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Background

Mathematics language is connected to students' conceptual understanding of content knowledge and skills (Capraro & Joffrion, 2006), and developing mathematics vocabulary may promote conceptual thinking about numbers and operations (Dunston & Tyminski, 2013). Vocabulary terms such as *more than*, *divide*, and *variable* are all connected to the symbolic representations (i.e., –, ÷, *x*) students use to solve problems in elementary and secondary school. As emphasized by the communication process standard by the National Council of Teachers of Mathematics (NCTM, 2000) and the standards for mathematical practice of the Common Core (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), explicitly connecting mathematics language to mathematical concepts may help students better comprehend mathematics. Focused instruction on mathematics vocabulary may be especially beneficial for students who experience mathematics difficulty, as these students regularly demonstrate lower mathematics performance compared to typical performing peers (Andersson, 2010).

Monroe and Panchyshyn (1995) break academic language related to mathematics into four categories: (a) technical words which have one meaning (e.g., *trapezoid*), (b) subtechnical words which have multiple meanings (e.g., *degrees*), (c) general words common in everyday language that have some type of meaning in mathematics (e.g., *simplify*), and (d) symbolic words in which amounts are represented by abstract numerals or symbols (e.g., *plus* as +). Existing research on mathematics vocabulary has focused primarily on technical words (e.g., Harmon, Hedrick, & Wood, 2005; Pierce & Fontaine, 2009) and symbolic words (e.g., Gilmore, McCarthy, & Spelke, 2007), which are often found in upper elementary and secondary grades.

Several researchers provide suggestions for teaching mathematics vocabulary. Selection of vocabulary is important, and Baumann and Graves (2010) suggest teachers focus on general and domain-specific vocabulary as well as vocabulary related to interpretation of mathematics symbols. After vocabulary is selected, Pierce and Fontaine (2009) suggest providing student-friendly definitions for vocabulary. Students should have extended opportunities to encounter words and engage in meaningful processing of such words. Bay-Williams and Livers (2009), as well as Monroe and Orme (2002), suggest providing explicit instruction of mathematics vocabulary and opportunities for students to encounter mathematics vocabulary in everyday and context-related situations.

Most suggestions for teaching mathematics vocabulary are based on vocabulary acquisition strategies from reading (Rubenstein & Thompson, 2002), and only a handful of studies evaluate mathematics vocabulary instruction empirically. For example, Monroe and Pendergrass (1997) provided mathematics vocabulary instruction to 58 fourth-grade students. Half of students received mathematics vocabulary instruction related to measurement using definitions, and the other half received instruction using graphic organizers. Students in the graphic organizers conditions performed significantly higher at posttest. This collection of research indicates that mathematics vocabulary is important, that mathematics vocabulary should be taught, and that students may have difficulty with mathematics vocabulary.

Research Questions

The empirical evidence related to instruction of mathematics vocabulary for all students is scant. While several researchers provide suggestions for selection of mathematics vocabulary terms for instruction and other researchers provide frameworks for providing mathematics vocabulary instruction to students, these suggestions are based on strategies for teaching reading vocabulary, have not been validated for mathematics, or have not been researched for students who experience difficulty with mathematics. In the present study, we investigated the mathematics vocabulary knowledge of students at grades 1 to answer three research questions:

1. What are the relationships among general word knowledge, mathematics fluency, and mathematics vocabulary for first-grade students?
2. What are the influences of general word knowledge and mathematics fluency on mathematics vocabulary?
3. What is the mathematics vocabulary performance of students with MD or reading difficulty (RD) compared to students without MD or RD?
4. Which mathematics vocabulary items cause the most difficulty for students?

Method

Participants

Participants (*N* = 104) were sampled from six first-grade classrooms in a school district in a state in the Southwest. The district's student population consisted of 9.8% African American, 0.6% Asian American, 61.7% Hispanic, 25.9% Caucasian, and 2.0% other. In the district, 13.6% of students demonstrated limited English proficiency, 65.9% of students were economically disadvantaged, and 11.0% of students received special education services. We collected the following demographic information for each student: age, gender, race/ethnicity, special education status, English learner (EL) status, and retained status (see Table 1 for demographic information). The average age of participants at the time of assessment was 7 years, 4 months.

