

# Paper Title: Formative intervention research to enhance equitable mathematics instruction: Results and feedback from integrating multiple data sources

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# Formative intervention research to enhance equitable mathematics instruction: Results and feedback from integrating multiple data sources

Abstract: in this paper, various stakeholders engaged in a formative intervention research initiative to support equitable mathematics instruction and enhance mathematical learning opportunities for African-American students share data from the first two years of the project. Data include surveys and interviews of teachers' beliefs regarding equitable mathematics instruction, classroom observations, and surveys of teachers' and researchers' perceptions of the design and effectiveness of the professional development initiative. We describe what each set of results indicates about equitable practices in teaching mathematics and how we have used the results to inform revisions to the professional learning experiences and support teachers to enact ambitious and equitable mathematics instruction.

## Introduction

A problem of practice continuing to plague urban schools is the gap between the mathematical achievement of minority students and their white peers. However, researchers who examine issues of race and equity in mathematics teaching and learning suggest directing less attention toward achievement gaps and greater attention toward *opportunity gaps* created when specific students or groups of students are provided different or impoverished opportunities to learn mathematics (Gutiérrez, 2008). Efforts to reduce opportunity gaps will subsequently generate reductions in achievement gaps.

In this paper, we describe a formative intervention research project designed to abate opportunity gaps in African-American students' learning of mathematics within the context o a large urban school system. Consistent with Perry (2013) and Jackson and colleagues (2013), we consider *opportunity gaps* (specific to mathematics) as differences in students' *opportunities to learn* mathematics due to the presence or absence of key components of ambitious mathematics instruction (cf. Franke, Kazemi, & Battey 2007). Hence, we consider paramount to students' opportunities to learn mathematics cognitively challenging instructional tasks (Stein, Grover & Henningsen, 1996; Tarr, Reys, Reys, Chavez, Shih, & Osterlind, 2008), rigor in task implementation throughout the lesson (Stigler & Hiebert, 2003; Stein & Lane, 1996), and rich mathematical discussions (Boaler & Staples, 2008; Stein, Engle, Smith, & Hughes, 2008).

Teachers' perceptions of the successes and challenges of African-American students in the mathematics classroom can impact the opportunities for learning presented to students in ways that may support or inhibit ambitious mathematics teaching and learning (Ladson-Billings, 2009; Martin, 2009). Martin (2012) cautions about mindsets based upon beliefs of African-American students' "cultural differences or deficits, limited mathematical knowledge and problem solving skills, family background and socioeconomic status, and oppositional orientations to schooling" (p. 48). For example, Martin (2007) coined the term "missionary" to refer to the mindset of "saving African American children from themselves and their culture" (p. 13). Similarly, Milner (2010) identifies instructional consequences associated with the constructs of: 1) color-blindness; 2) cultural conflicts; 3) meritocracy; 4) deficit mindsets; and 5) contextneutral mindsets. Regarding productive mindsets, Ladson-Billings (2009) describes teachers' ways of being, and the relationships teachers seek to form with African-American students, as critical in fostering students' learning. She describes culturally relevant teachers as possessing characteristics that empower themselves to positively impact students (e.g., the belief that all students can succeed, seeing themselves within the school community and the community surrounding the school, celebrating students' cultural identities, establishing relationships and connectedness with students, encouraging a community of learners and collaborative learning, and belief in developing knowledge by scaffolding student's learning).

In this study, we investigate how to use data on ambitious mathematics instruction (via classroom observations), teachers' beliefs and mindsets (via surveys and interviews), and reflections on professional learning experiences (via surveys) to support formative intervention research aimed at enhancing equitable mathematics instruction and African-American students' opportunities to learn mathematics. We present data from a Mathematics-Science Partnership (MSP) project set in the context of a large urban school district. Typical to other MSPs, this project has as a central premise that enhancing participating teachers' mathematical knowledge can generate subsequent changes in teachers' beliefs and classroom practices regarding the effective teaching and learning of mathematics, which can then enhance students' mathematical learning and achievement. Towards this goal, participating middle and high school mathematics teachers engage in a four-week summer professional development (PD) workshop and ongoing school-year follow-up. This project, however, differs from other MSPs in two important ways: 1) the focus on equity and systemic change within the context of a specific large urban school

district, in order to better understand and eradicate these issues more broadly; and 2) framing our work as formative intervention research.

## **Underlying Assumptions and Perspectives**

We view the work of the project as operating as a kind of formative intervention research (Engeström, 2011). In his characterization of the methodology, Penuel (2014) described three key commitments: 1) *focusing on a problem of practice*—a challenge encountered by participants in their life or work activities; 2) *stimulating participants to produce innovations*— by first introducing a challenging situation, problem, or set of obstacles, and then triggering a process for overcoming those challenges or obstacles by engaging participants in design work and supporting their implementation; and 3) taking as the primary goal the *expansion of agency of participants*—to "enable new forms of collective activity to emerge through direct engagement with the contradictions embedded in practice" (p. 100).

