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# As Common Core Takes Hold: Changes in Teachers' Mathematics Curriculum Use 

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#### Abstract

This paper lays out a case study of the changes in teachers' mathematics curriculum use in one large urban district in the U.S. from three years before the adoption of Common Core State Standards for Mathematics (CCMS) to three years after. Using six years of district leader and teacher interviews, the paper not only lays out the change in teachers' mathematics curriculum use, but also shows how those changes in curriculum use relate to changes in district policy. The findings show that while the Common Core mathematics standards were designed to create a more coherent, deeper and mathematically rich set of standards so that students were developing greater mathematical understanding while using more real-world problems, the lack of coherent curricular materials and the district emphasis on student achievement scores corresponded with teachers' use of a greater range of materials leading to less coherence of the curriculum materials used by teachers.


Problem Statement
Common Core State Standards in Mathematics (CCSSM) were written to create a more focused and coherent curricular approach to math education in the U.S. (Common Core State Standards Initiative, 2015). This more coherent curricular approach, designed to improve the quality of student learning and understanding of mathematics, was motivated by comparative international data, which demonstrated that greater coherence of mathematics standards - or the intended curriculum -- corresponded to higher student achievement (Schmidt, Wang, \& McKnight, 2001). In addition, enacted curriculum - the content and skills which teachers actually teach in their classroom -is overwhelming shaped by the content of the textbook and other curricular resources utilized by teachers, with researchers finding correlations as large as .95 (Schmidt, 2002). In determining which curricular materials are available and utilized by teachers, district-level leadership is influencial (Remillard, 2005). What is more, instructional materials are theorized to be only one element of districts' efforts to create a coherent instructional system, which is reflective of the explicit goals for student learning and the design of the instructional system determined at the district level (Cobb \& Jackson, 2011). In this way, the district-level goals and design for student learning is likely to strongly influence the coherence of the enacted curriculum.

## Research Questions

1) While transitioning to CCSSM, how do district leaders and the teachers in one district respond to the changes in standards with regards to curriculum use?
2) What kinds of resources do the teachers utilize and how do those relate to the goals and guidance of district leaders?

## Literature Review

For the purposes of this study we define curriculum materials as the materials that with which teachers and student have contact - either physically or online - during the course of
teaching and learning mathematics. This is an adaptation of Remillard, Herbel-Eisenmann, and Lloyd's (2009) definition which says that curriculum materials are only those with which teachers have physical contact. In the last seven years, much has changed with online lesson resources and as our the teachers in our study illustrate many teachers are not physically touching lessons that they implement.

Use of common curricular materials provides a coherent system of mathematics learning that builds on ideas over time, rather than separate topics (Newmann, Smith, Allensworth \& Bryk, 2001; Schmidt, Wang, \& Knight, 2005). While in some cases, teachers might appreciate the autonomy of designing or choosing their own curriculum, other teachers may prefer the structure and certainty of a mandated curriculum (Ben-Peretz, 1990). Prior research has found that novice teachers in particular are often overwhelmed by the demands of developing their own curriculum and often appreciate more explicit guidance in curriculum and instruction (Kaufffman, Johnson, Kardons, Liu, \& Peske, 2002; Remillard \& Bryans, 2004). Teachers are unlikely to have the time and all of the necessary expertise to design a rigorous curriculum evaluate the appropriateness of a vast array of curricular options in order to select and cobble together a curriculum. Prior research has established that the ways in which teachers evaluate the potential and quality of curricular materials varies with teacher pedagogical content knowledge and teacher experience (Ben-Peretz, 1990). Absent more specific guidance on curricular selection, teachers' selections of curricular materials may be excessively influenced by teachers individual preferences, what they believe will be "fun" and engaging classroom activities, or what they believe their students are capable of learning based on their demographic characteristics (Ball \& Feinan-Nemser, 1988).

Within school, common materials can be shared and used by teachers during collaboration and school-level PD. In one of the primary theoretical lenses for understanding teacher learning and practice as situated in a school setting - the communities of practice perspective - a shared body of materials or resource repertoire is an essential element of a community of practice (Wenger, 1998). School level teacher collaboration or teacher professional learning communities have in some studies been associated with greater student achievement (Gallimore, Ermeling, Saunders, \& Goldenberg, 2009; Goddard, Goddard, \& Tschannen-Moran, 2007; Ronfeldt, Farmer, McQueen, \& Grissom, 2015). However, schools where teachers exercise independence in their selection and implementation of curricula have experienced much greater difficulty in establishing productive teacher collaboration routines than those with greater within-school curricular coherence (Fernandez, 2002).

Within District, common curricular materials provide a commonly available resource for districts to design PD around. Also, common curricular materials and pacing helps to minimize disruption for students with high mobility rates, who move between schools (Williams, 1996).

In addition, there are potential isuses of inequity that may be magnified without common curricular materials across a district. There is much literature which suggests that important measures of teacher quality (including years of experience) are not equitably distributed between schools, with less experienced teachers more likely to teach schools with disproportionate numbers of at-risk students (Boyd, Lankford, Loeb \& Wyckoff, 2003; Clotfelter, Ladd, \& Vigdor, 2004). In the absence of district-level guidance and mandates which might represent attempts to establish coherence in curriculum and instruction, stakeholders at the school level can
be given a substantial degree of latitude to innovate and establish a coherent approach to curriculum and instruction most appropriate to the school-level context (Bryk et. al 2010). Indeed, since the 1980s, decentralization of educational decision-making has been a prominent theme in education reform in the US (Murphy \& Beck, 1995), but also globally (Astiz, Wiseman, \& Baker, 2000). However, while in some settings, devolution of decision-making to the schoollevel has been associated with overall gains within a district, these gains are not distributed equally across all schools; even while student learning may have increased in the aggregate, in school communities with low rates of social capital and high rates of students at risk of failure, student performance often stagnated or declined under decentralization policies (Bryk et al, 2010)

At the same time, teachers' utilization of curricular resources does not occur in a vacuum, but is influenced by the organizational context. Specifically, district-level leadership and policies profoundly influences the availability and utilization of curricula in the classroom (Remillard, 2005). Indeed, districts' curricular policies are one key part of their efforts to create a coherent instructional system, reflecting districts' explicit goals and design for student learning (Cobb \& Jackson, 2011). In this way, the district-level goals and design for student learning are likely to strongly influence the coherence of the enacted curriculum (see Figure 1).