Table 1
Demographic Information for Participants

	N	%
Gender		
Male	55	52.9
Female	49	47.1
Race/ethnicity		
African American	12	11.5
Asian	1	1
Caucasian	37	35.6
Hispanic	54	51.9
Special education	3	2.9
English learners	1	1

Measures

We administered three measures to all students: general word knowledge, mathematics fluency (i.e., predictor of overall mathematics knowledge), and mathematics vocabulary.

General word knowledge. To measure students' general word knowledge, we administered the Word Decoding subtest of the Gates-MacGinitie Reading Tests, Level 1 (GMRT; MacGinitie, MacGinitie, Maria, & Dreyer, 2000). The Word Decoding subtest is designed to analyze the decoding and vocabulary knowledge of students. The examiner reviewed two practice items, and students had 20 min to answer 43 questions where the student selected a term (out of four answer choices) that matched a picture. Students received 1 point for each correct answer. The maximum possible score was 43. As reported by Maria and Hughes (2008), Kuder-Richardson Formula 20 for reliability was .93 for the spring of first grade; reliability for this sample (i.e., Cronbach's α) was .91.

Mathematics fluency. To measure students' mathematics fluency, we administered the Math Fluency subtest of the Woodcock-Johnson III Normative Update (WJIII; Woodcock, McGrew, & Mather, 2007). We selected WJIII Math Fluency because it is brief, appropriate for first-grade students, and because mathematics fluency is a strong predictor of overall mathematics competence (e.g., Jordan, Kaplan, & Hanich, 2002; Mabbott & Bisanz, 2008). On Math Fluency, students solved addition, subtraction, and multiplication number combinations (i.e., single-digit addends, minuends, subtrahends, and factors) for 3 min. The score was the number of correct answers. The WJIII Math Fluency has adequate split-half reliability ($r = .90$), test-retest reliability ($r = .95$; Woodcock et al., 2001), and criterion validity ($r = .67$), as measured against the Kaufman Test of Educational Achievement (Kaufman & Kaufman, 1985). Cronbach's α for this sample was .96.

Mathematics vocabulary. To measure students' vocabulary knowledge specific to mathematics, we developed Mathematics Vocabulary Grade 1 (Powell, 2015). We selected the mathematics vocabulary terms presented on the measure by conducting a thorough search of two common first-grade mathematics textbooks and the glossaries, *Go Math!* by Pearson (Dixon, Larson, Burger, & Sandoval-Martinez, 2014) and *EnVisionMATH* by Houghton Mifflin Harcourt (Charles et al., 2014), as well as identifying terms presented in the CCSS (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) and the Curriculum Focal Points of the NCTM (2006). Based on this search, we identified 118 novel first-grade mathematics vocabulary terms and selected 64 terms (of which 52 were featured in the first-grade glossaries) to create the measure.

Table 2 features included and excluded first-grade vocabulary. We selected terms for the Mathematics Vocabulary measure using one of four methods. First, the term appeared within both textbook glossaries (i.e., *Go Math!* and *EnVisionMATH*). Of the 64 terms, 31 were present in both glossaries. Second, the term appeared in a combination of one textbook glossary and a standard (e.g., *Go Math!* and CCSS). This inclusion criterion applied to 21 novel terms. Of the

52 terms identified within glossaries and standards, we also located 31 of the terms in kindergarten-level glossaries for *Go Math!* and *EnVisionMATH*. We had concerns about creating a Mathematics Vocabulary measure that would be too difficult for first-grade students, especially students with MD. The inclusion of terms, of which approximately half were introduced in kindergarten, allowed us to alleviate concerns about floor effects. Third, the term was an opposite of a selected term. *Longest* was included in kindergarten glossaries but not at first grade. Because *shortest* was included at first grade, we opted to include *longest*. Therefore, we selected 1 term because it was an opposite term. Fourth, we identified 11 terms to alleviate concerns about ceiling effects. *Go Math!* and *EnVisionMATH* included these 11 terms at second grade but not within the first-grade glossaries, and each term was related in some manner (e.g., place value, geometry, numbers and operations) to already-selected first-grade Mathematics Vocabulary terms.