In this project, the overarching problem of practice is confronting and reducing racial inequities in students' opportunities to be successful in mathematics. Consistent with formative intervention research, no particular program is being implemented in order to address this problem; teachers are not "trained" to enact particular strategies. Rather, facilitators attempt to engage teachers in activities to potentially disrupt certain mindsets, perspectives and practices, and invite teachers to respond by developing and attempting innovations in their classrooms and schools. In particular, project leaders work to integrate three foci: mathematics, racial equity, and learning and teaching. In the paragraphs that follow, we first highlight the assumptions underlying each of those three foci in terms of facilitators' goals and supports for teacher learning, and then provide a characterization of the theory of teacher learning that (implicitly) guides the work—assumptions on the part of the project's leadership, which then inform the research and evaluation teams' data collection and analysis. We conclude the section by describing the role that the research and evaluation teams try to play in providing feedback to support the effort.

## **Mathematical Perspectives and Practices**

A key pair of assumptions underlie the project: (a) not all mathematics teachers have had opportunities to engage in the kind of mathematical practice characteristic of the work of professional mathematicians, and (b) affording such opportunities might work to disrupt teachers' current conceptions of mathematics and lead to redefining the goals for students in classrooms. A key component of the professional learning experiences is therefore a "mathematics immersion" experience for teachers (Antonellis & Cuoco, 2001; Stevens et al., 2007), in which facilitators pose a series of "low threshold, high ceiling" problems (i.e., problems that are relatively easy to get into but can lead to complex mathematics) and then support participants in collaborating to invent and discuss their strategies, and to eventually pose and pursue their own spin-off problems. In this project, the mathematics immersion comprised the beginning half of each day over three weeks of the summer workshop.

Such open-ended, "authentic" mathematical engagement serves to reframe mathematics as a dynamic activity to which humans contribute, rather than a fixed set of knowledge that humans acquire. Advocates of such professional development argue that the opportunities to step into the role of a learner and to build a mathematical community of colleagues around these pursuits can support teachers in engaging their students in a similar approach to mathematics (e.g., Gates et al., in press). Facilitators therefore try to leverage new definitions of mathematics to support teachers in problematizing typical forms of instructional practice, a focus we return to below.

#### **Racial Equity Perspectives and Practices**

Another fundamental assumption underlying project efforts is that, to a significant extent, inequity in opportunity is produced in classrooms, in interactions between and among teachers and students. The five interrelated constructs of Milner's (2012) "opportunity gap explanatory framework" provide a means of interrogating instructional practice with respect to student opportunity. It could be that teachers:

- a) enact *color blindness*, viewing their students as "all the same" in learning mathematics and failing to acknowledge and affirm their racial (and other) identities (Aguirre, Mayfield-Ingram, & Martin, 2013; Nasir, 2011);
- b) introduce *cultural conflicts*, operating primarily from their own cultural ways of knowing (e.g., taking for granted—and failing to clarify—the contextual features of a mathematical task, thereby limiting students' access; Jackson et al., 2013);
- c) adhere to a *myth of meritocracy*, the idea that their students' (and their own) success is only earned, and that failure is solely a result of poor choices;

- approach students with *low expectations and deficit mindsets*, which may influence the extent to which they engage students in rigorous mathematical activity (Wilhelm, Munter, & Jackson, in press); and/or
- e) enact *context-neutral mindsets and practices*, failing to recognize deep-rooted realities embedded in a particular place, such as a school in a particular community.

Project facilitators engage teachers in activities (e.g., readings, discussions, reflective dialogue) intended to uncover and disrupt such perspectives and practices as those listed above, and introduce new language where appropriate. To aid in this endeavor, early in the experience, officials from the district's equity office lead a series of sessions on the Courageous *Conversations* protocols for engaging in race talk (Singleton, 2014). This is followed by introducing ideas about student and teacher identity (Nasir, 2011); tenets of culturally relevant pedagogy (Ladson-Billings, 1995); counternarratives about students' and parents' participation and success in school (Battey & Franke, 2015; Perry, Steele, & Hilliard, 2004); and historical accounts of the development of schooling and school mathematics, and arguments that these systems have been built in ways that marginalize certain communities and student populations, specifically African-American students (Moses & Cobb, 2001). Similar to the mathematics work described above, facilitators then work to support teachers as they use these ideas and lenses to reflect on their current instructional practices and relationships with students in their classrooms. This work occurred during half-day sessions over three weeks of the project, with a typical daily schedule consisting of mathematics immersion work in morning and equity-focused work in the afternoon.

# **Instructional Perspectives and Practices**

During the last week of the summer workshop, after wrestling with the ideas and activities described above over the first three weeks, teachers are invited to respond by engaging in design work (Penuel, 2014)—developing plans of action for introducing innovations into their classroom practice in the upcoming school year. These innovations represent potential "solutions" to the problems that teachers have newly identified or reframed. To further stimulate teachers' design efforts, over time certain instructional practices are introduced, including practices for orchestrating whole-class discussions of students' ideas and strategies (Smith & Stein, 2011) and practices for facilitating productive group work and addressing issues of status and power, drawn from research on complex instruction (Cohen, 1994; Horn, 2012). Again,

teachers are not "trained" to employ these or any other specific instructional practices. Facilitators do, however, try to support teachers in identifying potential changes that will foster classroom norms and cultures more conducive to the kind of mathematical exploration in which they themselves engaged as learners during the mathematics immersion experiences, and in ways that are racially equitable.