Figure 1: Remillard (2005) Framework of components of teacher-curriculum relationship (p. 235).

## Methodology

## Sources of the Data

The data for this research project came from the Middle School Mathematics and the Institutional Setting of Teaching (MIST) project, which aimed to understand how large urban districts bring about instructional change in mathematics at the district level. Each year (20072015) MIST collected data from the districts, two districts from 2007-2011 and two districts from 2007-2015. For this study we used data from District D only, years 2008-2014, three years before the district implemented CCSM and three years after the district implemented CCSM.

We chose this district because it was the one of the four districts that we followed before and after CCSM adoption. District D is a large, urban district in the United States.

The data for this study came from three primary sources 1) teacher interviews, 2) district leader interviews, and 3) MIST project documents that synthesized the project data, including interviews and surveys, each year.

## Teachers

The participants for the study included the teachers from seven randomly selected middle schools in the district, which was about a quarter of the middle schools in the district. Within those schools we randomly selected five mathematics teachers to participate. Because we wanted to keep our average at five teachers per school, at some schools there were a few more or a few less depending on teachers' willingness to participate. For the six years of this study, we coded 192 unique teacher interviews, with an average of approximately 33 teachers interviewed per year of the study.

The primary data is from teacher interviews. The interviews, which were approximately 40-50 minutes, asked teachers about their experiences in the district that year. The questions primarily focused on gathering data about district initiatives aimed at improving middle school mathematics.

## District leaders

In addition, to understand the district perspective, this study used interviews with district leaders. We interviewed district leaders who were in charge of academics twice per year. This included the head of academics for the district and most of the assistant superintendents, as well as the head of curriculum and instruction and the district-level support specialists for mathematics. Each interview lasted about 40-50 minutes and each time we interviewed 5-13 district leaders, with the average number of district leaders being nine. For the purposes of this study we used 103 unique district leader interviews.

## Synthesis Reports

In fall the goal of our interviews with district leaders was to understand the district's theory of action in bringing about change in their district in middle school mathematics that year. After interviewing the district leaders about their strategies and plans for bringing about change in middle school mathematics, we coalesced their ideas into one organized document called District D's Theory of Action and we shared it with them for feedback to ensure that it captured their year's goal(s) in middle school mathematics and their strategies for reaching those goals. If they agreed that it did capture their goals and strategies for the coming year, then we used it to design our interview protocol for interviewing teachers in the winter. If we received feedback that it needed to be changed (usually just a tweak in wording or a misunderstanding of a piece of a strategy) we changed it to align with their thinking. Our goal with the Theory of Action was to synthesize and understand their thinking and district plans for the year - including the goals and strategies - for what and how to improve middle school mathematics in their schools. For more information on Theory of Action see Cobb and Jackson, 2011.

In late winter, we interviewed the district leaders again to ask how their strategies and plans as shared in the fall had worked out. At that same time, we interviewed teachers (which are part of the data included in this study), principals, and instructional coaches to understand what had happened in the district that academic year in relation to the plans the districts leaders
had laid out in the fall. These interviews, along with some other data including a survey, were amalgamated into a document called the District Feedback and Recommendation Report, which was shared with the district in the late spring.

The annual Theory of Action and the District Feedback and Recommendation reports, with their synthesis of all role group interviews, provided a global picture of the district useful to this analysis.

## Analysis

Qualitative
The teacher interviews were used as the primary source of data to investigate how teachers' talk about their curriculum use changed over the course of time, three years before and three years after CCSM adoption in the district.

To answer our research questions, we coded, using NVivo, the teacher interviews to gather data on teachers' use of curriculum resources (e.g. textbooks, workbooks, internet sites), what types of district support were provided for the use of curriculum resources, how the teachers interpreted the district support regarding curriculum resources, and how those resulted in change of curriculum resource use over time at the district, school and teacher level (three years before CCSSM implementation and three years after).

To investigate the relationship between district leader goals and curriculum use at the school and teacher level, we coded the Theory of Action reports for each of the six years for the districts' middle school mathematics goals and the strategies the district said they would use to support teachers' curriculum resource use. The annual District Feedback and Recommendation report was coded for how the district goals were understood by the teachers and realized by the district in relation to curriculum resource use.

## Quantitative

In this analysis, we sought methods to describe the variation in curricular resources cited by teachers and how this variation changed over time. We found two approaches to be particularly helpful and appropriate for this analysis: a series of histograms illustrating frequencies, and a measure of within school variation in curricular resources.
Tabulating frequencies was straightforward: we read through teacher interview transcripts and recorded their responses to the question in the interview protocol about which curriculum resources they used to teach their regular mathematics courses. While middle school mathematics was the focus of this study, teachers selected for participation often also taught classes outside the scope of this study, such as ninth grade Algebra I for advanced students or support math classes which students needing remediation or extra assistance would take in addition to their mainstream sixth, seventh, or eighth grade math class. Teachers responses about curriculum for these kinds of classes were omitted for this analysis. While the tabulation of results was in general fairly straightforward, it did require some familiarity with the teachers' resources, especially to identify when the same curricula was referred to using slightly different terms (e.g. "Connected Mathematics Project" is also referred to as "CMP"). Additionally, this task also required some decisions regarding meaning and categorization, specifically in regards to aggregating responses referencing "websites" "the internet," "online", and "Google" under one broad category, while individual, specifically named
websites (e.g. illustrativemathematics.org, teacherpayteacher.com) were each assigned their own category.