Items varied in student response method, which included multiple choice, short answer, drawing pictures, and matching. For question development, we created three levels of questions for each vocabulary term (recall, comprehension, use in complex tasks; Haladyna & Rodriguez, 2013). We worked to make each question as readable as possible for a first-grade student even though measure items would be read aloud by the examiner (Abedi & Lord, 2001). We created the final version of Mathematics Vocabulary Grade 1 with 64 unique terms. We included an additional 13 third-grade terms on the measure to understand differences across grade levels, but these items proved too difficult for first-grade students and were removed from all analyses. The 64 items equally represented recall, comprehension, and use in complex tasks item response formats. During administration, the examiner read the each item prompt aloud, and students were provided ample time to respond to each item. Administration was with each whole class and took place in the general education classroom. Administration time was approximately 35 min. Items were scored as incorrect or correct (i.e., 0 or 1) with a total raw score of 64. Cronbach's α for this sample was .87.

The mathematics vocabulary terms represented broad areas of understanding in mathematics including geometry, operations with whole numbers, simple fraction concepts, comparison, and place value. Each term was expressed either explicitly or implicitly within the CCSS mathematics domains. An explicit association between a term and a standard was one that specifically used the selected vocabulary term to address or teach a mathematical concept. For example, the term *sum* appeared in the Operations and Algebraic Thinking domain at first grade (CCSS, 2010) as "Addition of three whole numbers whose *sum* is less than or equal to 20" (p. 15). This single standard also included the terms *less* and *equal*. In contrast, implicit terms were terms that were not specifically addressed in the standards, but we inferred classroom teachers may have used the terms to teach a mathematical concept. For example, the cluster of standards under the first-grade Operations and Algebraic Thinking domain (CCSS, 2010) is defined by "Represent and solve problems involving addition and subtraction" (p. 15). Although the term *minus sign* was not explicitly addressed in this cluster of standards, we inferred that teachers may use the term when introducing and reviewing subtraction concepts and procedures. Consequently, we coded each term with an explicit or implicit association to a CCSS domain. The terms represented the following domains: Operations and Algebraic Thinking (n = 20; e.g., *addition sentence, equal*), Geometry (n = 18; e.g., *circle*, *triangle*), Measurement and Data (n = 17; e.g., *hour*, *number line*), and Number and Operations in Base Ten (n = 9; e.g., *ones*, *tens*).

Beyond the CCSS, we organized mathematics vocabulary in two ways based on previous categorizations from literature. First, we identified whether mathematics vocabulary terms were classified as technical (i.e., specific to mathematics), subtechnical (i.e., dual meanings, one of which is mathematics related), general (i.e., non-mathematics term), or symbolic (i.e., vocabulary for numerals or symbols; Monroe & Panchyshyn, 1995). Pierce and Fontaine (2009) and Harmon et al. (2005) discussed Monroe and Panchyshyn’s (1995) four mathematics vocabulary categories as a viable method for deconstructing mathematics vocabulary. These four categories are more specific than those for academic vocabulary (e.g., domain-specific versus general; Baumann & Graves, 2010). The first author classified each term, and the second author re-coded 100% of terms. Reliability of classification was 92%. We rectified all discrepancies based on discussion. Second, both authors worked together to categorize mathematics vocabulary according to the 11 difficulty categories of Rubenstein and Thompson (2002). This was necessary due to 56.4% of vocabulary terms categorized from the Monroe and Panchyshyn (1995) cataloging as subtechnical; we wanted to learn more about the specifics of these subtechnical terms. The 11 categories of Rubenstein and Thompson provided more specificity, and other researchers, such as Riccomini, Smith, Hughes, and Fries (2015) and Witzel, Riccomini, and Herlong (2013), have also organized mathematics vocabulary according to these 11 categories.

Table 2
First-Grade Vocabulary Terms for Inclusion and Exclusion

Included terms		Excluded terms	
add	more	balance	missing part
addend	multiply	bar graph	near double
addition sentence	nickel	break apart a ten	needs
circle	number line	charity	number sentence
column	octagon	closest 10	o'clock
cone	odd	compare	order
counting back	ones	corner	part
counting on	outside	curved surface	pattern
cube	penny	data	picture graph
cylinder	pentagon	digits	quarter of
decagon	quarter	doubles minus one	rectangular prism
difference	quarters	doubles-plus-1 fact	related facts
dime	rectangle	doubles-plus-2 fact	save
divide	rhombus	earn	scale
double	row	edge	side
equal	separate	estimate	solid figure
equal shares	shortest	expanded form	sort
equal sign	skip counting	fact family	spend
even	sphere	fewer	standard form
greatest	square	flat surface	survey
half	subtract	half hour	T-chart
heptagon	subtraction sentence	hundred chart	tally chart
hexagon	sum	in all	unequal shares
hour/hour hand	take away	income	value
hundreds	tally	join	vertex
inside	tens	length	wants
least	thousands	measure	whole
less	trapezoid		
longest	triangle		
make 10	triangular prism		
minus sign	unequal parts		
minute/minute hand	zero		