## **Feedback for Ongoing Design**

Revisiting Penuel's (2014) criteria for formative intervention research, we can now provide more specific illustration of the work of this project: 1) *focusing on a problem of practice*— enhancing opportunities to learn mathematics for African-American students in teachers' classroom, school, and district; 2) *stimulating participants to produce innovations*— challenging teachers' existing conceptions of mathematics and how one engages in mathematics, disrupting teachers' mindsets regarding their African-American students, and providing access to ambitious instructional strategies, and then supporting teachers to design and implement new strategies; and 3) *expansion of agency of participants*— supporting teachers in directing their own change efforts toward achieving equitable mathematic learning opportunities for African-American students in their classrooms, schools, and district.

Throughout this work, as components of the MSP project, research and evaluation teams collect a variety of data to assess the impact of the project and provide formative feedback to project leaders. In particular, the research team follows the annual feedback cycle model developed by Cobb and colleagues in the *Middle School Mathematics and the Institutional Setting of Teaching* (MIST) project (Cobb & Jackson, 2011, 2012; Cobb & Smith, 2008). Each school year, the research team conducts interviews with district leaders in order to understand the current "theory of action" for secondary mathematics, including the district's goals, initiatives for reaching those goals, and underlying rationales. That theory of action serves as a backdrop to mid-school year interviews with teachers, principals and districts leaders, of which the focus is understanding the institutional setting of teaching for racial equity as experienced by those tasked with enacting the district's initiatives. Based on analyses of these interviews, the research team provides feedback to project and district leadership, including findings specific to the aims of each and recommendations for refining their respective designs. Throughout this work, a key purpose is to support multiple stakeholders in understanding the project within a broader set of

district initiatives, with the assumption that situating the project in those terms may yield mutual benefit.

In this paper, we examine data on perceptions of the professional learning experiences, teachers' beliefs and mindsets, and teachers' classroom practices in order to address the following research questions:

- 1. What do surveys indicate about teachers' and project leaders' perceptions of the professional learning experiences?
- 2. What do surveys and interviews indicate about teachers' productive and unproductive mindsets in teaching mathematics to African-American students?
- 3. What do classroom observations indicate about students' opportunity to learn mathematics in a large urban school system?

At this stage in the project, results will be used to assess professional learning experiences and teachers' mindsets and classroom practices (in aggregate) to provide project leaders feedback to enhance formative intervention research efforts (e.g., revise professional development experiences in ways that further disrupt teachers' conceptions of mathematics, race and equity in the teaching and learning of mathematics, and effective mathematics instruction; support teachers' design and implementation efforts towards these goals; and expand the agency of project participants). We also identify successes and challenges in conducting formative intervention research that aims to support teachers to enact equitable and ambitious mathematics instruction.

## **METHODS**

At the time of this analysis, teachers (n = 36) had participated in the Year 1 professional development (PD) activities identified in Figure 1, including: 1) a 4-week workshop (Summer 2014) focused on developing mathematical knowledge and understanding historical inequities contributing to the "educational debt" (Ladson-Billings, 2006); 2) weekly school-based working groups (AY 2014-2015) aimed at building relationships with students and enhancing instructional practices; and 3) monthly mathematics symposia (AY 2014-2015). A key goal of the first year of collaboration with teachers was to disrupt deficit-oriented perspectives and support the development of more productive explanations of students' struggle. As described in the previous section, the Year 1 professional learning activities were divided into the component of mathematics immersion, explorations of race and equity issues, considerations of instructional

strategies, and time for design work. We now describe the instruments, subjects, and analyses for each data source.

Sum	ner Professional Deve	elopment Workshop	School-Year Follow-Up			
	(Year 1)	)				
	Weeks 1-3	Week 4	Weekly	Monthly		
AM	Mathematics	Instructional	Design and	Cohort meetings		
	Immersion	Strategies;	Implementation			
PM	Exploration of Race	Design work	work: 10 hrs	Mathematics		
	and Equity Issues		paid professional	Colloquia		
			development			
			time weekly			

Figure 1. Year 1 Professional Learning Experiences for Participating Teachers

## Subjects, Data and Analyses

Data include surveys of teachers' and researchers' perceptions of the effectiveness of the professional development (PD) experiences, surveys and interviews of teachers' beliefs regarding equitable mathematics instruction, and classroom observations. In addition to sharing the findings from each data set, we also describe what each data set indicates about successes and challenges in supporting teachers to enact equitable and ambitious mathematics instruction, and highlighting how we use results to inform revisions to the on-going work.

**Professional Development Surveys**. The purpose of the professional development (PD) survey was to assess teachers' and project leaders' perceptions of the Year 1 and Year 2 professional learning experiences intended to support teachers to enact equitable and ambitious mathematics instruction. Desimone's (2009) model of teachers' learning and instructional change provides a framework for how PD experiences generate change in teachers' knowledge/beliefs, which then lead to changes in teachers' instructional practices, and subsequently enhance students' mathematical learning. As PD is the catalyst in the model, and we sought to disrupt teachers' existing conceptions of mathematics, race and equity, and effective mathematics teaching, assessing teachers' and project leaders' perceptions of the PD was an essential aspect of the formative intervention research in this project. Evaluating

professional learning experiences also adheres to NSF and American Evaluation Association guidelines for formative evaluation (implementation, monitoring, and progress).

