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Author(s): Linda Venenciano

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Developing Empirical Thinking, Davydov's Theory in Curriculum Design

Linda Venenciano

Lhirashi@hawaii.edu

University of Hawaiʻi at Mānoa

The widespread adoption of the Common Core State Standards or CCSS (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) and development and release of tools that emphasize mathematics instruction as a progression has fueled discussions about curriculum and its potential to address the standards in a meaningful way. Thoughtful design of mathematics curriculum involves more than reorganizing a collection of topics. Curriculum is the primary instructional tool used by teachers and can therefore potentially convey theoretical assumptions about instruction and learning.

In a cross-cultural analysis of curriculum materials, Remillard, van Steenbrugge, and Bergqvist (2014) found that curriculum had a cultural context that appeared to relate to different educational traditions. For example, the two U.S. teacher's guides examined in the study, Everyday Mathematics and Math in Focus, were found to provide teachers with more guidance in the details for instruction than the two Swedish traditional curriculum teacher's guides they contrasted it with in their study. The researchers found that degree of guidance appeared to be linked with cultural tradition, where the dominant instructional mode of the culture was reflected in the mode by which textbook authors communicated with teachers. That is, if teachers played a more directive role in leading instruction, curriculum guidance to the teacher tended to be directive as well. Thus, if culture indeed matters in the written style of curriculum guides, how does a curriculum developed in Russia influence teaching and learning in the U.S.?

In this paper we first describe a U.S. elementary grades curriculum project that embodies learning theory developed by V.V. Davydov and his colleagues in mathematics curriculum. Then, we begin to consider the impact of culturally foreign curriculum by examining the long-term effects on the development of logical reasoning and algebra preparedness in U.S. students.

Theoretical underpinning about student learning provide us with useful guidance for designing materials for teachers, to the extent that the materials themselves are educative for the teacher's own professional growth. The Measure Up (MU) elementary curriculum project explored the implementation of a theory put forth by V.V. Davydov (1966, 2008). Davydov considered the elaborate interlacing of psychological factors and pedagogical issues with concepts fundamental to the structure of mathematics in the design of school curricula (Venenciano & Dougherty, 2014). According to Davydov, the presentation of mathematical relationships should be grounded in concepts that are both fundamental and basic. Recalling young children's informal play with measureable quantities (e.g., with sand or water) as a starting point for what children explore helps us gain insight to what they learn to be fundamental and basic concepts. As children make observations through play and other everyday activities, they begin to spontaneously interpret and formulate tentative understandings about how quantities compare, what is the same (equal) or which quantity is larger (greater than another). In order to advance children's theoretical knowledge and move them from concrete and perceptual states of mind to conceptual, abstract, and logical thinking states, Davydov theorized that instruction needed to target the development of empirical generalizations.

Davydov (2008) argued that a goal of school mathematics was to begin with developing young children's consciousness and thinking about generalizations. Generalization can be thought of as the process of searching for a common or unifying feature among a class of objects. Where some features among the objects may vary on a somewhat superficial level, the ability to generalize enables children to recognize a key invariant among the objects. The focus on an invariant feature (e.g., the mass of objects) helps students discriminate the features or attributes of other objects and decide how the objects compare. To compare quantities by an attribute helps students think systematically about the processes of comparing and generalizing. However, rather than instruct children to interpret the generalization as a result of comparing individual quantities, Davydov proposed the reverse. That is, instruct children to compare nonnumeric quantities as a means for learning about the generalization, and then use number examples as specific instances of the generalization.

Davydov's work has led to the development of experimental curriculum (Davydov et al., 1999) in Russia and in the early 2000s, the development of a U.S. version at the Curriculum Research & Development Group at the University of Hawaiʻi. Translations of the Russian work resulted in what can be described as directive lessons. Using a framework from Remillard, Van

Steenbrugge, and Bergqvist, (2014) to describe curriculum instructional guides for teachers, directive lessons are characterize by the nature of communication with the teacher where instructions for the teachers' actions are described, but they may not necessarily include information for the teacher to understand the purpose for the actions. Davydov's curriculum was written in a directive manner, instructing teachers to demonstrate measuring with quantities to achieve a specific outcome and to use specific representations to capture this outcome, however it also included educational supports to inform the teacher about the mathematical goals. Taken together, these helped to ensure that the presentation of the measurement activities led to outcomes that guided students to thinking about quantitative generalizations.

An additional source of information that informed the design of MU was research about the theoretical underpinnings, the mathematics, and the rationale for the curriculum design examined the Russian experimental curriculum in conjunction with. This research was educative for the MU teacher-researchers and provided rationale for the delivery of the mathematics.