Procedures. The first author and a graduate student in an education-related field collected all data; both administrators had experience in standardized test administration procedures. Each examiner read verbatim from a test administration protocol during administration. Each measure was administered in whole-class format, and data collection occurred in the late spring of the academic year approximately 4 weeks before the end of the school year. Two coders (an undergraduate and a second graduate student in education-related fields) entered 100% of student responses for each measure. This was conducted on an item-by-item basis into two separate electronic databases. Inter-scorer reliability of the measures was 98.55% for the GMRT, 99.25% for WJIII, and 95.46% for *Mathematics Vocabulary Grade 1*. The discrepancies between the two databases were compared and rectified to reflect the student's original response and to ensure 100% accurate scoring.

Results

Correlations

Pearson correlations were calculated to determine the relationship between students' mathematics vocabulary knowledge, general word knowledge, and mathematics fluency (see Table 3). All correlations between measures were positive and significant. Correlations were moderately high and ranged from .51 to .69. The strongest relationship was between general vocabulary knowledge and mathematics vocabulary knowledge (r = .69).

Table 3
Means, Standard Deviations, and Correlations

	Raw Score		Correlations	
Variables	*M*	*(SD)*	WD	MF
GMRT Word Decoding (WD)	27.66	(8.56)		
WJIII Math Fluency (MF)	31.07	(14.26)	.513	
Mathematics Vocabulary (MV)	36.30	(8.07)	.694	.584

Note. GMRT = Gates-MacGinitie Reading Tests; WJIII = Woodcock-Johnson III.

Regression Analyses

Although not the primary purpose of this paper, we examined group differences by gender and race regarding mathematics specific vocabulary knowledge in first grade based on prior research indicating mathematical performance differences based on gender or race (e.g., Carr, Steiner, Kyser, & Biddlecomb, 2008; National Center for Education Statistics, 2013). We performed mean comparisons using ANOVA ($p < .05$) to examine these differences. There were no significant differences between males and females, similar to Lachance and Mazzocco (2006), but there were significant differences between Caucasian and non-Caucasian first-grade students ($p = .004$). Given these differences, we controlled for race in the regression analyses; however, once the general vocabulary variable (i.e., GMRT Word Decoding) was added to the model, race was no longer significant and only accounted for 1% explained variance above and beyond the GMRT Word Decoding. In order to create the most parsimonious model, we preceded without the inclusion of race in either model. Because students were nested within classrooms, we also ran our analyses with consideration to the hierarchical nature of the data. We compared the models with and without classroom as a level 2 variable. After calculating the difference in −2 Log Likelihood, we determined the difference (0.193) with 1 *df* was not significant. For simplicity of the regression analyses and interpretability of results, we did not include classroom as a variable in either model.

We used block entry regression analyses to examine the influence of general word knowledge and mathematics fluency on the mathematics vocabulary knowledge of first-grade students. This method allowed us to consider the effect of each variable separately and examine the change in total variance accounted for between models. We used the results from the correlational analyses and the reliability of the measures for this sample to select predictors of mathematics vocabulary knowledge at each step of the regression analyses. The GMRT Word Decoding and WJIII Fact Fluency had adequate reliabilities for this sample (> .80), and because the GMRT Word Decoding had the strongest relationship with Mathematics Vocabulary, it served as the predictor in Model 1. Table 4 features the regression analyses for Models 1 and 2.

In Model 1, general vocabulary accounted for 49% of the variance in Mathematics Vocabulary scores at first grade, and general vocabulary was a significant predictor ($p < .001$). For every 1-point increase in general vocabulary knowledge, there was a 0.66 increase in Mathematics Vocabulary. In Model 2, the addition of mathematics fluency added 6.7% explained variance with the model accounting for 55% of variance in Mathematics Vocabulary scores; both general vocabulary ($p < .001$) and mathematics fluency ($p < .001$) were significant predictors. For every 1-point increase in general vocabulary, there was a 0.51 increase in Mathematics Vocabulary, and for every 1-point increase in mathematics fluency, there was a 0.17 increase in Mathematics Vocabulary.