Quantitative and qualitative data were collected and analyzed using a mixed methods design. Surveys were administered in summer 2014 and 2015 to leadership team members (e.g., Principal Investigators in the project, PD designers and facilitators). Teachers responded to surveys at the ends of Weeks 1, 3, and 4 during the 2014 and 2015 summer PD sessions. Response rates in 2014 were low. Out of 36 teachers who attended the sessions, survey completion rates were 24 in Week 1 (67%), 21 in Week 3 (58%), and 15 (42%) in Week 4. Completion rates for the survey in 2015 were considerably higher. Out of 28 teachers who attended these sessions, 25, 27, and 24 teachers responded to the surveys in Week 1, 3, and 4, respectively, an 86% rate or higher. Other forms of data from the PD include tallies of teacher participation during PD sessions, videos, and documents (e.g., schedules for each day and week of the PD, handouts, presentations, questions for discussion, and exhibits of teachers' writing/work generated for various activities).

**Equity Survey**. We administered a survey of teachers' beliefs regarding equitable mathematics instruction, intended to provide an indication of: 1) teachers' awareness of and attribution for success and challenges in African-American students' learning of mathematics; and 2) teachers' mindsets that may be productive or unproductive in supporting the mathematical learning of African-American students. Seminal literature from experts in the field was used to develop categories and items within the survey (e.g., Jackson & Wilson, 2103; Ladson-Billings 2009, 2013; Martin, 2007; Milner, 2010). The literature supported the fact that mindsets of teachers have a significant impact on the success or failure of students in the mathematics classroom (Ladson-Billings, 2009; Martin, 2009). The survey contains eight categories of items highlighted in the literature: *Beliefs about Student Achievement and Motivation, Culturally Responsive Teaching, Perceptions of Challenges of African-American Students, Mathematics as a Civil Right, Missionary Beliefs, Beliefs about Critical Thinking, and Student Behavior.* 

The survey was administered in spring 2015, between the Year 1 (2014-2105) and Year 2 (2015-2016) PD activities focusing on mathematics and culturally responsive/relevant pedagogy. Twenty-two mathematics teachers responded to the survey. Fifteen (68%) self-identified as White, three (14%) self-identified as African American or Black, one (5%) self-identified as

Asian, and three (14%) declined to self-identify. The average years of teaching experience for all 22 teachers was 12.5 years. Teachers identify a focal class for this professional development program. The average percentage of African American students in teachers' focal class was 55%, with a range of 0 - 100%.

Quantitative analyses were conducted on the eight dimensions of the Equity Survey. For six of the dimensions, teachers' responses were examined as a set: *Beliefs about Student Achievement and Motivation, Culturally Responsive Teaching, Student Behavior, Missionary Beliefs, Beliefs about Critical Thinking, and Civil Rights.* Responses for the remaining dimensions, *Perception of Challenges of African American Students* and *Success of African American Students in the Classroom* were analyzed item by item. Coefficient alpha was obtained for each dimension to determine its internal consistency. Descriptive statistics and chi-square analysis were conducted to compare results within and across dimensions. In order to determine the overall score/rating for this dimension, responses to negatively worded items were recoded so that the total score/rating for the respondents would represent the extent to which teachers have positive beliefs about that particular dimension. The options for each item ranged from 1 to 5. Scores closer to 1 represent positive beliefs and scores closer to 5 represent negative beliefs.

**Interviews.** In the teacher-interview data, we employed Jackson and Gibbons's (2014) framework to characterize teachers' explanations and attributions for the mathematical performance of African-American students as either *productive* (i.e., framing problems as locating within instruction and aspects of the school institution), *unproductive* (i.e., framing problems as locating within students' inherent deficiencies, lack of motivation, or unsupportive family or community), or *mixed* (i.e., including instances of both productive and unproductive explanations). Our analysis follows NCTM's (2014) framing of teachers' beliefs as either "productive" or "unproductive," with the latter potentially acting as "obstacles" to innovations being taken up in instructional practice.

Beginning with the school year preceding the first summer of professional development, the research team has conducted annual interviews with all participating teachers. In this analysis, we include interviews conducted with 13 mathematics teachers (a subset of the 22 teachers completing the equity survey described previously), in both 2014 and 2015 (before the Year-1 summer workshop and during their first school year as participants). To assess the productivity of teachers' explanations for students' struggle, we conducted semi-structured

interviews, in which we asked about perceived challenges of teaching mathematics, and explanations for why racial achievement differences exist. We investigated whether that goal was met with this guiding question: to what extent and in what ways are mathematics teachers' (a) explanations for differences in students' mathematics achievement scores and (b) descriptions of their challenges as mathematics teachers different after participating in professional development intended to disrupt deficit views?

Aimed at helping us situate the project's efforts within the institutional settings in which teachers and students work, the interviews touch on a range of topics, including teachers' responses to district initiatives, instructional leaders' expectations, and the supports they have been provided. Additionally, we use the interviews to assess teachers' evolving perspectives on mathematics instruction and students with two previously developed instruments, which we describe in the following paragraphs. In both cases, each transcript was transcribed and coded independently by two research assistants [CS or CH author], with the PI [CM author] coding 20% of the transcripts to ensure that at least 80% exact agreement across all codes was maintained. Discrepancies were resolved thorough discussion and consensus.

<u>Vision of High-Quality Mathematics Instruction (VHQMI)</u>. As described by Munter (2014), the VHQMI instrument includes a set of interview questions and corresponding rubrics used to assess the sophistication of teachers' articulations of high-quality mathematics instruction with respect to four dimensions of classroom practice: role of the teacher, mathematical tasks, student engagement in classroom activity, and classroom discourse (which consists of five sub-dimensions: patterns and structure of talk, the nature of talk, teachers' questions, students' questions, and students' explanations). To help in examining change over time, we calculated a single VHQMI score for each teacher each year by averaging scores across all rubrics, but also view individual scores as qualitative characterizations of teachers' instructional visions.