Because two of our schools (Cottonwood and Aspen) dropped out of our sample in Year 7 of our study, we decided that when constructing measures of diagrams which illustrate change over time, we would either exclude these from analysis or treat them as a separate sample, so that changes over time would not be confounded with changes in the sample of participating schools.
A measure of within-school qualitative variation: A Minimal Commonality Index
Given the potential benefits of teachers utilizing the same curriculum - especially within a school - we sought an appropriate measure of within-school, between-teacher variation in curricular resources cited. The data for this analysis is categorical, and there are a number of measures of categorical or qualitative variation used in both social and life sciences to describe diversity, concentration, and isolation. Because we felt that one of the most important likely benefits of shared curricular resources at the school level was its potential to facilitate collaboration, we felt it appropriate to draw upon measures developed particularly to describe linguistic diversity, especially given that they are often used to make inferences about the degree or frequency to which beneficial communication between individuals in a given population can and does take place.

However, the structure of our data was not amenable to most of these well-established measures of categorical variation or diversity. In applications of most of these kinds of measures, individuals typically belong to one and only one mutually exclusive group. However, when teachers in our study responded to the interview protocol question regarding curricular resources utilized, their responses cited not only one particular resource, but more often a set of multiple curricular resources. Consequently, we adapted a measures commonly used to describe population diversity to describe our data, referred to as the ethnolinguistic fractionalization variable (ELF) ${ }^{1}$, which can be interpreted as the probability that two randomly selected individuals from the population are from the same ethnolinguistic group and can speak and understand the same language (Alesina, Devleeschauwer, Easterly, Kurlat, \& Wacziarg, 2003).

Some research using this measure has found evidence to support the hypothesis that higher rates of mutual-intelligibility in a population might be associated with improved social cohesion, effective governance, and economic growth (Easterly, Ritzen, \& Woolcock, 2006). In a similar way, we suggest that it might be reasonable to believe that the use of common curricular materials among teachers might, all things being equal, contribute to more frequent and productive collaboration between teachers. However, in adapting the measure for this analysis, instead of constructing a measure of diversity, we elected to formulate a measure of similarity or commonality, which could be interpreted as the likelihood that two math teachers

[^0]selected randomly from a given school would share at least one common curricular resource in their self-reported sets of curricular resources they report utilizing.

In order to describe this measure more formally, we define, for teacher $t$ in school $j$ in year $y$, set $\mathrm{C}_{\mathrm{t} 1, \mathrm{j}, \mathrm{y}}$ as a set of curricular resources the teacher $t l$ cites utilizing in response to the interview protocol question $\qquad$ . For example,

$$
\begin{equation*}
\mathrm{C}_{\mathrm{t} 1, \mathrm{j}, \mathrm{y}}=\{\text { "Connected Mathematics Project", "Engage NY website", ...etc. }\} \tag{1}
\end{equation*}
$$

In school $j$ in year $y$, let variable $s$ be defined as equal to 1 if, in year $y$ interviews for teachers $t 1$ and $t 2$, there is at least one curricular they both cite using, with $s$ equal to zero if there is no curricular resource they both cite using. More formally,

$$
s(t 1, t 2, j, y)=\left\{\begin{array}{l}
1 \text { if } C_{t 1, j, y} \cap C_{t 2, j, y} \neq \emptyset  \tag{2}\\
0 \text { if } C_{t 1, j, y} \cap C_{t 2, j, y}=\varnothing
\end{array}\right.
$$

Where there are a total of $n_{j, y}$ math teachers participating in our study in school $j$ in year $y$, the probability that any pair of participating math teachers chosen at random share at least one curricular resource in common, can be calculated using the following expression:

$$
\begin{equation*}
\frac{2\left(n_{j, y}-2\right)!}{n_{j, y}!} \sum_{t 1=1}^{n_{j, y}-1} \sum_{t 2=t 1+1}^{n_{j, y}} s_{t 1, t 2, j, y} \tag{4}
\end{equation*}
$$

which is the average value of $s$ over all possible pairs of math teachers in school $j$, where

$$
\begin{equation*}
\frac{2\left(n_{j, y}-2\right)!}{n_{j, y}!} \tag{5}
\end{equation*}
$$

is the multiplicative inverse of the number of combinations of math teachers in school $j$ in year $y$, taken two at a time. We can then aggregate this measure from the school-level to approach the district level, characterizing the subsample of $k$ schools in the district participating in this study, using the following expression:

$$
\begin{equation*}
\frac{1}{k} \sum_{j=1}^{k} \frac{2\left(n_{j, y}-2\right)!}{n_{j, y}!} \sum_{t 1=1}^{n_{j, y}-1} \sum_{t 2=t 1+1}^{n_{j, y}} s_{t 1, t 2, j, y} \tag{6}
\end{equation*}
$$

In this expression, each participating school is weighted equally in constructing this measure, regardless of variation in the total number of participating math teachers across these schools. This measure does have some limitations. We had to eliminate from consideration vague and general responses (e.g. "the internet", "teacher created materials", "things I pick up here and there"), although teachers who cited these ambiguous sources also generally included more specific materials in their responses. Furthermore, this method of identifying the probability of cases where teachers share at least one curricular resources in common does not take into account the potential diversity within a given teacher's self-reported list of materials. For example we might expect that two teachers who both cited "Carnegie Math" and only Carnegie Math would represent a case which, everything else equal, would be more likely to realize some of the benefits of curricular uniformity. In comparison, a case where one teacher utilizes only Carnegie Math, while for the other teacher, Carnegie Math is only one of five curricular resources utilized might be relatively less likely to benefit from all of the potential benefits of a uniform curriculum in the same way or to the same extent. However, these case are treated the same, for the purposes of this measure. For that reason, we might consider the statistics generated by this measure to be interpreted as a relatively liberal measure of commonality or commonality of resources. For this reason, in describing these findings, we will describe the statistic generated by this feature as a minimal commonality index (MCI).

