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Attending to Precision in Statistics

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Abstract

The Common Core Mathematical Practices are processes and proficiencies that mathematics educators strive to develop in their students. In this article, we present a case study of three teachers, that examines the prevalence of Mathematical Practice 6: Attending to Precision, in a professional development statistics course for in-service teachers largely focused on open-ended activities. We also illustrate how the elicited practice, Attending to Precision, may differ in the context of statistics as compared to mathematics.

Keywords: statistics education, professional development, common core, mathematical practices, teacher preparation

Attending to Precision in Statistics

Based on the NCTM process standards (NCTM, 2000) and the strands of mathematical proficiency specified in the National Research Council's report Adding It Up (2001), the Mathematical Practices (MPs) are "processes math educators at all levels should seek to develop in their students" (NGA, 2010). The MPs are also closely aligned with Polya's steps to problem solving (Polya, 1945) and the Mathematical Habits of Mind (Cuoco et.al, 1996) that describe processes or methods involved in mathematical problem solving and reasoning. Since the MPs are problem solving procedures to be implemented by students in the classroom, both researchers and teachers alike find value in considering ways to better engage students in problem solving and the MPs (Marrongelle, Sztajn, & Smith, 2013). To better prepare teachers to engage students in the MPs in their own classrooms, it is important for teachers themselves to have time to use MPs in their own mathematical problem solving.

In this paper, we study how MP6: Attending to Precision, appears in a series of statistics professional development (PD) workshops for teachers. In particular, we focus on how teachers interact with MP6 in the context of statistics. Using observations and video of the PD, teacher written work, and voice recorded reflections as evidence, we give a descriptive analysis of accounts where MP6 occurred in the PD and document evidence of how this practice appears in a statistical context compared to a mathematical context. We then reflect on the results of this case study analysis and note the implications of this examination on mathematics teacher preparation and PD as well as K-12 education.

Why Statistics?

Along with algebra and geometry, the Common Core State Standards in Mathematics (CCSSM) places a large emphasis on statistics particularly in the middle and high-school grades. The middle grades standards focus on understanding variability and sampling variability while the high school standards largely focus on bivariate data analysis. Although statistics has been included as an important branch of the CCSSM (NGA, 2010), the Mathematical Education for Teachers II (CBMS, 2012) and the Statistical Education of Teachers (SET) report (Franklin et al, 2015) both note that statistics is a large area of need for teacher preparation and PD. As teacher preparation and professional development begin to tackle preparing teachers to teach the statistics content, teacher educators must consider how teachers will engage in the MPs within the statistics content domain.

The Guidelines for Assessment in Statistics Education K-12 Report (GAISE) outlines the statistical content that should be taught in primary and secondary schools (Franklin et al, 2007). GAISE discusses statistics as a problem solving process that includes: formulating a question, collecting data, analyzing data, and interpreting results. While thinking statistically, one progresses through this process. Statistical thinking has been defined as the type of thinking that statisticians use when approaching or solving statistical problems. Statistical thinking has been described as understanding the need for data, the importance of data production, the omnipresence of variability, and the quantification and explanation of variability (Cobb, 1992). Statistical thinking has a focus on variability in data that sets it apart from mathematical thinking (Franklin et al., 2007).

Several papers have linked how the GAISE report can inform the teaching of statistics content proposed in the CCSSM (e.g., Groth & Bargagliotti, 2012). Groth 2013 also links statistics to Mathematical Knowledge for Teaching (MKT) by offering a map of Statistical Knowledge for Teaching (SKT). Similarly to MKT, SKT includes common and specialized knowledge that teachers need to support students' learning. In statistics, this knowledge must be connected to the statistical problem solving process put forth in GAISE and make distinctions between statistical thinking and mathematical thinking. While there are many obvious connections between mathematics and statistics, there are some, more subtle, differences between the two disciplines (Hannigan, Gill, and Leavy, 2013). However, as mathematics teachers in K-12 primarily teach statistics, it is necessary to consider how the MPs play out in the statistics content domain. Teacher preparation and PD in statistics thus should encourage the GAISE framework, the development of teachers' SKT, and the engagement of teachers in the MPs.

