding other solutions, she had him recall basic facts that summed to 11 and drew the conclusion for him about the pattern among his solutions. When her children found all solutions, she suggested an extension question that required only a yes-or-no answer: "Do you think we'd have more solutions if we had more pennies?" and concluded for them that "if we change the rules of the problem, it would come up with a different answer" (Group H Video, p.23).

However, in Weeks 5 and 6 she maintained cognitive demand at *procedures with connections* by tailoring her questions to attend to children's particular strategies. She asked, "How did you know to draw your strings that way?" (Group H Video, p. 70) to help pupils notice and articulate a pattern. She then elicited a detailed explanation of a child's reasoning, methodically reconstructed the child's diagram, confirmed her accurate representation of his work, and then posed follow-up questions to help him identify an error and draw his *own* conclusion: “Why did you just say you don’t need to do that? So do you notice anything about that?” (Group H Video). After seeing her team successfully encourage pupils to attend to one another’s work, Casey implemented the same strategy in subsequent weeks, asking a pupil, “Do you want to explain to her [his partner] what you did after that?” (Group H Video). Then Casey directed them to work together: “Do you have all of these answers? Can maybe you guys maybe compare your answers and see which ones he’s missing?” (Group H Video). Encouraging her pupils to work together is seen in the spikes in the cognitive demand from Week 3 on.

In concert with her initial struggles facilitating discussions, Casey also did not critically reflect on her work with pupils each week. Her reflections in the first block did not address children’s mathematical thinking; they generally focused on keeping the children on-task, coordinating collaboration, and pacing. Reacting to her fellow teammates’ analysis of their group’s work helped develop Casey’s awareness of her own difficulties and Casey’s improved reflections paralleled the improvements in her implementations in the second 3-week block. In Week 4, she showed a burgeoning awareness of her focus on answers and not explanations when she responded to Nadia’s commentary:

> Nadia’s concern about not fully understanding a student’s reasoning is valid. I think we assume they understand how they solved the problem and that we do too. But, sometimes we don’t know why they solved a problem a certain way or how they even got to the answer! I am guilty of hearing an explanation and just nodding my head or saying “good job!” when I don’t even know what is going on. I didn’t notice that I did that until Jordan pointed it out. (Casey, reflection data, p. 25)

By Week 5, Casey was finally picking up on the same issues as her group members and reflections were improving. After discussing her own struggles with how to help their pupils, she related her struggles to those Kate experienced:

> I totally understand the need to jump in! I felt like I wasn't helping the girls solve the problem at all. I just stared at their white boards while they attempted to solve the problem wrong in a variety of time consuming ways. I didn't know if it was beneficial or detrimental to jump in and tell them they're doing it. (Casey, reflection data, p. 30)

Like Casey, Dana too struggled early in the study. At times, she achieved high cognitive demand when she focused on eliciting justifications and repeatedly pressing for complete justifications. She consistently posed why questions after several pupil responses "So you don't think any others [solutions] would work? ….Like, could I have 8 chocolate and 4 vanilla?.... Why wouldn't that work? ….Right, and that's because why?" (Group I Video). Yet these segments were followed by highly teacher-led discussion In her first implementation of the Cupcakes Task, she interjected early on to correct her child's misconception. Rather than giving her child (C7 in the transcript below) time to attempt a guess-and-check strategy, she tried to guide her to see that her initial guess would not work. She reduced the task to a series of short answer questions that guided the child through Dana's way of thinking about the task.

> Dana: So how many boxes of vanilla [cupcakes] do you have here?
>
> C7: I have 5.
>
> Dana: And you have how many chocolate boxes?
>
> C7: Oh, …5 [Adds another box of 4, for a total of 50 cupcakes].
>
> Dana: So do you know how many cupcakes that give you the total?

C7: Ok. This is 20 [5 boxes of 4]. That is … ok … add together [with the 30 from 5 boxes of 6]… 50 … um … that same

Dana: What could you do to …

C7: … I still have 10 boxes. But I have …

Dana: You still have how many, how many boxes have you used?

C7: I have 10 boxes here, and I have 50 cupcakes. Right there. I'm close to 58 cupcakes.

Dana: You are?

C7: And so, I think I'm going to go … I think I might go up, maybe try to going up. I don't know. I just want to try it.

Dana: So now you have too many [C7 has 60 cupcakes].

C7: Yeah.

Dana: So, you know that you're close. How many boxes do you have total between these two right now … before you erase that?

C7: I have 60 [cupcakes]. 12 [boxes]. (Group I Video transcript, pp. 3–4)

This example was typical of her initial implementation of problems: establish cognitive demand at the *memorization* level with Dana drawing conclusions for the child and only pushing for high cognitive demand at the end of the problem when the child had obtained a correct answer.