## Findings

## Quantitative

## Frequencies

The pattern revealed by the series of histograms is striking. In Years 2 through 4, no other curricular resources besides Connected Mathematics Project (CMP) were mentioned Even excluding non-specific broad categories of curricular materials (e.g. those referencing websites/Google/internet/online, teacher created materials, or other unspecified supplemental resources), the number of unique curricular resources increases from to 5, 11, and 30 in Years 5, 6 , and 7 , respectively.

Some curricular resources most frequently mentioned are worth considering individually. For example, review of teacher interview transcripts revealed that College Preparatory Mathematics (CPM) was a text used at the high school level prior to CCSS-M adoption. With the adoption of CCSS-M some of the learning standards which had previously been addressed in ninth grade algebra were now slated to be taught in eighth grade. As a response, teachers teaching grade 8 mathematics began using both CPM and CMP to address the new standards for their grade levels. For these reasons, we see the number of teachers in our sample utilizing CPM rising in our sample from zero percent in Years 2 through 4 to 25 percent in Year 7. CMP, adopted districtwide prior to CCSS-M adoption, is cited as a curricular resource by over 90 percent of teachers interviewed in Years 2 through Year 6. Although it is still the most frequently cited curricula in Year 7, only 64 percent of teachers cite using it.
Over the six years of data, another source of curricula which shows the most longitudinal growth in terms on the proportion of our sample utilizing it is the broad category encompassing teachers references to "Google", "websites", "the internet", or "on line". By Year 6, a quarter of all teachers mention this vague resource; one year later, half of teachers in our sample reference this broad category as a source for curricular material

## Looking at central tendency measures

While the growth of a "long tail" over time in this frequency distribution is dramatic, it is worth noting that while the sample of schools in the histogram sample is constant over time, the sample of teachers was not constant overtime. As a consequence, it may be that the introduction of a large number of curricular resources in these later years could be due to one or two teachers who may be particularly enthusiastic about drawing from a large number of diverse resources. An investigation into central tendency statistics of the yearly distributions of the number of curricula referenced by each teacher reveals that, in general, teachers in this sample are, overtime, referencing more curricula in response to this question; for example, by Year 7, the median number of curricula cited by our participants in reference to our question is three, with the average number being slightly higher. This suggest that, as a whole, teachers in our sample are coming to rely on more numerous and more diverse resources, especially when compared to patterns of curricular usage prior to CCSS-M adoption (i.e. Years 1-3).
Quantitative Description: Minimal Commonality Index
Our application of the Minimal Commonality Index (MCI) applied to our longitudinal data of our seven participating schools revealed some district patters. Prior to CCSS-M adoption, in Years 2 through 4, no schools deviated from a MCI score of 1.0. In other words, in each of these schools in those years, the set of curricular materials utilized by each participating
teacher shared at least one element in common with the respective set of curricular materials cited by the other participating teachers. When we compare this figure with the histograms generated earlier, the situation becomes clear: everyone cited utilizing Connected Mathematics and only Connected Mathematics.

However, after CCSS-M adoption, schools began to take on different trajectories. Two schools, Elm and Hawthorn, maintain a MCI score of 1.0 through CCSS-M adoption. Otherwise, the MCI score for our other participating schools all decline at least once beginning in year 5. All but Elam and Hawthorne show declines in their last year of participation in our study, which is Year 6 for Magnolia and Aspen, and Year 7 for Cypress, Birch, and Laurel.

We caution that while we have adapted diversity indices like the ethnolinguistic fractionalization (ELF) index in order to fashion a minimal commonality index for the purpose of describing our data in this context, we cannot make any claims about any inferences which may be drawn above and beyond its primary interpretation. While it is accurate to interpret these values as the probability that any two random participating math teachers selected from a given school utilize at least one common curricular resource - as determined by responses to a specific interview protocol question - it is not the purpose of the paper to explore any other properties of the measures (e.g. its sensitivity to sample size, confidence intervals when making inferences about the larger population from which the sample is drawn, etc). However, we view it as a helpful manner to quantify the diversity in teachers' curricular choices within a school, and to provide a complement to and starting point for the qualitative analysis which follows.

## Qualitative

Before Common Core - Study Years 2-4
Pre-CCSSM th district was saying that their goals for middle school mathematics were "to support teachers' development of inquiry-based instructional practices that engage students' natural curiosity, develop deep understanding of mathematics concepts, and emphasize critical thinking and problem-solving skills that entail high-level thinking" (District D, Theory of Action, Fall 2008, Fall 2009). To do this their primary strategy in these pre-CCSSM years focused on successfully implementing CMP2 in all middle schools (year 2), improve CMP2 implementation (year 3 and 4) (District D, Theory of Action, Fall 2008, 2009, 2010).

As Figure 2 illustrates in year 2-4, the years before CCSSM adoption, teachers who answered the question regarding which resouces they used to teach mathematics, all answered Connected Mathematics for their regular mathematics courses. In fact, the curriculum program that they were using, appeared to be synonymous for some teachers with the class. When asked what she taught Cindy replied, "I teach one reading class, two Algebra 1 classes, and two Connected Math classes" (Cindy, Cypress Middle, January, 5, 2011).