MP6: Attending to Precision

In Mathematics, Attending to Precision focuses on precision in mathematical communication. Demonstrable attributes of this practice involve students trying to use clear definition and clarifying their reasoning. Also when eliciting this practice, students calculate accurately and efficiently, with particular attention to precision appropriate for the problem context (NGA, 2010). Precision in mathematical problem solving has also been seen as being precise about mathematical language used (Cuoco, Goldenberg, & Mark, 2010).

The SET report, commissioned by the American Statistical Association (ASA), has a chapter dedicated to interpreting the MPs under a statistical lens. The following presents the report's statistical interpretation of MP 6:

> **6. Attend to precision.**
> Statistically proficient students understand that precision in statistics is not just computational precision. In statistics, one must be precise about ambiguity and variability. Students understand that the statistical problem-solving process begins with the precise formulation of a statistical question that anticipates variability. Precision is also necessary in designing a data collection plan that acknowledges variability. After the data have been collected, students are precise about choosing the appropriate analyses and representations that account for the variability in the data. As students interpret the analysis of the data, they are precise with their terminology and statistical language. Students recognize that the precision of this estimate depends partially upon the sample size – the larger the sample size, the smaller the margin of error. As students interpret statistical results, they connect the results back to the original statistical question and provide an answer that allows for the variability in the data.

As mathematics teachers are charged with teaching both mathematics and statistics, it is important for teachers to gain a deep understanding of the underlying and foundational concepts of both the content areas. Although MP6 is important in both math and statistics, the way students exhibit this practice in the two domains may differ. For example, students when rounding a number to a certain place value may exhibit mathematical precision, whereas students may exhibit statistical precision as they select an appropriate sample and sampling method for their study. Teachers need to be made aware of such potential differences in the content domains. Thus, we look at how teachers employ MP6 while working on statistics.

Case Study Design

To examine teacher use of MP6 in statistics, we focus on a case study of three teachers who participated in a thirty-hour professional development (PD) course for secondary math teachers focused on statistics. The PD was delivered as a graduate course using materials developed through Project-SET. Project-SET was a NSF-funded project that aimed to develop innovative curricular materials to enhance teachers' statistical knowledge for teaching (SKT).

The materials and the PD focused on the teaching and learning of two fundamental topics in statistics— sampling variability and regression. Several studies have shown that teachers have trouble with the concept of variation (Hammerman & Rubin, 2004, Confrey & Makar, 2002, Makar & Confrey, 2004, Bargagliotti, & Franklin, 2013) and regression (Casey & Wasserman, 2015, Casey, 2010), and both topics are fundamental in the study of K-12 statistics in the CCSSM (Franklin et al., 2007, Burrill & Biehler, 2011; Garfield & Ben-Zvi, 2004). As part of the project, Project-SET developed a PD program. Two iterations of the PD were carried out—this study draws from the second iteration of the PD.

Professional Development. The PD was delivered in twelve 2.5-hour class meetings. During each class meeting, teachers worked on a series of open-ended tasks, which often involved the use of dynamic statistical software (the PD used StatCrunch), and took the majority of the class time to complete. Although the focus of the PD was on sampling variability and regression, the activities encouraged teachers to engage in a wide range of other statistics concepts along the way in a deep, rather than broad, approach to the learning of statistics as recommended by the CCSSM and GAISE frameworks (Bargagliotti et al., 2014). The PD also addressed pedagogical knowledge by discussing students' thinking, as well as current research and approaches to the teaching and learning of statistics in K-12. In this study, we focus on the way MP6, Attending to Precision, was utilized by the in-service teachers to complete the Project-SET open-ended statistics tasks.

Participants. Four teachers took the PD. Three of the teachers were full time high school teachers teaching in public schools and the fourth participant was an international student studying to become a teacher. The three full time high school teachers were those included in this case study. Only one of the full time teachers had ever taught statistics before but had done so five years prior to taking the Project-SET PD. The other two teachers had begun implementing the CCSSM within their classrooms and thus had taught some statistics units throughout the year, however, neither had ever taught a full course in statistics.