<u>Views of Students' Mathematics Capabilities (VSMC).</u> We also used the interviews to assess teachers' views of students' mathematics capabilities. As described by Jackson, Gibbons, and Dunlap (in press), the VSMC instrument includes a series of interview prompts and rubrics that help to characterize the (un)productive nature of teachers' diagnoses of sources of students' difficulty in mathematics and prognoses of support. Teachers' diagnoses, or explanations, of sources of students' difficulty that alluded to instructional practice or learning opportunities (e.g.,

inequitable instruction; institutional racism) were coded as *productive*, while those that attributed difficulty to inherent traits of students or parents (e.g., lack of motivation; lack of parental support) were coded as *unproductive*. Those explanations that waivered between productive and unproductive diagnostic framings were coded as *mixed*. Teachers' prognoses, or descriptions of supports for students experiencing difficulty, were assessed in a similar way. Descriptions of supports that were aimed at supporting students in successfully participating in rigorous classroom activity were coded as *productive*, while those that were aimed at less ambitious mathematics goals (e.g., remediation in order to pass a test) were coded as *unproductive*. Those descriptions that waivered between productive and unproductive prognostic framings were coded as *mixed*.

**Classroom Observations.** We observed 21 mathematics teachers, 14 middle school (MS) and 7 high school (HS), each during two lessons in spring 2015. Two pairs of MS teachers "team-taught," resulting in observations of 19 classrooms and 38 lessons. Lessons were videotaped (30/38; 79%) or observed (8/38; 21%) by two trained observers. Lessons were rated using the *Instructional Quality Assessment* observation tool assessing instructional tasks, task implementation, and mathematical discussions (Boston, 2012). On the IQA, scores range from 0-4 in each dimension, with scores of 3 and 4 indicative of ambitious mathematics instruction (e.g., cognitively demanding tasks, rich mathematical discussions). Ten lessons (26%) were consensus-scored for rater-calibration and 28 lessons (74%) were scored independently by two trained raters. Exact-point agreement was 84% overall, with discrepancies resolved through discussion. Analyses include descriptive statistics and one-tailed Mann-Whitney tests for differences between MS and HS lessons.

We also assessed the quality with which teachers set up, or "launched" mathematical tasks, using a series of "<u>Tasks As Set-Up</u>" rubrics developed by Jackson et al. (2013). Modeled after the IQA rubrics, with scores ranging from 0-4, the setup rubrics are intended to capture the extent to which teachers create a shared context so that all students can engage meaningfully in a task, including (a) ensuring that students are familiar with contextual features of a problem-solving scenario and (b) establishing an understanding of features of the problem that are to be mathematized, all while maintaining the cognitive demand of the task. Contextual features refer to the non-mathematical aspects of the task. For both of these rubrics, scores of 3 and 4 indicate evidence of collective involvement in explicating such aspects of the task.

In the next section, we describe the results of the analyses.

#### RESULTS

#### **Professional Development Survey**

Teacher survey results for Year 1 PD sessions (2014) indicated that while the focus on mathematics was important, teachers wanted more time to make connections to their curriculum. Teachers also hoped to learn more about equity *in the classroom*, but noted this was lacking in the sessions. Teachers indicated that they chose this particular MSP project over others available to them because of the equity component, and believed that equity would be infused throughout. Others indicated that readings were difficult, sometimes "pretentious," and hard to follow.

The majority of leadership team members (91%) reported some of these sentiments, especially wanting more time for discussions and planning surrounding equity in math teaching and learning. However, results for emphasis on the math component varied, 69% indicated that the math component should receive the same amount of emphasis in leadership team monthly meetings, while only 31% indicated less time for math. Several project leaders also suggested the need for mapping out the goals and objectives for each morning and afternoon session of the second summer of PD.

Teacher survey results for the Year 2 PD sessions (2015) were more positive. The large majority of teachers found the equity component and reading/discussion component to be valuable, commenting that the book chapters they read (e.g., Milner and Nasir books) were "enjoyable and helpful in improving their teaching practices." They also found the speakers to be "enlightening, informative, and inspiring" during the discussion sessions. Finally, more than 80% of teachers said the teaching strategies component of the PD was valuable, which is quite an improvement over the 2014 PD survey where only 50% of teachers indicated it was valuable and another 27% said it was nonexistent. Hence, the connections to practice (in mathematics, equity, and instructional strategies) teachers were hoping to explore in Year 1 met their expectations in Year 2.

## **Equity Survey**

Equity Survey results indicate moderate to strong consistency in all survey dimensions except Civil Rights, as shown in Table 1. Coefficient alpha was obtained for each dimension to determine its internal consistency. Reliability ranged from .491 for the 7 items in "beliefs about student achievement and motivation" to .828 for the 24 items in the "culturally responsive teaching" dimension. Only the Civil Right dimension was problematic, where the reliability coefficient could not be produced due to model assumptions not being met.