First Year of Common Core Adoption - Figuring Out the Changes - Study Year 5
The very first year of CCSSM adoption, the goal for the district changed from supporting teachers to develop inquiry-based instructional practice to engage students to "ensur[ing] that all students receive instruction that is aligned with the new Common Core standards and that all students meet learning goals measured by student achievement on state assessments" (District D, Theory of Action, Fall 2011). The district is still focused on instruction and their primary strategy for meeting that goal was to "support teachers to improve instruction" but the methods for doing that were not defined within their goals and the defintion of good instruction had more leeway compared to when the district goal had been
synthesized into "inquiry-based instructional practices that engage students' natural curiosity, develop deep understanding of mathematics concepts, and emphasize critical thinking and problem-solving skills that entail high-level thinking." This pre-CCSSM goal provided a picture of the type of instruction the district was hoping to encourage pre-CCSSM. In contrast "improve instruction" does not define what type of instruction is valued by the district. This switch in district views and the new vagueness is evident to the teachers.

In all schools in District D, the Connected Mathematics Project (CMP) textbooks were consistently cited as the primary classroom resource for teacher interviewed, prior to Common Core implementation (i.e. Study Years 2-4). The first year of common core implementation in the district, year 5 of the study, all teachers reported still using CMP as a resources for teaching mathematics, however the district-designed curriculum guides moved some CMP units to different grade levels because CCSM moved topics to different grade levels compared to the district's previous standards. Teachers spent time figuring out the changes and trying to adjust in district-designed professional development (PD), schoolbased math teams and on their own.
...at the beginning of the year, [math department] meetings focused on the [curriculum guides] and... looking at this is what 6th grade's covering now, this is what 7th, this is what's 8th so that we have a good idea, especially with the new standards coming out, we kind of didn't know. We said, "Well hey, I used to teach this. Why don't I teach it anymore?" Well, that's in this grade now. (Samantha, Aspen Middle, January 4, 2012)

The district provided curriculum guides that year that mapped out the required sequence of Common Core mathematics standards, along with pacing, and noted only the units within CMP books that would help teachers meet the standards. Also, some teachers felt that the CMP units didn't address a standard or covered a standard that was now in a lower grade level, which the curriculum guides did not acknowledge. "[We use the curriculum guides to decide]... how long it should take, what are the key standards, and, you know, "Well, what if the book doesn't cover the standard?" - we were talking about that today" (Winona, Elm Middle, January 9, 2012).

A few teachers expressed frustration at this lack of one coherent resource and the missing support for figuring out this new system. One teacher sums it up as a patchwork curriculum.

Our curriculum is kinda, here's the standards and make sure you're meeting 'em kinda thing. Whereas in the past it's been, do this book, do this lesson, do this book, do this lesson. Now it's kinda, big change, yeah. Kinda like they let the reins free, but yet I don't feel like I have a lot of support on what I need to be doing. I feel like I'm, it's, 'Where's Waldo? Where should I start now? Where should I go?'

Interviewer: Yeah. So I'm wondering if you use [district] tools when you're planning?
Teacher: I just use the curriculum [guides].... they give me, kinda the standards that we should be hitting and suggestions for books and lessons that we should do. Like, oh you might wanna go to Brain Pop and use this lesson or you might wanna, do this page in the workbook or you might wanna do a supplemental page out of the CMP book and... you know, different things like that, suggestions, which is helpful, but I don't know. It just seems like it's such a patchwork thing. (Barbara, Cypress Middle, January 6, 2012)

Second Year of Common Core Adoption - Finding Additional Resources - Study Year 6

The district shifts away from its previous focus on instruction in year 3 in its theory of action for improving middle school mathematics. They district leaders' goal for middle school mathematics synthesizes into "implement the CCSSM, with a primary focus on the content standards, and create school-specific responsive targeting of student needs aimed at reducing gaps between the proficiency benchmark and performance of under-achieving groups of students" (District D, Theory of Action, Fall 2012). The primary strategy cited by the district for reaching these goals is increasing principal autonomy and responsibility for school performance. Only one of the three district strategies this year focuses on teachers and it focuses on developing data-drivent professional learning communities to increase student achievement. The district focus on improving teacher instruction is no longer present. There is no longer a strategy or a focus in the district's goals about how teachers' classroom practices may be improved to reach the goal of increased student acheivement. There is not a discussion at the district level about district-wide curriculum materials, or a district-wide curriculum program that may support teachers implementation of CCSSM. This new lack of focus on instruction corresponds with the district offering no district-wide mathematic- specific professional development. Similar to year one of CCSSM adoption, this lack of focus on particular curriculum matters is felt by the teachers however, as they become further away from the directive to teach CMP2, finding ones own curriculum materials to meet the CCSSM becomes a theme.

Despite a district's lack of focus on CMP2, in the second year of the district's adoption of Common Core, CMP2 is still the most commonly cited curricular material of the teachers, although not all teachers are using it. In their interview, teachers were asked how they used Common Core when planning for instruction. All teachers in this second year of adoption acknowledged planning was different because of the new standards and required additional resources.

The issue is some of our new standards doesn't really meet Connected Math (CMP) all the way, so we do, you know, pooling resources from other places, but Connected Math is the primary source. (D2, January 25,2013 )
[When planning] you say, "Okay. Well, we used to teach this, but it's no longer in our standards. So, we're gonna have to chuck that activity and move over to this one," and, you know, it's changed some of the things that we do, for sure. (D1, January 11, 2013)
[When we plan] we'll take the standard [as stated in the district curriculum guides] and then go to, to the Connected Math books and see what matches best. And then we fill in any gaps with our additional resources that we have, whether it be our workbook here or just, you know, another textbook or online resources. (D1, January 11, 2013)

This theme of teachers using online resources emerges and stays as a common thread throughout the interviews.