Documentation of Evidence of the Use of MP6. To understand how teachers interact with Attending to Precision in the context of statistics, we examine three types of evidence from the PD— (1) teacher written work on in-class activities and assessment tasks, (2) video of the PD, and (3) voice recorded reflections made by the participants at critical points in the PD.

Video. To understand how participants engaged with MP6, author 2 observed the PD with the intent to monitor the use of mathematical practices in problem solving. Author 2 notated instances of teachers demonstrating attention of precision or lack thereof as defined by the SET report. Subsequently, those instances were viewed in the video recordings by both authors and then transcribed. This enabled the authors to create an "inventory" of instances where Attention to Precision arose in the PD as well as the frequency of such instances. The videos provided a means to confirm and elaborate on the observed instances of MP6 in the PD made by author 2.

Written Work. Throughout the PD, the participants were assessed using the open-ended assessment tasks designed by the Project-SET team to gauge whether teacher SKT progressed. The assessment tasks were adopted and elaborated on from items of the Illustrative Mathematics™ Project (IMP, www.illustrativemathematics.org). Items from the IMP underwent a double-blind review process before being released on the website. The Project-SET team chose released items that discussed aspects of sampling variability and regression and then developed assessment tasks around them. The team opted to use the IMP items to ensure that the materials were aligned to existing, well-agreed-upon expectations. In addition, because the IMP tasks are directly aligned with the CCSSM, the results gave insight into whether the teachers were prepared to teach the concepts in the CCSSM.

Throughout the PD, 10 open-ended individual activities were given. For each activity, the instructor for the PD (author 1) gave a brief introduction to the material, and then participants were given approximately 120 minutes to problem solve and complete the activity. All work and sample creations were done with the aide of statistical software. After, the teachers completed the activities; they shared their responses with the group.

Author 2 analyzed each of the 10 in-class activities for alignment with the CCSS mathematical practices. Also, author 2 rated and coded each of the 10 in-class activities for depth of knowledge (DOK) level (Webb, 1997). The DOK framework allowed the authors to assess the cognitive rigor of the activities in terms of the context of the task. Thus, the teacher's cognitive ability level is not being assessed but instead the level of engagement the tasks provide were being notated. The activity used in the PD ranged from DOK level 1 to level 3.

The activities were the main focus of the PD program and the teachers spent the majority of class time working through the activities, typically one per class period. Thus many of the instances of Attending to Precision notated by Author 2 could be directly mapped on to specific activities eliciting the practice. Teacher work on the activities was collected, photocopied, and handed back to the participants at the end of each class period.

Voice Recordings. At the end of each of the two major units of sampling variability and regression covered in the PD, the participants were asked to reflect on their understanding of the topics covered in the course. In particular, the teachers were asked to record their answers to the following questions: (1) what are the big take-away ideas of what you have done so far? Are there any open questions that you want answered immediately? (2) What math practices have you engaged in during this process? If possible, give examples. (3) Which math practices did you find yourself engaging in the most and why? To do this, teachers had 15 minutes of class time to think through their answers and then 5-10 minutes to make a voice recording of their answer. These voice recordings served as a means to document teacher perceptions of their own understanding of the tasks and the practices. All voice recordings were then transcribed.

Results

The case-study results showed that three "types" of precision naturally emerged within the PD that matched some ideas articulated in the SET report— (1) precision in quantifying variation, (2) precision in language, and (3) precision in explaining variation. Note, this study was conducted prior to the release of the SET report so these findings suggest that the description of MP6: Attend to Precision given in the SET report can in fact be observed in a statistics classroom. We will focus our discussion around the three types of precision we found; and for each type of precision, we will present examples highlighting how teachers engaged with MP6 in the context of statistics. Throughout our discussion we will note similarities and differences of how MP6 appears in a statistical context as compared to a mathematical context.