Table 4
Summary of Regression Analyses

Predictor	*B*	*SE B*	β	*t*	*p*	R^2	ΔR^2
Model 1							
Intercept	18.088	1.940		9.325	< .001		
GMRT Word Decoding	0.661	0.067	0.697	9.826	< .001	.486	
Model 2							
Intercept	16.779	1.844		9.157	< .001		
GMRT Word Decoding	0.509	0.074	0.538	6.874	< .001		
WJIII Fact Fluency	0.172	0.044	0.304	3.881	< .001	.553	.067

Note. GMRT = Gates-MacGinitie Reading Tests; WJIII = Woodcock-Johnson III.

Performance Differences for Students with Mathematics or Reading Difficulty

To understand whether mathematics vocabulary performance was differential for students with and without mathematics and reading difficulty, we identified students as experiencing mathematics difficulty or reading difficulty. Then, we ran the analyses to learn of any significant performance differences based on difficulty status.

We identified students with *mathematics difficulty* (MD) as students performing at or below the 25th percentile on WJIII Math Fluency. In the literature related to mathematics disability and difficulty, the 25th percentile is a commonly utilized cut-off score (Bodovski & Farkas, 2007; Desoete, Ceulemans, De Weerdt, & Pieters, 2012; Geary, Hoard, Nugent, & Bailey, 2012; Vukovic, 2012). On Mathematics Vocabulary, the average score for students (n = 26) with MD was 29.8 (SD = 6.7); the score for students without MD (n = 78) was 38.5 (SD = 7.4). Students with MD performed significantly lower than students without MD, $F(1,102) = 28.50$, $p < .001$. The effect size, calculated as Hedge's g, was 1.20.

In a similar manner, we identified *reading difficulty* (RD) as students performing at or below the 25th percentile on GRMT Word Decoding. Researchers have used the cut-off score of the 25th percentile in other reading disability or difficulty studies (Fletcher et al., 2011; Simmons et al., 2014; Speece & Ritchey, 2005). Students with RD (n = 26) demonstrated an average Mathematics Vocabulary score of 28.7 (SD = 6.9), whereas as students without RD (n = 78) scored 38.8 (SD = 6.8). Students without RD outperformed students with RD, $F(1, 102) = 42.36$, $p < .001$; Hedge's g was 1.46.

Of the 26 students with MD, 13 of those students were also identified as experiencing RD (i.e., MDRD). This comorbidity has been identified in the literature (Fuchs et al., 2008). The performance differences between MD and MDRD, as well as RD and MDRD, were not significant.

Descriptive Performance on Mathematics Vocabulary Items

Following the correlational and regression analyses, our goal was to examine the differences in student responses on the Mathematics Vocabulary measure. We categorized the mathematics vocabulary terms in the following manner: (a) introduced in kindergarten, first-grade, or second-grade glossaries; (b) explicitly or implicitly addressed by the CCSS; (c) CCSS domain area; (d) technical, subtechnical, symbolic, or general as outlined by Monroe and Panchyshyn (1995); and (e) falling under the 11 mathematics vocabulary difficulty categories of Rubenstein and Thompson (2002). After we coded each term within a category (e.g., 0 = implicit; 1 = explicit), we examined differences between different categories of terms by calculating the average percent of students that got the items correct.

First, we analyzed the differences in accuracy based on whether the mathematics vocabulary term was introduced in kindergarten, first, or second grade. For items initially introduced in kindergarten, accuracy was 67.1%. The accuracy for terms introduced in first grade (i.e., not contained within the kindergarten glossaries) was 52.1%. As expected, the accuracy for second-grade items, included on Mathematics Vocabulary to control for ceiling effects, was 35.2%.

Second, we investigated performance differences in accuracy between terms that were either implicitly (n = 18) or explicitly (n = 46) addressed in the CCSS. We detected no meaningful differences. Items with implicit terms were answered correctly 58.4% of the time, and items with explicit terms were answered correctly 56.1% of the time.