I use a lot of CMP, or the Connected Math Program, for my regular math and my [advanced class]. And I will be using College Prep Mathematics, or CPM, just for [my advanced class]. But, I mean, we've got online resources that we use a lot, too, where we just find something and we'll Google it. (D5, January 23, 2013)

Third Year of Common Core Adoption - Feelings of Frustration and Freedom Study Year 7
In the district's third year of Common Core adoption, the district's primary goal narrows down to "raise student achievement through the implementation of the CCSM, with a focus on the content standards" (District D, Theory of Action, Fall 2013). The strategies focus on 1) improving principals' roles as instructional leaders who use school-level data to drive decision making, 2) strengthening support for teachers' use of data through improved use of professional learning communities and instructional coaches, and 3) simplifying and streamlining periodic assessments and curriculum tools. If this goal and these strategies are compared to the preCCSSM goals and strategies, there is distict change in district focus from improving classroom instruction to instructional managment ideas such as using data and organizing PLCs. Both of these ideas while good leave undescribed what should be going on in classrooms during instructional time in order to meet the goal of raising student acheivement. This focus on student scores with a lack of definition or description about classroom practice leaves it open for teachers to determine. This lack of definition leads to feelings of frustration in some teachers, while others it provides a sense of freedom, as evident in the teacher interviews shared below.

In the district's third year of Common Core adoption teacher sstill most commonly cite CMP as a resource. However, this year even the district-designed curriculum guides moved away from using it as a resource.

Last year, [the curriculum guides] had the standards, and then there was a column that told you specifically what CPM 2 lesson went with what standard. So it was more specific, but not all the time did those really line up with the standard ver-, you know, they did the best they could to line them up with common core. And, but this year, it, there isn't that. There's a lot more freedom. So I think that's part of the adjustment that we as teachers have to make is that there, you know, it doesn't say, "Go to page 15 for Standard 6.NS.1," you know? It is - we have to find what aligns better. So the freedom's good. I like the freedom. But at the same time, it's kinda like it went from, "This is what you do," to, "Oh you can do anything." (laughs) [Diana, Cypress Middle, January 23, 2014)

As the teachers moved away from a common district textbook to finding additional materials to meet the needs of Common Core two different themes emerge within in the district's teachers freedom and frustration. Often, these emotional responses manifest within the same teacher interview. Diana who spoke of freedom above laments how it has turned her from using rich, CMP tasks to a "worksheet queen."

This year, (laughs) it's, this year really it's been very difficult trying to find things that align with the Common Core. Now I, I'm a fan of Common Core. I love the Common Core. What I don't like about it is that, you know, we all preach the textbook or whatever, and I don't even have to have a textbook. That's not the thing. It's just it is very difficult to find the resources to, you know, like the workbooks that go along with it. What I liked about CMP is that, you know, they could do a little exploring on their own,
and kinda make it more in depth, but, you know, sometimes I, I feel like I've become a worksheet queen. [Cypress Middle, January 23, 2014).

When asked what were her primary resources for teaching, Linda responded,
That's another thing that I struggle with because they quit the textbook adoptions a long time ago. And so, the textbook that I have one class set up, is a 2004 textbook. At this point it's obsolete. The last time they gave us money for textbooks was to buy CMP 2, and that also is at this point, obsolete because of the Common Core. We need new textbooks and so basically, every day I'm coming up with just whatever I can find off the internet or whatever I can find from this old textbook or stealing stuff that I'm really not supposed to use...to come up with, to come up with six weeks of material to teach something because I have no textbook. (Cypress Middle, January 23, 2014)

When asked the same question, Nora said:

Well, we were switching over to the new standards, I really had to pull things from a lot of different places. There are some really good online resources that I like to use. I use a lot of the, the SMART Exchange lessons where, you know, they go along with the SMART interactive system. And, and I also have some, some Coach and Buckle Down Common Core books that I like to use. And, then there's just some, some other websites. Like I, I go online a lot because the Connected Math, it's not aligned, at least the version that we have. I just kinda have to pull resources. (Birch Middle, January 23, 2014)

The first theme within the teacher interviews is frustration at the time and energy required to find, adapt and use curriculum materials on one's own.

Well, we don't really have a textbook we work out of with the new standards. Before, Connected Math had been used, but it doesn't really align with the standards, so we're kind of cutting it together. Myself and the other 7th grade math teacher have been using a book called Ready Common Core to kind of do some of our more textbook-like stuff, and then everything else I pull from NCTM Illustrations. I pull from Engage New York. I pull from just anything and everything, a Stations book that I found means, I hate to use the word cobbled together, but in a way it kind of is because we don't have any kind of set book. (Teresa, Birch Middle, January 23, 2014)

The second theme that emerges simultaneously is one of teachers enjoying the freedom to make these curricular decisions. However, these same teachers recognize both the frustration and creative freedom lack of a common text provides.

There's a lot more freedom, I feel like, to teach the standards. Last year with the maps they pretty much said use Connected Math section blah, blah, blah with this standard. And then I would look at it. I'm like this doesn't even go with this standard. Why would we want to use it? And I hardly ever followed that. But now they just give you the standard, give you the learning target, and it's basically use whatever you want to teach it, which I like and dislike. I like that I'm not bound down to a book that's not aligned to the Common Core. I dislike that I don't have a book that is aligned to the Common Core, and that I have to pull from all kind of different things. I mean it's time consuming. (Tyler, Birch, January 23, 2014)

The wide use of the internet by teachers as resource is evident.
I've got a few different books that I purchased what I use. There's Common Core, Math Standards, there's Hands-On Activities. And then there's Ready Common Core Math Instruction. These things I purchased myself. There's also like stuff online so EngageNY is good. We'll pull some of the Connected Math lessons and then there's some, some illustrations online that we can get to through the, the [district] website. I've got a whole bunch of links saved. We use a lot of Brain Pop, the little quick Brain Pop videos and the quizzes that go along with those. I've got Education.com, Illuminations Lessons, Concourse Cheats, Illustrative Mathematics.org, CPM, IXL. [Jack, Birch, January 23, 2014).