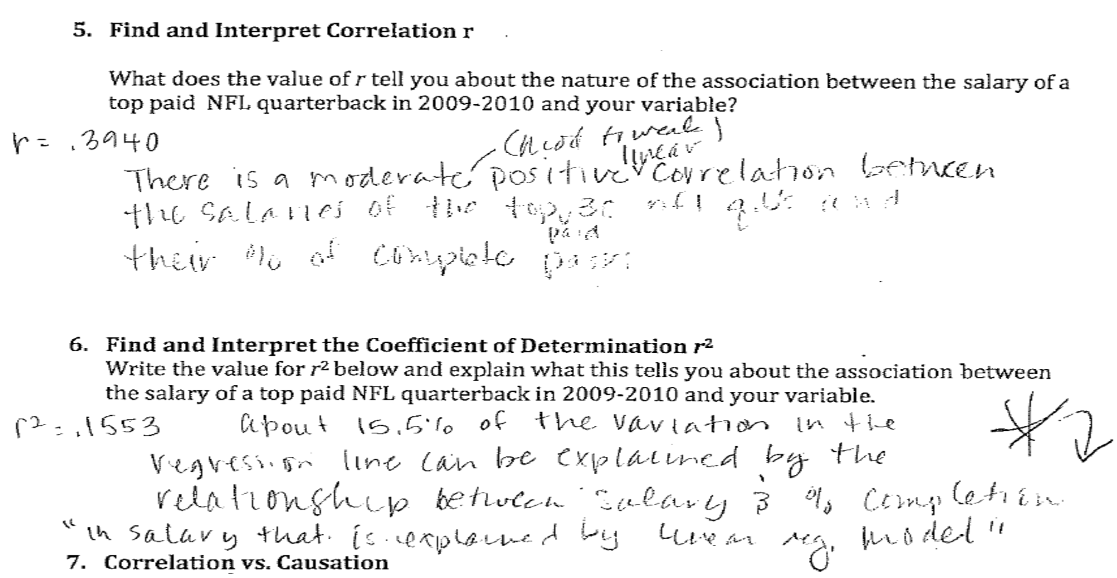
Precision in Quantifying Variation

The first type of precision we identified teachers working with was *precision while quantifying variation*. As statistics is the study of patterns in data, there is a need to do statistical analyses and interpret analyses in the presence of variability in data. While doing this, often times one is required to quantify the variation present. For example, one may need to determine how accurate a result is or one may need to determine how well a model fits the data in the presence of variability. We present an example to show how teachers in our PD worked through this type of precision as well as provide examples of their reflections quantifying variation.

Curve Fitting Example. The importance and need for developing students' abilities to apply the mathematics they are learning to solve complex, real-world problems has been long studied (Verschaffel, L., De Corte, E., 1997; Lesh, R. & Doerr, H.M., 2003; Doerr, H. M., & English, L. D., 2003; Lesh, R., & Lehrer, R., 2003). As the CCSSM emphasize mathematical modeling as both a practice and content standard in the curriculum, teachers have to be prepared to engage students in modeling in a deep, non-superficial way (NGA, 2010). One example of modeling is statistical modeling and curve fitting. In high school mathematics, teachers are charged with instructing lessons on curve fitting. Teachers have to be prepared to have rigorous discussions with students on the different types of models that could fit the data and why the model they chose is best. Typically in mathematics, after a model is chosen, we do not consider the variation present in the data to see if the model we chose was optimal or how the variation may affect the model. However, in statistics, quantifying variation through the determination of whether the model is a good fit is an essential component of analyzing any curve of best fit. The following activity illustrates this.

In a lesson on linear regression, the case study teachers were given data containing the highest paid NFL quarterback salaries from 2009-2010, their pass completion percentage, total number of passing touchdowns and average number of yards per game in 2009-2010. Utilizing technology, teachers began the activity by visualizing the data and estimating the relationship as