She continued to struggle to establish a high demand in later sessions, until she was assisting a struggling teammate, Alice. In the second 3-week block, Dana as Alice became more directive, Dana intervened to elicit descriptions of the child's representation and to ask him to map his work back to the original task. She elevated the cognitive demand to *procedures* and ultimately *connections*. As Alice reverted to advancing her strategy rather than following the child's thinking, Dana continued to skillfully intervene at key moments: "What does the problem say?", "How did you get from 33 to 16?", "Which places in the problem did you take half?"

(Group I Video). When she saw Alice try to force a guess and check strategy, she advised, “He’s working backwards. I think you should go back” (Group I Video).

Dana showed important improvement here as she asked a few careful questions, stepped back to let Alice take the lead, and stepped in again as she saw Alice continue to struggle. Dana not only had to carefully consider when to intervene on the child’s thinking, she also had to understand what Alice was trying to accomplish and when to intervene in Alice’s work. Dana described this experience:

> We ended up spending about 20 minutes trying to get to the root of John’s thinking and encourage him to see his mistake….We kept hitting a roadblock because John’s mind was fixed on his idea of what the problem was asking and it was hard for us to dissect his thoughts. I found myself wanting to just explain and clarify; it was so hard attempting to get him to realize his own mistake.” (Dana, reflection data, p. 39)

Although their child continued to stumble over the same misconception of what the whole was in the task and never reached the correct solution for the problem and pupil was never able to correct his misunderstanding, neither Dana nor Alice corrected his thinking; instead, they were comfortable ending the session with the pupil not having resolved the issue. Dana described her improvements in asking questions.

> I have found I tend to ask yes or no questions when my students are not able to elaborate, as an easy way out. I also noticed I could be pretty repetitive with my questions when I am not getting the answers I want. However, when I was able to ask more open-ended questions and students responded, I found asking questions

without an answer already in my mind left room for student ideas rather than my own. (Dana, reflection data, p. 57)

Conclusions

Over the duration of the study, both Dana and Casey increased their attention to student thinking by using purposeful questioning. Both developed a set of generic questions for eliciting student thinking in any context and were able to adapt those questions to attend to particular aspects of pupil solutions. For both, their improvement may be due at least in part to collaborative teaching, repeated enactments, and collective reflection. The cognitive demand of Casey's implementations was pulled up by her team, whereas Dana elevated the cognitive demand in trying to bolster a teammate. Additionally Casey's reflecting with her teammates and Dana's observations a teammate's difficulties made each PST aware of her missteps. This approximation of practice incorporated a type of assisted performance (Feiman-Nemser, 2001; Mewborn & Stinson, 2007), collaborative teaching and collective reflection, which created opportunities for teacher learning that would otherwise have been missed. Though Casey and Dana's improvement was targeted to a specific skill (focusing on teacher questioning) and they yet had room for further improvement in establishing and maintaining high cognitive demand, these results point to the potential importance of collaboration in fostering novice teachers early practice in regards to facilitating discussions.

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Appendix A

Problem Solving Tasks

Generalizing and Explaining Patterns

Task 1: Telephone Club

Your class made telephones out of strings and juice cans. Each group of students has to work together to make a phone club that connects every person to every other person. If a group had four people, how many strings would be needed to connect every member of the group to every other member of the group? What if you used 28 strings, how many people would be in a group?

Task 2: 6 Numbers

Can you put the numbers 1-6 in the triangle shown so that each side adds up to the same amount?

Making Organized Lists

Task 1: 12 Pennies

Place 12 pennies in 3 piles with no two piles having the same number of pennies.

Task 2: *Clock 6s*

How many times in a 12-hour period does the sum of the digits on a digital clock equal 6?

Working Backwards

Task 1: Crayons

Mary has some crayons. Doug had 3 times as many as Mary. But Doug gave 4 to the teacher and now John has 2 more crayons that Doug. John has 7 crayons, how many does Mary have?

Task 2: Puppies

The pet store advertised that they had lots of new puppies on Monday. The owner took 1 puppy for his son. Then on Tuesday he sold half of the rest of the puppies to a farmer with lots of land. On Wednesday a mom took a half of the puppies that were left for her children. When you got to the pet store on Thursday there were only 4 puppies left to choose from. How many puppies were there on Monday?

Reasoning Algebraically

Task 1: Cupcakes

A baker makes chocolate and vanilla cupcakes. He packages the vanilla ones in boxes of 4 and the chocolate ones in boxes of 6. He made 38 cupcakes and used 8 boxes. How many boxes of vanilla and how many boxes of chocolate did he make? (alternate version: 58 cupcakes and 12 boxes)

Task 2: Tickets

Amy and Judy sold 19 play tickets altogether. Amy sold 5 more tickets than Judy. How many tickets did each girl sell?