The MU learning activities are structured to develop mathematical knowledge through everyday empirical experiences with common measureable quantities (e.g., area, length, volume, and mass). The design of the MU curriculum promotes a culture for teaching and learning mathematics by instructing the teacher to use measurement as a context for class activities, observations, and representations of their observations. For example, in Grade 1, generalized ideas (length J > length H by a difference, length A) develop into specific cases (7 > 3 by a difference of 4). MU presents generalizations as a means for focusing on the quantitative relationships. This instructional presentation is opposite from the more typical approach to school mathematics that begins with specific cases (e.g., number facts) then builds toward generalized statements. From Davydov's perspective, the genetic development of number is achieved through work with continuous quantities. Schmittau and Morris (2004) contrasted the Davydov curriculum with work pursued in the generalized arithmetic strand of early algebra and stated, "Thus, while children in the U.S. have pre-algebraic experiences that are numerical, Russian children using Davydov's curriculum *have pre-numerical experiences that are algebraic*" (p. 61).

Research Design

As a means for assessing Davydov's theory into an educational curriculum that develops students' empirical or evidence-based thinking, we hypothesize that students' from the MU mathematics program develop logical reasoning and algebra preparedness capabilities. We propose a conceptual model to investigate the contributing effects of logical reasoning abilities on algebra preparedness. For our study, we use the term logical reasoning to encompass a set of characteristics, relevant to the study of school mathematics, believed to be the result of having developed children's consciousness and empirical thinking. With the understanding that a construct, such as logical reasoning, cannot be directly observed or measured, we rely on indications of the construct observed in how students solve mathematical tasks. These characteristics include reasoning from cause to effect, generalizing, solving and analyzing problems as part of a case of problems, considering and satisfying necessary and sufficient conditions, thinking deductively, and applying if-then reasoning. We refer to algebra preparedness as the ability to interpret letter representations in flexible ways, including the ability to use the letter as assigned values or as objects, evaluating the letter, interpreting the letter as representing a set of unspecified values.

We propose a system of direct and indirect relationships and use structural equation modeling or SEM to examine the construct validity of our proposed conceptual model. SEM allows us to examine the extent to which the observed variables serve as valid and reliable measures of each factor. Confirmatory factor analysis (CFA) is used to examine how well each observed item is associated with its proposed construct.

This is a quasi-experimental design, specifically, a post-test design with non-equivalent groups. Our sample consists of 129 Grade 5 and 6 students from the school. Forty students attended the school during their elementary years and had MU as their school mathematics for 40–45 minutes every day. Since MU began with Grade 1 students, if a student entered the school during Grades 2–5, supplemental tutoring on foundational MU concepts was provided. The MU group received from one through five years of the curriculum. The comparison group consisted of 89 students who did not have any MU experience. These students received various types of math instruction, including traditional (e.g., direct instruction) and reform (e.g., cooperative learning strategies and problem solving) in their elementary school settings.

Data Collection and Analysis

Two assessments were used in this study. The first assessment was intended to serve as measure of logical reasoning and the second assessment was intended to serve as a measure of algebra preparedness. The logical reasoning assessment contained items from the 1999 administration of the Trends in Mathematics and Science Study (TIMSS), previously identified by researchers (Tatsuoka, Corter, & Tatsuoka, 2004) as indicators of logical reasoning. To measure algebra preparedness we selected the Chelsea Diagnostic Math Tests–Algebra subtest (CDMTA) as our instrument because the tasks were designed to clearly target the interpretation of letters.

To analyze the data, we propose a system of direct and indirect relationships and use structural equation modeling (SEM) to examine the construct validity of our proposed conceptual model. Three control variables were used in this study. These were student's age, prior achievement as measured at Grade 5 on the Stanford Achievement Test, v. 9, and MU treatment. The data suggest that for each group, prior achievement and age were normally distributed. To address concerns that the Grade 5 measure might confound our ability to separate the MU effect, we conducted preliminary analysis and found that MU students had slightly lower prior achievement, $M = 5.0$ ($SD = 1.5$), compared with non-MU students, $M = 6.3$ ($SD = 1.9$).

Students' age and previous mathematics test scores were substituted for the absent pretest measure. Preliminary analyses confirmed that both variables were significant predictors ($p < .05$) of algebra preparedness. Furthermore, controlling for these variables, MU defined in terms of years of exposure (i.e., 0, 1, 2, 3, 4, 5) was statistically related to preparedness; that is, each added year of exposure was worth a 0.20 standard deviation (*SD*) increase in preparedness. We also specified MU as a dichotomous indicator (1= participation versus 0 = no participation) and found it was significantly related to preparedness, with participation accounting for a 0.17 *SD* increase in preparedness. We settled on coding MU as an ordinal variable with 0 = no participation, 1–3 years coded 1 = low participation, and 4–5 years coded 2 = high participation. The preliminary effect size on preparedness was 0.19 *SD* for each added level of exposure.