> $y = 44375.9 + 244351.32x$, where x is percentage of complete passes and y is salary. In mathematical modeling, once this equation or model is created we would then accept the given model and use it to make predictions relating the salary and percentage of complete passes, without consideration of the variation of the data points. However, in statistics, one is required to consider and explain the variation present in the data. For example, Teacher 3 does so by computing the correlation coefficient, r, and the coefficient of determination, r^2, and then connects these values to the relationship between salary and percentage of complete passes. Teacher 3 writes: *So the big take away ideas of what we have done with regression is that once you have a model for your data, for me, there are really four pieces of information you need to determine if the model is a good model. Before I thought you looked up the "r", the correlation coefficient. But now I know that it is really important that you have r, r^2, the residual plot, and the confidence interval. And that they each individually tell us information that we can't get from the others. For example, r tells you if your data is positive or negative and how close to a linear fit the data is. However, the confidence interval tells you if your data is significantly linear or not. Also, we can use a transformation if the residual plot is actually telling us, "no this model isn't good". We can use transformations to open up that data if needed. So if we have an okay r and an okay confidence interval, but our residual plot has some sort of pattern, we can look at that pattern and see if we can get an even better r, r^2, and confidence interval.*


5. Find and Interpret Correlation r

What does the value of *r* tell you about the nature of the association between the salary of a top paid NFL quarterback in 2009-2010 and your variable?

r = .3940

There is a moderate (Good to weak) positive linear correlation between the salaries of the top paid 30 nfl q.b.'s and their % of complete pass

6. Find and Interpret the Coefficient of Determination r^2

Write the value for r^2 below and explain what this tells you about the association between the salary of a top paid NFL quarterback in 2009-2010 and your variable.

r² = .1553 About 15.5% of the variation in the regression line can be explained by the relationship between salary & % completion

"in salary that is explained by linear reg. model"

7. Correlation vs. Causation

Figure 1. Written work from Teacher 3

At the culmination of the regression unit, while taping the voice recording, teacher 1 articulates this type of precision by stating:

> *So the big take away ideas of what we have done with regression is that once you have a model for your data, for me, there are really four pieces of information you need to determine if the model is a good model. Before I thought you looked up the "r", the correlation coefficient. But now I know that it is really important that you have r, r^2, the residual plot, and the confidence interval. And that they each individually tell us information that we can't get from the others. For example, r tells you if your data is positive or negative and how close to a linear fit the data is. However, the confidence interval tells you if your data is significantly linear or not. Also, we can use a transformation if the residual plot is actually telling us, "no this model isn't good". We can use transformations to open up that data if needed. So if we have an okay r and an okay confidence interval, but our residual plot has some sort of pattern, we can look at that pattern and see if we can get an even better r, r^2, and confidence interval.*

This illustrates how this teacher attended to this type of precision. In statistics, there are many interrelated ways to quantify variation present in data. In the single variable regression case, this type of precision can be seen through understanding how these different ways (r, r^2, confidence interval for the coefficient, residual plot) can quantify the variation and how they are related. While interpreting results, then the teacher must be able to precisely interpret these in order to decide whether the model is a good model of the data or not.

Confidence Interval Example. Precision in quantifying variation really came to light in the PD when introducing confidence intervals. For example, while completing an activity leading up to the formal construction of a confidence interval, teachers struggled with how to precisely describing the potential error in their estimates. An exemplary statement given by Teacher 1 was:

> *There must be a way to measure how certain we are about our inferences—I think this might be called the standard error. If we have a representative sample, we can feel pretty certain about our inferences. If our sample is not representative of the population, then we can't be very sure about the inferences we can make.*

This teacher is alluding to the necessity to quantify the variation present in order for inferential statements to be made. Because variability is in some sense “imprecise” and variation is naturally present in data, in statistics one must be careful of articulating how the quantification of that variation. Teacher 2 who writes further exemplifies this:

5. I am 95% confidant that the population mean falls within this interval because I know that the sampling distribution for sample means of the salaries is normally distributed and centered at the population mean. I also know that approximately 95% of the sample means fall within 1.96 standard errors of the population mean. Therefore, I used my sample mean as the center of my interval and added and subtracted 1.96 standard errors to get my upper and lower limits of the confidence interval.

6. When a population parameter is unavailable, we are able to make an inference about this parameter with one sample because of the theory behind sampling distributions. Regardless of shape of the population distribution, when sample size is at least 30, the sampling distribution is normal and centered at the population parameter. As sample size increases, the standard error of the sampling distribution gets smaller. This is because the standard error is calculated by dividing the population standard deviation (we are using sample standard deviation because n>30) divided by $\sqrt{n}$. Therefore, as n increases, we are dividing the same number by a larger number, making the standard error smaller.