Construct	Number	Reliability	Mean	SD	Range
	of items				
Beliefs about Student Achievement and	7	.491	2.03	.43	1.00-2.71
Motivation					
Culturally Responsive Teaching	24	.828	2.03	.35	1.33-2.71
Success of African-American Students	8	.635	1.90	.43	1.25-3.13
Missionary Beliefs	8	.644	2.14	.44	1.13-2.88
Beliefs about Critical Thinking	4	.699	2.18	.50	1.00-3.00

Table 1. Analysis of Internal Consistency of Equity Survey

NOTE: In the Civil Rights dimension (5 items), model assumptions were not met. This dimension is currently under revision.

The two dimensions of the survey focusing on teachers' awareness of and attribution for success and challenges in African-American students' learning of mathematics were intended to be analyzed at the item level. The results of the *Success* section showed that all teachers (100%) agreed that regardless of ethnic or cultural backgrounds, students can experience success in their classrooms. However, 52% agreed that African-Americans are successful in mathematics when they have an innate ability to perform mathematics, indicative of a meritocratic mindset. Similarly, 43.5% believe that only students who show strong math ability should have access to upper level math courses.

Table 2 provides results for *Perceptions of Challenges of African-American Students*, where teachers responded to the item stem: "The challenges faced by African-American students in my mathematics classroom are due to \_\_\_." Two patterns in this section were particularly striking. First, approximately half (50-65%) of teachers agreed with items attributing challenges and lack of success *to characteristics of students* (e.g., disinterest in mathematics). Second, teachers overwhelmingly disagreed (85-90%) with items attributing challenges facing African-American students *to characteristics of the teacher, classroom, or school* (e.g., low teacher expectations, lack of pedagogical or content knowledge). These results indicate

unproductive mindsets (e.g., deficit perspectives of students) that may impact teachers' "ways of being" (Ladson-Billings, 2006) while teaching mathematics to African-American students.

In contrast, a number of results indicate productive mindsets and recognition of classroom or systemic issues facing African-American students. Results in Table 1 showed that, on average, the teachers have relatively positive beliefs with regard to the five dimensions, with mean values ranging from 1.90 to 2.18. The Beliefs About Student Achievement and Motivation section revealed that 95.6% of teachers agree that African-American students can be successful in mathematics. In the *Civil Rights* section, 95.7% of teachers believe that it is their responsibility to ensure that all students have a strong mathematics knowledge base. In the *Missionary Beliefs* section, 95.7% of teachers feel that they are able to connect with and form trusting relationships with their African-American students, and only 8.7% believe they cannot expect much from their African-American students because students lack the skills to tackle rigorous tasks. The Beliefs About Critical Thinking section showed that all teachers (100%) feel that their African-American students are capable of solving math problems using critical thinking. Also, 91.3% of the teachers believe that African-American students are able to meet the expectations for higher order math skills. In the Student Behavior section, 56.5% of the teachers believe that African-American students are disciplined more often that students of other races. Also, 43.5% percent feel that African-American students are disciplined more harshly than students of other races. 87% of the teachers felt that they made an effort to understand their students' cultural backgrounds in their math classroom. In addition, 65.2% believed that being a part of the community surrounding the school is an important part of teaching.

Item	Percent Agreement/Disagreement
low teacher expectations	82% disagree
limited teacher pedagogical knowledge	86% disagree
limited teacher content knowledge	91% disagree
negative experiences in math class	82% agree
negative experiences in school	82% agree

Table 2. Results for the Equity Survey dimension, "The challenges faced by African-American students in my mathematics classroom are due to..."

large class sizes		77% agree	
history of low achievement		64% agree	
disinterest in math		59% agree	
lack of math skills		68% agree	
lack of effort in math		59% agree	
opposition to authority		50% agree	
Lack of math ability	27% agree	27% neutral	41% disagree
Limited school resources	23% agree	41% neutral	36% disagree
Culturally irrelevant material	46% agree	32% neutral	18% disagree
Lack of relevant math content in lessons	36% agree	27% neutral	36% disagree

## **Teacher Interviews**

Interview results point to a shift toward more productive explanations of achievement differences (i.e., locating more responsibility within classrooms and schools). Four (31%) teachers' explanations were more productive in year two, and none became less productive. However, almost all of those teachers still attributed challenges they encounter to the diversity and/or perceived deficiencies among students. While potentially productive shifts were detected in two cases (dropping a cultural deficit argument; adding a sense of racial awareness), identified challenges remained fairly consistent, with students' lack of prerequisite knowledge and/or motivation the most frequently identified.

Although it is generally believed that changing teachers' views on the sources of students' struggle is difficult (Battey & Franke, 2013), findings from this study suggest that the project's current approach shows promise. The shift in the nature of explanations for differences in achievement suggests that Year 1 efforts to introduce new frameworks and perspectives may have had some positive impact. However, the lack of change in how teachers frame their professional challenges suggests that teachers may not yet be using new frameworks to reframe old problems, which points to the need for refining our design, possibly drawing on more "practice-forward" (Thompson, Windschitl, & Braaten, 2013, p. 605) arguments for supporting teachers' learning.

## **Classroom Observations**

The results of classroom observations are provided in Tables 3-6. Results in Tables 3-5 illustrate two important patterns. First, MS lessons exhibited high means, percentages of high-cognitive demand scores, and numbers of teachers with high scores in Tasks, Implementation, and Questioning. MS lessons scored significantly higher than HS lessons in Tasks (z=2.86; p<.01), Implementation (z=3.04; p<.01), and Questioning (z=3.6; p<.001). Second, discussion-based rubrics scored low overall. However, MS lessons *containing a discussion* (non-zero scores) posted moderate means for Discussion, Teacher-Press, and Student-Providing.