Next, we evaluated differences in accuracy according to which Common Core mathematics domain each vocabulary term was associated: Measurement and Data (n = 17; 72.3%), Geometry (n = 18; 57.4%), Numbers and Operations in Base Ten (n = 9; 49.3%), and Operations and Algebraic Thinking (n = 20; 46.3%). Differences between vocabulary terms within the same Common Core domain were also examined. For example, within Geometry terms, students correctly identified 51.3% of plane figures and 56.9% of solid figures. More than 70% of students correctly identified the plane figures *circle*, *rectangle*, *rhombus*, *square*, and *triangle*, and more than 60% of students recognized *cone*, *cube*, and *sphere*. Less than 30% of students identified regular polygons with more than four sides (e.g., *hexagon*, *octagon*), and less than 30% of students knew specific a prism name (e.g., *triangular prism*). For another Geometry item, students were prompted to shade a *row* within an array and shade a *column* within an array of the same dimensions. Interestingly, 98.1% of students correctly shaded a *row*, while only 25.0% of students correctly shaded a *column* of the rectangle.

Other differences also existed between terms that are commonly taught together and fell under the same CCSS domain area. For example, while 21.2% of students correctly identified the term *sum*, less than 2% of students correctly identified the term *difference*. In another set of items, students were instructed to match place value terms with the numerals in a four-digit number and less than 20% of students correctly matched the terms *thousands*, *hundreds*, and *tens*, and 41.4% correctly matched the terms *ones* with the correct numeral. Meanwhile, there were few differences (i.e., less than 5% point difference) in accuracy between opposite terms such as *greatest* and *least*, *inside* and *outside*, and *shortest* and *longest*.

We also looked at the average accuracy per item to identify terms that were the easiest and most difficult for first-grade students to identify. More than 90% of students correctly identified the terms *inside*, *outside*, *longest*, *shortest*, *more*, *penny*, *row*, *take away*, *tally*, *triangle*, and *zero*. The group of terms that most students were able to correctly identify was not surprising, as many of them appear in everyday contexts or reading outside of mathematics material, and all but three of the terms (*inside*, *outside*, and *tally*) were featured in kindergarten glossaries. Accordingly, many of these terms should be familiar to most first-grade students. In contrast, less than 10% of students correctly identified the terms *counting on*, *decagon*, *difference*, and *multiply*. *Decagon* and *multiply* were second-grade terms included to control for ceiling effects. Both *counting on* and *difference* are explicitly stated in the CCSS; *counting on* was included in one textbook glossary, whereas *difference* was included in two of the glossaries.

Fourth, we examined the average accuracy for vocabulary terms grouped according to the Monroe and Panchyshyn (1995) categories (i.e., technical, subtechnical, symbolic, general). We categorized the majority of terms as subtechnical (n = 39) and technical (n = 15). On average, students correctly identified 56.4% of subtechnical terms and 42.0% of technical terms. We categorized seven terms as general and three terms as symbolic. The average correct response rate for general terms was 91.1%. In fact, 63.5% of students correctly identified all seven general terms. On the other hands, the response rate for symbolic terms was 54.5%. With the three symbolic terms (*equal sign*, *minus sign*, *zero*), 96.2% of students correctly produced the numeral “0” when provided with the written term, but only 41.4% of students correctly matched the term *equal sign* with the symbol, (=), and even fewer students (26.0%) correctly matched the term *minus sign* with the symbol, (–).

Fifth, we determined accuracy for vocabulary terms grouped according to Rubenstein and Thompson’s (2002) categories of mathematics vocabulary difficulty. We conducted this second round of analysis to understand more about which mathematics vocabulary cause difficulty for students. With the Monroe and Panchyshyn (1995) categorization, over 60% of terms fell into one category, and information from such a category may not be helpful for educators. Most terms (n = 52) were associated with only one category of difficulty while the remaining 12 terms were associated with two or three categories. There were no substantial differences in average accuracy between terms that were in only one category (56.0%) versus terms in more than one category (59.9%) of difficulty. There were, however, differences between individual categories of difficulty. For example, students responded correctly to 50.0% or more of terms in categories 1 and 2. Category 3 was characterized by terms that appear only in mathematical contexts and had a lower average for correct responses (40.4%). Some terms (e.g., *longest*) did not align closely with the categories identified by Rubenstein and Thompson (2002) because they were general words. Consequently, we created a 12th category for general English language terms that are used in everyday English and mathematical contexts where the usage does not differ between contexts. Five terms were included in this category, and students correctly identified the terms 91.3% of the time. Terms were associated with categories 4, 5, 6, 9, and 10 less frequently, and each category contained fewer than 6 terms. Furthermore, the majority of terms in categories 4 and 6 were also associated with category 2 (mathematics terms that are shared with everyday English). Average accuracy for the less frequent categories of difficulty ranged from 21.2% to 75.8%.

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