Figure 2. Written work from Teacher 2

This example of a write-up highlights the manner in which statistics requires one to quantify variation through “confidence” in one’s results and “errors” and precision about statements. In addition, the description this teacher gives of the Central Limit Theorem illustrates how to use the theorem to make sense of variability in data and in turn quantify it.

It should be noted that this type of precision (precision in quantifying variability) is unique to statistics. In other words, in mathematics, there is not a focus on variability and thus the need to quantify variability in order to make sense of analyses is not typically a mathematical exercise. For this reason, it is particularly important for teachers to gain experience with difficult statistical concepts such as sampling variability and regression in order for them to practice quantifying variability in data and analyses.

Precision in Language

As noted in the SET report interpretation of MP6, when analyzing data, one must be *precise with terminology and language*, specifically related to the quantification of variability precision discussed above. In mathematics, precision in communication and language also exists. In fact, many mathematics teacher educators have noted that it is a type of precision we want to emphasize to teachers (Silver, Kilpatrick, Schlesinger, 1990). Mathematics teachers are required to develop student’s precise communication of mathematics by having them form and interpret mathematical definitions and explanations. For example, in elementary school geometry, students are asked to classify and sort two-dimensional shapes based on properties related to the shape’s angles and sides utilizing Venn diagrams. Classifying and sorting shapes requires students knowing precise definitions of what term defines which shape and what properties distinguish one shape from the other. Thus, precision in language is a common thread throughout both statistics and mathematics. While this type of precision may “look different” in each of statistics and mathematics, it is important in both. Here, we give an example of precise language related to confidence intervals.

Confidence Interval Example 2. At the culmination of the unit on sampling variability, the teachers were asked to carry out a small project. For this project, they had to set up a statistical question and then investigate the question by collecting data, analyzing the data, and then interpreting the results to ultimately answer the posed question. Here is an example of teacher work:

1. My big "overarching" question is, "What city has the safest restaurants in Los Angeles County?" One question I will investigate to help make this decision is about my hometown. What is the average health code score of a restaurant in Glendale, CA? The population is restaurants in Glendale, the parameter is average health code score, and I will collect data from publichealth.lacounty.gov.
2. I will collect a sample of 40 and find the mean health code score from this sample. The sample scores are shown to the right.
3. The mean of this sample is 93.575 (standard deviation 3.536)
4. The sampling distribution is normally distributed, centered at the population mean.
5. Standard Error=$\frac{3.536}{\sqrt{40}}$= 0.559091. So the 95% confidence interval is 93.575±2(0.559). I am about 95% confident that the population mean of Glendale restaurant health code scores is between 92.45 and 94.69.
6. I am able to make an inference about the population when I only have one sample because of the Central Limit Theorem, which says that the sampling distribution is normally distributed centered at the population mean with standard error $\frac{\sigma}{\sqrt{n}}$ where σ is the standard deviation of the population and n is the sample size.
7. I am about 95% confident that my inferential statement is correct. My sigma is actually the sample standard deviation (not the population standard deviation) so I might not be exactly 95% confident. As we saw when simulating confidence intervals for the population mean income of Mira Beach, this percentage might not be completely accurate because we are assuming that the population standard deviation is close or equal to the sample standard deviation.

Figure 3. Written work from Teacher 1

In #7 of the work, teacher 1 differentiates between two ways to compute the standard error—one computation done by using the standard deviation of the sample and one done by using the population standard deviation. Teacher 1 notes that in this case, because the population was not known, she used the sample standard deviation. This estimate would give her some error in her computations. However, teacher 1 takes this into consideration and is able to explain the connection of these computations to her confidence rating of the inferential statement. This teacher also gives a clear statement of the Central Limit Theorem in order to justify reasoning about population inference in response #6. Note that her language describing the Central Limit Theorem is very precise. Overall, this example illustrates the precision in language needed in statistics, even when discussing imprecise things such as errors. Teacher 1 justifies the 95% confidence interval using precise percentages for the interval as well as using terminology such as "not exactly" and "about" to show the difference in the illustration and the exact value of the confidence interval. In the same culminating activity, teacher 3 connects the precision of his inferential statement to the size of the sample:

6) I'm able to make an inference about the population after only one sample, because we know 97% of our sample fell into this range and we also know from previous testing that if the sample size is big enough, the population mean will be the same as the sample mean. We also know that the sample mean +/- 2(standard error) will give us a confidence interval of approximately 95%.