The district-wide trends by year (Figures 2 and 3) demonstrate that in the first year of CCSS-M implementation (i.e. Y5) all teachers still use CMP as a primary resource, however, as the teachers moves forward from figuring out the new standards in year one, they realize they need different resources and are given more freedom to find those resources by the district, as evidenced by the districts mention of different resources on the curriculum guides.

## School-level Data

In our study schools, the school district responded to CCSM implementation by moving from a prescriptive approach to curricular implementation (one that specified the sequence and pacing of the learning standards to be addressed in instruction, along with the corresponding curricular materials to be utilized) to a more loosely-managed approach (i.e. where sequence and pacing of learning standards were specified, with no specification given for curricular materials to be used). Schools responded to this new organizational structure in one of three ways. In some schools, teachers and administrators established a consensus and secured the necessary resources to adopt a new curriculum school-wide. In contrast, some schools were not able to secure the consensus and resources necessary for a complete school-wide adoption of a new curriculum, but instead piloted a curriculum at a more limited scale, with this piloting sometimes occurring at only one grade level. However, X of the seven schools we followed for this study did not secure the consensus or resources necessary for either a full- or more limited piloting of a curriculum; in these schools, teachers generally worked in grade-level teams or as individuals drew upon internet resources, materials from prior textbook adoptions, and some materials
identified by instructional coaches to cobble together a set of curricular resources to address the sequence of Common Core aligned student learning goals described in the district's curriculum maps.

## Common Trend of Curriculum Use

Four schools follow the same trend of curriculum use by using a commonly shared textbook for years one through five and dropping off in years six and seven.

## Whole-school Adoption of Another Textbook

Two of the schools show that for all years of the study $100 \%$ of the teachers at the school if matched with another teacher would be using a common curricular resource or text. These two schools, D2 and D3, teachers in year two of Common Core adoption, worked with their colleagues and principal to test and purchase a new Common Core-aligned curriculum. One of these school purchased Connected Mathematics 3 (CMP3) and the other purchased Carnegie Learning Math.

Although this results in teachers within a school having a common textbook on which to focus instruction and common planning discussions it also leads to new challenges within a district that is not supporting those new textbooks.

The biggest challenge is more aligning Carnegie pacing with [the district's] pacing. Carnegie, Carnegie does it their way and [the district through the scope and sequence on their curriculum guides] does it their way (laughs) and I have to follow [the district] so that, that's a challenge in and of itself because you're hopping around to different chapters all the time. Literally you're, like chapter 15, then chapter 2 , then chapter 6 , then chapter 16. I mean it's crazy. [Bill, Hawthorn, January 21, 2014)

## Pilot Testing another Textbook

In the first year of Common Core adoption, Magnolia Middle School chose to test out a new Common-Core-aligned curriculum, called Digits. The principal bought a license for one team of teachers at each grade level to pilot. This accounted for Magnolia being the only school that showed a drop off in year five of having a common curriculum to share for collaboration.

Interviewer: Is there anything we haven't discussed that you feel is important for us to understand in your work as a middle grades math teacher in the school and the district?

Right now it's just really challenging for math just because of the new standards and not really having a curriculum to follow, but luckily we did purchase the Digits. I think that's been the, I think that's been our savior. ...Based on my PLC conversations with the 6th grade teachers, we are really enjoying Digits. [Valerie, Magnolia Middle, January 10, 2012)

## Discussion/Implications

What may be the implications of a non-district-wide curricular material coherence, meaning curriculum materials pulled from multiple sources in which are different for each teacher in each school? Does it affect student learning outcomes? The answer to this gets back to value of a coherent curriculum and why it matters.

First, content standards alone do not provide a curriculum that ensures high-quality lessons. In looking at the content standards of high achieving countries, Schmidt, Wang \& McKnight's 2005 study of data from the Third International Mathematics and Science Study (TIMSS) found that the way mathematical topics are introduced and how topics are integrated over grade levels matters to student learning. As written the CCSSM do not specify within a grade level the order. However, that being said, in 2012 Schmidt and Houang analyzed the Common Core State Standards for Mathematics and found them to be more coherent, focused and rigorous than the average individual standards states were implementing before CCSSM implementation.

As the teachers in District D noted, lessons to teach a common core standard may be in various places in the current textbook or not there at all. This is backed up by Schmidt \& Houang (2014) who found in their analysis of mathematics textbook use after CCSSM implementation, "diffuse or scattered treatment of the standards makes it difficult for teachers to construct focused and coherent instruction" (p. 61). The CCSSM include practice standards however these have not been the focus of CCSSM's adoption (Schmidt \& Houang, 2014). If how these lessons are implemented matters there needs to be a greater focus on figuring out how the mathematics topics are introduced and how to create high quality lessons. This requires time, leadership, and mathematical expertise, which may be most likely found at the district level.