As listed in Table 6, the results of assessing the quality with which teachers launched mathematical tasks suggest that teachers could be doing more to set tasks up so that more students have access and can engage meaningfully. Although a few teachers devoted attention to mathematical relationships in their launches, we found that almost no attention was being paid to ensuring that students developed a shared understanding of problems' contextual features.

					Teacher	Student	Teacher	Student
	Task	Implementation	Discussion	Questioning	Linking	Linking	Press	Providing
Mean <sup>a</sup>	3.18	2.66	1.63	2.63	1.24	1.21	1.58	1.63
Median	3	2	2	3	1	1	2	1
Number of Lessons (	n = 38) w	ith:						
Score = 1	1	0	8	6	11	16	7	8
Score = 2	9	19	8	10	13	5	9	8
Score 1 or $2 = Low 0$	Cognitive	Demand						
	10	19	16	16	24	21	16	16
	(26%)	(50%)	(42%)	(42%)	(63%)	(55%)	(42%)	(42%)
Score = 3	10	9	6	10	2	4	9	6
Score = 4	18	9	5	11	1	2	2	5
Score 3 or $4 = \text{High C}$	ognitive I	Demand						
	28	18	11	21	3	6	11	11
	(74%)	(47%)	(29%)	(55%)	(8%)	(16%)	(29%)	(29%)
Score = 0 (construct	0	1	11	1	11	11	11	11
not observed)	(0%)	(3%)	(29%)	(3%)	(29%)	(29%)	(29%)	(29%)
Number (%) of Clas	sroom (n	= 19 classrooms; 21	l teachers) <sup>b</sup> w	ith:				
At least 1 Score = Hig	gh Cogniti	ve Demand						
	15	12	8	14	3	5	9	7
	(79%)	(63%)	(42%)	(74%)	(16%)	(26%)	(47%)	(37%)
At least 1 Score= 0 (c	onstruct n	ot observed)						
	0	1	10	1	10	10	10	10
	0%	(5%)	(53%)	(5%)	(53%)	(53%)	(53%)	(53%)

# Table 3. Lesson Observations Data Overall (n = 19 classrooms; 38 lessons)

<sup>a</sup>Means are provided for interpretive value.

<sup>b</sup>Two pairs of teachers taught together in the same classrooms, resulting in observations of 19 unique classrooms.

	Task	Implementation	Discussion	Questioning	Teacher	Student	Teacher	Student
					Linking	Linking	Press	Providing
Mean <sup>a</sup>	2.57	2.00	0.86	1.71	0.86	0.79	0.79	0.79
Mean non-0 scores	2.57	2.15	1.50	1.85	1.50	1.38	1.38	1.38
Median	2	2	1	1	1	1	1	1
Number (%) of Lesso	ons $(n = 1)$	4) with:						
Score of 3 or $4 = $ High	Cognitive	e Demand						
	6	2	0	3	0	1	0	0
	43%	14%	0%	21%	0%	7%	0%	0%
Score = $0$ (construct n	ot observe	d)						
	0	1	6	1	6	6	6	6
	0%	7%	43%	7%	43%	43%	43%	43%
Number (%) of Teac	hers (n =	7) with:						
At least 1 Score = H	ligh Cogni	itive Demand						
	4	2	0	3	0	1	0	0
	57%	29%	0%	43%	0%	14%	0%	0%
At least 1 Score= 0 (co	onstruct no	ot observed)						
	0	1	5	1	5	5	5	5
	0%	14%	71%	14%	71%	71%	71%	71%

<sup>a</sup>Means are provided for interpretive value.

					Teacher	Student	Teacher	Student
	Task	Implementation	Discussion	Questioning	Linking	Linking	Press	Providing
Mean <sup>a</sup>	3.54	3.04	2.08	3.17	1.46	1.46	2.04	2.13
Mean of								
non-0 scores	3.54	3.04	2.63	3.17	1.84	1.84	2.58	2.68
Median	4	3	2	3	1	1	2	2
Number (%) of Lesso	ons $(n = 24)$	4) with:						
Score of 3 or $4 =$								
High Cognitive	22	16	11	18	3	5	11	11
Demand	92%	67%	46%	75%	13%	21%	46%	46%
Score = 0								
(construct not	0	0	5	0	5	5	5	5
observed)	0%	0%	21%	0%	21%	21%	21%	21%
Number (%) of Class	rooms <sup>b</sup> (:	n = 12) with:						
At least 1 Score =								
High Cognitive	11	10	8	11	3	4	9	7
Demand	92%	83%	67%	92%	25%	33%	75%	58%
At least 1 Score= 0								
(construct not	0	0	5	0	5	5	5	5
observed)	0%	0%	42%	0%	42%	42%	42%	42%

 Table 5. Lesson Observations Data for Middle School Teachers (n = 14 teachers; 12 classrooms; 24 lessons)

<sup>a</sup>Means are provided for interpretive value. <sup>b</sup>Two pairs of teachers taught together in the same classrooms, resulting in observations of 12 unique classrooms.