7) I am confident of my inferential statements after only one sample, because my sample was nearly ¼ of the population and I applied a confidence interval of 2 standard error.

Figure 4. Written work from Teacher 3

In these examples, teachers demonstrate how precision in language interplays with statistical inference. These two teachers' work highlights how precision in statistics is tied to being precise about variation through precision in language.

Precision in Explaining Variation

Along with quantifying variation and precision in language, we also identified the *precision needed in explaining variation*. While all three types of precision identified are obviously related, this type of precision was highlighted in a slightly different way. The idea of explaining variation is in some sense more informal than the necessity to quantify variation. In fact, this type of precision is likely to be present in introductory activities as it challenges teachers to describe and explain instead of compute and interpret. Because explanation requires description, precision in language is naturally connected to this type of precision as well.

Sample to Sample Variation Example. The following example illustrates teachers struggling with how to describe variation present from sample to sample in a precise manner. Teachers were asked to open a bag of M&Ms and compute the percentage of blue M&Ms present in the bag. Using this information, they had to make a guess for the proportion of blue M&Ms in bags of M&Ms. Through this activity each teacher obtained different values and then has to attempt to argue why their value would be the best guess for the correct population proportion. The following transcript illustrates the teachers precisely articulating how the percentage of blue M&Ms varies:

Instructor/Author 1: Teacher 2 you got yours already, right? What's your proportion of blue?

Teacher 2: 16.7

Instructor/Author 1: Okay 16.7, so here's my argument to you: 16.7 is the correct percentage and it's my best estimate for the parameter, because my bag was a random sample. I believe that the different colors should be about evenly distributed and that they should have…Oh, and there's six colors of M&Ms in any bag and nine actually happens to be right around the exact percentage that would mean if they were all even. That's my idea for you. Given that one…who wants another one?

Teacher 2: But if you have two people and there're next to each other, it's not like you have your given anymore.

Instructor/Author 1: So let's do Teacher 3…I just have to convince you that that's an argument and that argument stands. So what was yours?

Teacher 3: 37.5

Instructor/Author 1: Do you have anything to convince us that 37.5 is pretty different?

Teacher 3: Yeah well we all know there's more blue, because the American flag has blue.

Teacher 2: It's more American. [laughter]

Instructor/Author 1: This is a tougher one to find a nice viable argument. My best argument would be something like "I have a random sample here, with my baggy, and I believe this is a random sample, so my best guess of that population parameter, which we don't know, which is the proportion of blue, this is my best guess and I think this is what it's going to be, because I don't have information further than that, so this is mine". That's what I would stick with.

Teacher 1: Yeah, so I am totally convinced of that, however when I open two bags, I am no longer convinced…right?

Instructor/Author 1: Well it depends…what did you get?

Teacher 2: 12

Instructor/Author 1: Out of? What did you have Teacher 3?

Teacher 3: 21 out of 57.

Teacher 2: 57
Instructor/Author 1: So if I'm [Student #4] and [Student #3], I'm pretty convinced that my 21, 22, is going to be a great approximation or great estimate for that population parameter. "Look it happened to me twice…I got about the same amount twice".
Teacher 3: And they are pretty close.
Instructor/Author 1: They are close too. So the issue becomes when you're against each other. Then your argument is, "Well, okay so I could have a value as extreme as 22 or a value maybe as extreme on the other end of 11, or whatever it was". So, I would say my best guess is somewhere in between those, but I'm going to stick with mine, because mine is the random sample.
Teacher 1: Mine is too Lady!
Teacher 2: Pick another random sample.
Teacher 1: Since we all got different ones, lets do more and see who's comes up more.
Instructor/Author 1: Yeah, so let's describe how the percentages of blues varies from one bag to the other. Let's actually, try to describe this. What's the variation that's happening? Let's get a description. Okay?