Second, what students are taught and the quality of instruction matters for student achievement. Schmidt, Burroughs, Zoido, \& Houang (2015) found that the SES of students affects students' opportunities to learn (OTL) as measured by the curriculum covered and this lack of OTL through curriculum explains why students' SES correlates to test scores. This means is not the students' SES background that results in lower achievement, rather it is that a student with a low SES background is generally taught a less rigorous course of study in mathematics. Research shows teachers rely upon the given math textbook to guide instruction (CITE) and if there is not a coherent, rigorous curriculum for teachers to access to guide that instruction, how does a school or a district support teachers to provide equal opportunities to students? How do they ensure that students have equal opportunities to learn? Teachers will always adapt lessons but starting out with high-quality textbooks is more likely to result in higher-quality lessons and greater student achievement (Silver \& Stein, 1996; Stein \& Lane, 1996, Tarr et al., 2008). To some degree, when a teacher cites drawing on the search engine "Google" for curricular material, it reveals a potentially problematic dynamic: while materials found on named internet sites may range from ostensibly high-quality, vetted, and research-based material (e.g. mathsnacks.com, developed by the New Mexico State Learning Games Lab) to those with unknown quality control mechanisms and with likely a range of quality of resources (e.g. teacherspayteacher), it may be inferred that when teachers cite drawing upon "the internet" and Google to locate curricular materials for their class, that they are potentially spending a nontrivial amount of time searching for, comparing, evaluating, and selecting curricular materials
from across the entire internet. Not only does it suggest that they consider the other curricular materials they have at their disposal to be inadequate, but it seems likely that the time spent in this search and selection may not be an efficient use of teachers time, especially if teachers experiencing similar deficiencies in available curricular materials are independently undertaking this search and selection process in order to address a common problem.

Third, these finding should also have implications for policy makers. Common Core Standards in Mathematics were written to create more focused, coherent standards to guide teachers. However, knowing how mathematics teachers generally rely on textbooks as their primary resource to turn standards into lessons and that high-quality textbooks are key to highquality lesson and greater student achievement, policy makers at the national, state, and local level needed to consider the timeline for roll out of the standards. Textbooks should have been designed and purchased before standards were implemented and certainly before they were tested in a way that reflects back on teachers. Our hypothesis is that textbook companies did not want to invest in these redesigns before they were sure the Common Core would be used. However, the result has been massive amounts of pushback in some districts and states including parents refusing to let their children be tested and teachers protesting Common Core aligned standardized tests (Cassidy 2015; Strauss, 2015; Ravitch, 2015). Before implementing new standards at a statewide scale there should be supports in place to guide teachers, parents and students. This issue is not entirely in the past as states and districts are still figuring out best ways to implement the CCSSM. Plus, we are hoping that errors of CCSSM implementation serves as a learning opportunity for policy makers - how will you support teachers, students and parents to understand new policy and how will you support teachers to teach in the ways that you are dictating before accountability measures are applied.

Figure 2: Proportion of teachers in reporting using a given curricular resource; in some years, teachers report utilizing more than one curriculum/curricular resource, such that proportions do not always sum to 1.00 . Sample limited to teachers in schools participating study Years 2 through 7 (i.e. Cypress, Elm, Hawthorn, Birch, \& Laurel)







Figure 3: Proportion of teachers in reporting using a given curricular resource; in some years, teachers report utilizing more than one curriculum/curricular resource, such that proportions do not always sum to 1.00 . Sample limited to teachers in the two schools exiting the study in Year 7 (i.e. Magnolia and Aspen)




Figure 4: Statistics for the number of curricula/curricular resources teachers indicate using in interviews (minimum, maximum, median, mean) by year for participating teachers in five schoos from Years 2 through 7 i.e. Cypress, Elm, Hawthorn, Birch, \& Laurel) (n(Y1-7)=22, 21, 22, 23, 25, 28).


Figure 5: Calculations of the Minimum Commonality Index for participating schools in our sample.


Note: Cottonwood and Aspen drop from sample after study yr 6; Elm and Hawthorn maintain a 1.0 value study_yrs 2-7

Table 6: By school, by year calculated values of our Minimal Commonality Index score, along with the by-year, by-school simple size of teachers from whose interview responses we drew to calculate the scores.

|  | Year 2 | Year 3 | Year 4 | Year 5 | Year 6 | Year 7 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | index | index | index | index | index | index |
|  | n | n | n | n | n | n |
| Cypress | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | $\mathbf{0 . 5 8}$ |
|  | 3 | 2 | 4 | 5 | 6 | 9 |
| Elm | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
|  | 3 | 4 | 3 | 5 | 5 | 6 |
| Hawthorn | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
|  | 3 | 4 | 3 | 3 | 4 | 2 |
| Birch | 1.00 | 1.00 | 1.00 | 1.00 | $\mathbf{0 . 6 0}$ | $\mathbf{0 . 4 0}$ |
|  | 4 | 4 | 4 | 5 | 5 | $\mathbf{5}$ |
| Magnolia | 1.00 | 1.00 | 1.00 | $\mathbf{0 . 8 0}$ | $\mathbf{0 . 7 0}$ |  |
|  | 3 | 5 | 2 | 5 | 5 |  |
| Laurel | 1.00 | NA | 1.00 | 1.00 | 1.00 | $\mathbf{0 . 7 0}$ |
|  | 3 | 1 | 4 | 5 | 5 | $\mathbf{5}$ |
| Aspen | 1.00 | 1.00 | 1.00 | 1.00 | $\mathbf{0 . 1 3}$ |  |
|  | 5 | 3 | 5 | 4 | $\mathbf{6}$ |  |
|  | $\mathbf{2 4}$ | $\mathbf{2 3}$ | $\mathbf{2 5}$ | $\mathbf{3 2}$ | $\mathbf{3 6}$ | $\mathbf{2 7}$ |

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[^0]:    ${ }^{1}$ However, the ELF is a measure of diversity, it can be considered part of a family of very similar measures of population diversity or sameness, which can be interpreted is the probability that a pair of individuals chosen randomly from a sample are in different (or in some statistics, the same) category. These measures often take different names in different academic disciplines. The approach to calculating the Simpson Index, the GiniSimpson Index, Blau's Index, the Gibbs-Martin index, and the Herfindahl-Hirschman Index all belong to this family of measures, although these measures often differ as to whether they are measures of concentration or diversity (i.e. the probability of encountering similarity versus the probability of encountering difference) and whether the pair of individuals sampled from a population at random are sampled with or without replacement. For a discussion, see Patil and Taillie (1982)