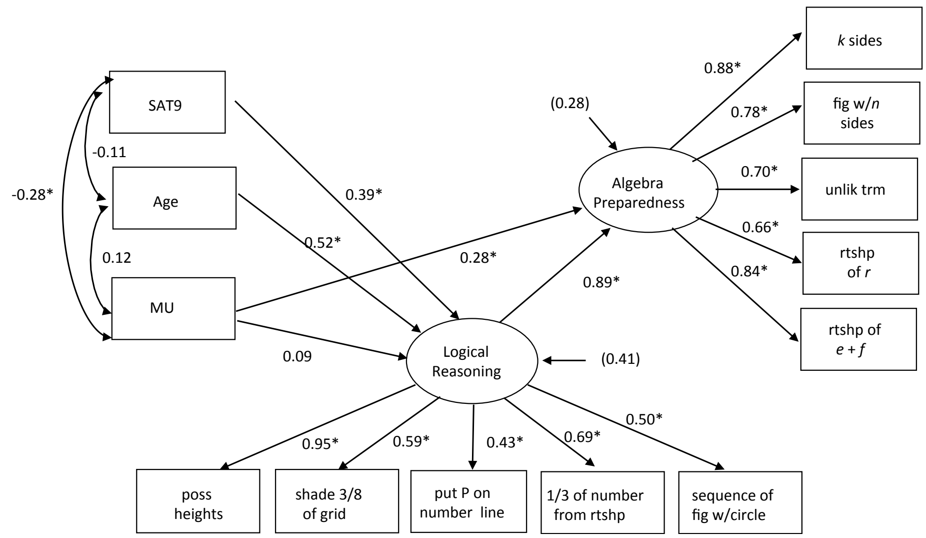

Figure 1. Standardized estimates of the final model.

Findings

Our findings (see Figure 1) resulted in a model that restricted the direct effects of age and SAT9 on algebra preparedness to 0. The model allowed a direct effect of MU on logical reasoning and direct and indirect effects of MU on preparedness. The factor loadings for the items were statistically significant. Significant direct standardized effects from MU to preparedness (0.26, $p < .05$) and from logical reasoning to preparedness (0.95, $p < .05$) were noted. More specifically for the MU variable, the results suggested increasing from 0 (no participation) to 1 (low participation from 1–3 years) resulted in a 0.28 *SD* increase in preparedness, controlling for other variables and, similarly, increasing from low MU participation to high participation (from 4–5 years) would increase preparedness by another 0.28 *SD*. Significant standardized direct effects on logical reasoning were also noted for age (0.52, $p < .05$) and prior achievement (0.39, $p < .05$). Our final model accounted for 72% of the variance in preparedness and 59% of the variance in logical reasoning. Significant standardized indirect effects were noted for age (0.23, $p < .05$) and SAT on preparedness (0.65, $p < .05$). The indirect effect of MU on preparedness was in the right direction but was not statistically significant (0.08, $p > .05$).

Discussion

From our investigations, we might conclude that our SEM results are a function of at least four factors: our proposed theory, the specific characteristics of the data and the research design (which can often eliminate some rival explanations), and the analytic approach we used to examine the data. In this study, we examined a common type of competing models; that is, the *full mediation* model (A→ B → C) versus the *partial mediation* model (i.e., where $A \rightarrow C$ directly and indirectly through the mediator *B*). We found that SAT9 scores and age were fully mediated by logical reasoning—that is, their influence on algebra preparedness was entirely through their positive effects on logical reasoning with SAT9 scores having an effect on logical reasoning about three times the size of age. We hypothesized that MU would likely directly affect algebra preparedness and logical reasoning and found partial support for this. Our results suggested that MU directly affected algebra preparedness; however, it did not directly affect logical reasoning. Therefore, we failed to reject this hypothesis as well as the hypothesis that its indirect effect on algebra preparedness would be statistically significant. We note that sample size and the observed strength of relationships between variables can influence the power to detect effects in the proposed model.

We examined the logical reasoning construct further by using additional items to define it, but in each case, we found the direct effect of MU was not significantly related to logical reasoning, although the direction of the effect was always positive, with standardized coefficients ranging from about 0.05 to 0.16 (with our reported effect of 0.09), and corresponding significance levels values decreasing to $p < .11$), suggesting that power might be one rival explanation. With a larger sample size by 25% (N=160), our power to detect the effect would be a bit stronger. With increased sample size, our guess is that MU would produce small statistically significant positive direct effects on logical reasoning and perhaps small indirect effects on preparedness.

This study is an attempt to develop insight into how Davydov's theory of empirical generalization may be realized in the implementation of curriculum. Our tenets for designing mathematics curricula promote the presentation of mathematics as sense-making at an accessible level for all learners. We believe our findings are consistent with Vygotsky's theory of learning leading development, a driving influence of Davydov's work. Though the development of logical reasoning may lag, it appears that MU students have learned mathematics that is consistent with the preparation for studying algebra. Additional research is needed to understand these curricular influences more fully.

In this paper we described theoretical considerations that led to the design of the MU curriculum. MU instruction reflected these considerations, as students in the program demonstrated that mathematics is about sense-making, via a measurement perspective and the development of empirical thinking. We believe these findings have implications for designing any mathematics curriculum and possibly influencing or supporting change in the culture for teaching elementary mathematics. That is, if instructions to the teacher are written to reveal the progression of the mathematics, teachers may be more likely to instruct their students in this same manner.

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