Using this as an introduction, the teachers proceeded to construct an approximate sampling distribution for the proportion of blue M&Ms. When Teacher 1 exclaims that "we should do more and see who comes up with more", this teacher is striving to have her explanation be more precise. In other words, teacher 1 is working towards gathering more evidence to make more precise statements enabling her to explain variation.

Often times in statistics, we strive to explain the variation present in data. This variation might be sample to sample variation like the one described in the transcription, or it might take the form of coming up with a model to explain the variation present in for example, test scores. During the voice recording after the sampling variability unit was completed, Teacher 2 highlighted precision in explaining variation (although it was not called that), when stating the following:

> *So explaining yourselves in the meaning of the problem and then analyze givens, restraints, relationships, and goals. Literally everything that we've done, we have had to say, "what a minute, what is this are we talking about population mean, sample mean. Is it a population parameter? What is it we are talking about? Is it sampling distribution, population distribution, sample distribution?" So we have to really keep our ducks in order, to understand where we are in the problem, in relation to everything else.*

Teacher 2 continues to specifically discuss her view of MP6 in the PD by recording:

> *Attending to precision is using clear definitions in discussion with others, so as we explained today. In class I kind of spit out the empirical rule in general terms and she wrote it on the board in the general terms that I said. Then we went back and contextualized it as to what it meant to our problem, because I said…for instance 68% of the data falls within one standard deviation of the mean. Then we went back and changed the data to the sample means. What data are we actually talking about? So we contextualized in the sense of the problem.*

Explaining variation precisely is specifically a statistical task. There is not a similar type of precision present in mathematics, thus highlighting the importance of having teachers do statistical tasks and activities that force them to think about statistical precision.

Reflections and Implications

Having teachers engage in the mathematical practices in the context of statistics is necessary for teachers to build these processes or habits in their SKT. The CCSS lists the math practices as general habits of mind, but as math teachers are required to teach a variety of content, i.e. statistics, algebra and geometry, teachers need a understanding of the intricacies involved in the various content domains and how those intricacies affect the emergence of the math practices in the different domains. In particular, even though a teacher may engage in mathematical practices and have facilitated these practices while doing mathematics, it is important for teachers to understand how these practices materialize in statistics. In this particular case study the focus was the practice of Attending to Precision (MP6). We found three dominant trends in Attending to Precision while working in the context of sampling variability and regression: (1) precision in quantifying variation, (2) precision in language, and (3) precision in explaining variation. We noted that precision in quantifying and explaining variation is a type of precision that is necessary for K-12 statistics, where precision in quantifying variation is seen in all of statistics and explaining variation is seen mostly in lower-level statistics. However, in K-12 mathematics, preciseness in quantifying or explaining variation is typically not considered. Precision in language is seen in both mathematics and statistics, but appears differently in both.

One of our main goals in producing this manuscript is to contribute to teacher educators' discourse about SKT. In particular, we hope that the work involved in the case study will start or continue conversations about the role of the mathematical practices in statistics in order to expand both pre-service and in-service teachers' content and pedagogical content knowledge. Specifically, we hope to encourage teacher educators to discuss and investigate best practices in incorporating the math practices into the different content areas and the content knowledge needed by both in-service and pre-service to understand the subtle differences in the appearance of the math practices as related to content. In addition to contributing to the practice of teacher education, such discussions can help refine theories, instruction and assessment methods of SKT.

In this manuscript we have only reported on Attending to Precision (MP6) in the context of statistics. In our study we also looked at other mathematical practices, such as MP3: Construct viable arguments and critique the reasoning of others, and we plan to analyze teacher work for trends and themes that emerge when teachers are utilizing that particular practice in statistics. Related questions that require further consideration include: In light of the SET report, how do the other MPs appear in statistics and what ways do the MPS appear similarly and differently in the content domains of mathematics and statistics? What formative and summative assessment techniques should teacher educators use to assess this knowledge of mathematical practices in statistics and the differing of use of these practices in the content domains of math and statistics? In what ways does having this knowledge affect teacher instruction practice? Exploring such questions will help to advance SKT by developing more effective means for promoting methodology and assessment.

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