	<b>Contextual Features</b>	Mathematical Relationships
	(n = 10)	( <i>n</i> = 17)
Mean <sup>a</sup>	1.6	2.41
Median	2	2
Number of Lessons wi	th:	
Score = 1	2	2
Score = 2	4	9
Score 1 or $2 = Low Co$	ognitive Demand	
	6	11
	(60%)	(65%)
Score = 3	2	3
Score = 4	0	3
Score 3 or $4 =$ High Co	gnitive Demand	
	2	6
	(20%)	(35%)
Score = $0$ (construct	2	0
not observed)	(20%)	(0%)

 Table 6. Lesson Observations: Tasks as Set-Up (n = 19 classrooms; 19 lessons)

#### **Discussion: Providing feedback to support formative intervention research**

Because the research and evaluation efforts of this particular MSP project occur within the frame of formative intervention research, we consider how to use the results to address the problem of practice (e.g., confronting and reducing racial inequities in students' opportunities to be successful in mathematics); specifically, what the results indicate about 1) the impact of the PD on disrupting teachers' initial conceptions of mathematics, race and equity, and effective mathematics instruction; and 2) teachers' actions and agency in enhancing students' opportunities to learn mathematics.

#### **Disrupting teachers' initial conceptions**

Based on the analysis of data from PD evaluation surveys, teacher interviews, and classroom observations, we expressed recommendations to enhance the project's impact on teachers' initial conceptions of mathematics, race and equity, and effective mathematics instruction. First, "in its mathematics-focused components, the project is re-enacting with its teacher participants...the very marginalization that it is ostensibly intended to disrupt....many participants became disengaged" (research and evaluation memo, 12/2014). This was especially true for teachers of color. Data from the summer PD showed that whole-group participation across all racial groups was higher in the afternoon discussions of race and culture; in particular, African-American teachers participated twice as often during the afternoon equity-focused activities compared to the morning activities focused on mathematics. Hence, teachers' conceptions of what mathematics is and how one engages in mathematics *did not appear to be disrupted as intended*. Rather, racial differences in participation and status in mathematics that teachers were hoping to eradicate in their classrooms (e.g., the problem of practice at the heart of the project) instead *appeared to be reinforced* by the mathematical activities in the PD.

Second, "the choice to work on mathematical ideas during PD that are outside the curriculum teachers are expected to teach... makes it unnecessarily difficult to draw explicit links between the teachers' mathematical activity in PD and the mathematical activity in which they hope to engage their students during the school year" (research and evaluation memo, 12/2014). Recommendations included decreasing the emphasis on mathematics disconnected from teachers' curriculum, and instead making explicit links to teachers' curriculum and classroom practices. By making key components of teachers' mathematical work explicit, the PD experiences might be more successful in disrupting teachers' conceptions of mathematics and

effective mathematics teaching. Fortunately, Year 2 Summer PD plans included an emphasis on problem-posing and cultural/historical mathematics and included time for teachers to apply the mathematical activities and instructional strategies from the PD to their own curriculum and classrooms.

Third, in the Year 1 PD survey (Summer 2014), many teachers wanted an increased emphasis on the equity component and indicated the articles related to race and equity were dry and too academic in nature. We recommended selecting articles and books for teachers to read during the Year 2 Summer PD by thoughtfully considering the relevance to teachers' work in their own classrooms and schools. While overall results of the Equity Survey and Teacher Interviews survey indicated a trend of positive results or productive mindsets, further work in disrupting unproductive mindsets was needed. As evidence, results on some dimensions and prompts in the Equity Survey imply a deficit or meritocratic mindset of many teachers, and/or support the notion of "achievement gap" which places the onus on students. Similarly, interviews indicated that almost all teachers attributed challenges to the diversity and/or perceived deficiencies of their students (e.g., students' lack of prerequisite knowledge and/or motivation). In considering productive versus unproductive mindsets, these results indicate the need to further disrupt teachers' beliefs regarding their role in creating more equitable learning environments that optimize the opportunity to learn mathematics for all students.

#### Supporting teachers' design and implementation efforts

Results of classroom observations indicate significant differences in opportunities to learn mathematics between MS and HS lessons, and lack of opportunities for rich mathematical discussions overall. As formative intervention research, data suggest directions and foci for PD efforts, such as supporting teachers to orchestrate mathematical discussions. Differences between MS and HS lessons may result from differences in the nature of tasks and instructional support provide by each curriculum, suggesting HS teachers require additional resources and strategies to enact ambitious instruction. Formative intervention research also requires using data to support teachers' *self-directed* efforts to enhance instruction, presenting a need to balance *directing* PD efforts towards specific instructional practices and *encouraging teachers to initiate* direction/foci.

At the and of the 2014-2015 school year, we also provided recommendations for supporting teachers throughout the school year, including: a) increasing teacher's attendance at

Mathematics Colloquia; b) implementing encouragement and accountability measures for teachers' school-based team meetings and projects; and c) making better use of monthly teacher Cohort Meetings for professional learning experiences that support teachers' development of equitable instructional practices in mathematics. Because our work occurs within the context of a school system, we also recommended refocusing efforts to involve school administrators, such as increasing efforts to involve district administration in project leadership meetings.

## Conclusion

As formative intervention research, our work explores how to utilize data to inform PD efforts and support teachers to implement ambitious and equitable mathematics instruction. We approach the critical issue of reducing opportunity gaps by drawing on teacher and district resources, in contrast to prevalent deficit perspectives of urban students, teachers, and schools. Most importantly, our work has scholarly significance in its timely and important focus on improving mathematical learning opportunities for African-American students, in considering what the data indicate about the successes, challenges, and supports necessary to enhance equitable mathematics instruction in the context of large urban schools more generally.

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