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Introduction

The professional noticing of children's mathematical thinking framework (Jacobs, Lamb & Philipp 2010) emphasizes interpretation of children's mathematical thinking which requires a particular type of teacher knowledge. In order to interpret a child's mathematical thinking, a teacher draws on her or his mathematical knowledge for teaching (MKT) (Ball, Thames & Phelps 2008). Thus, I claim that when engaging in professional noticing, teachers rely on their MKT. While others have touched on this intersection between MKT and professional noticing either by referring directly to MKT in the context of teacher noticing (Kaiser, Busse, Hoth, König, & Blömeke, 2015) or by designing noticing interventions focused on developing aspects of MKT (Flake, 2014; Schack et al. 2013; Vondrova & Zalaska 2013), this paper will be focused ways to integrate the measurement of MKT and teachers' professional noticing. Because MKT is content specific, measurement of teacher noticing is situated both in mathematical content and in the context of interventions designed to assist teachers in engaging with professional noticing.

Theoretical Frameworks

The basic premise behind the professional noticing framework (Jacobs et al. 2010) is that novices in any profession must learn to notice in ways unique to the profession. The professional noticing framework focuses on children's mathematical thinking and can be applied to teacher analysis of student work by studying the decision-making processes teachers use when evaluating students' responses. Jacobs et al. (2010) conceptualize professional noticing of children's mathematical thinking as comprised of three interrelated skills: attending to children's strategies, interpreting children's understandings, and deciding how to respond on the basis of children's understandings.

For this study, two aspects of MKT: specialized content knowledge (SCK), and knowledge of content and students (KCS) were chosen as foci for integration into the professional noticing framework (Ball et al., 2008). Because MKT is situated within the mathematical content being noticed, different mathematical concepts require different types of SCK and KCS. The content of focus for this study was multi-digit addition and subtraction; much is known about how children approach multi-digit addition and subtraction story problems as well as different strategies children tend to use (Carpenter, Franke, Jacobs, Fennema, & Empson,, 1998; Fuson, 2003; NRC, 2001). Knowledge of different strategies and the general progression of children's strategies is considered part of KCS. To address this KCS aspect of MKT and situate it within the professional noticing framework for this study, Jacobs et al.'s (2010) framework was modified to include an additional component, Identify Strategy Level of Sophistication (See Figure 1). For this study, as part of preservice student teachers' interpretation of their students' work, they were asked to identify their students' strategies' levels of sophistication prior to deciding how to respond instructionally. Incorporating a content specific type of interpretation that draws on KCS, namely identification of strategy sophistication, into the professional noticing framework is one way to simultaneously measure MKT while also analyzing professional noticing.

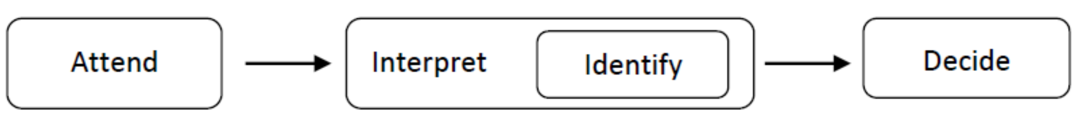

Figure 1. Professional noticing framework derived from Jacobs et al. (2010)

Including Identify into the noticing framework is the first method I employed for integrating the measurement of MKT and professional noticing. Another method involves considering the content specific role SCK plays in each of the four components of professional noticing (see Figure 1). To address the SCK aspect of MKT, SCK was mapped onto this study's professional noticing framework (Figure 1) using Ball, Thames & Phelps' (2008) table of mathematical tasks of teaching that draw on SCK (reproduced in Table 1) as well as well as their explanation that SCK encompasses teachers' knowing "features of mathematics that they may never teach to students, such as a range of non-standard methods or the mathematical structure of student errors" (p.10). For example, the mathematical teaching task "using mathematical notation and language and critiquing its use" was considered part of attend since it deals with noticing mathematically significant details, while the teaching task "evaluating the plausibility of students' claims" was considered a part of interpret since it is employed when developing interpretations of a student's work sample. "Knowing non-standard methods" was considered as either interpret or identify depending on the student teachers' discussion. For example, when the student teachers discussed a non-standard method and its level of sophistication, for the purposes of coding, it was considered a part of identify. Table 2 contains the subset of the mathematical tasks requiring SCK that became the framework for this study's integration of SCK and professional noticing, and was used to simultaneously measure both noticing and evidence of SCK.

Table 1.

Mathematical Tasks of Teaching that Draw on SCK (Hill, Ball & Schilling, 2008, p. 10).

Mathematical Tasks of Teaching that draw on SCK	
Presenting mathematical ideas	Responding to students' "why" questions
Finding an example to make a specific mathematical point	Recognizing what is involved in using a particular representation
Linking representations to underlying ideas and to other representations	Connecting a topic being taught to topics from prior or future years
Explaining mathematical goals and purposes to parents	Appraising and adapting the mathematical content of textbooks
Modifying tasks to be easier or harder	Evaluating the plausibility of students' claims (often quickly)
Giving or evaluating mathematical explanations	Choosing and developing usable definitions
Using mathematical notation and language and critiquing its use	Asking productive mathematical questions
Selecting representations for particular purposes	Inspecting Equivalencies

Table 2.

Mathematical Tasks Requiring SCK Mapped onto the Professional Noticing Framework.

Mathematical Tasks Requiring SCK **Ball, Thames & Phelps (2008)**	**Related Professional Noticing Component**
Critique notation and language	Attend
Evaluating plausibility of student claims	Interpret
Evaluate math expressions	Interpret
Know non-standard methods & common errors	Interpret/Identify
Ask productive math questions	Decide

Description of the Study

For the research study, one group of four first-grade student teachers and one group of three second-grade student teachers completed a set of three carefully sequenced Professional Learning Tasks (PLTs). The PLTs were facilitated by college supervisors. Each PLT focused on analysis of the student teachers' students' written work on multi-digit addition and subtraction story problems. Prior to each of the three PLT sessions, the student teachers were provided directions on what types of problems to pose to their students as well as how to choose different student work samples to bring to the sessions. Each of the PLTs were focused on developing the student teachers' SCK & KCS around multi-digit addition & subtraction. Table 3 contains information about the PLT sessions' MKT foci.

Table 3

PLT Sessions with description of MKT focus

PLT Session	PLT SCK/KCS Focus
One	Different types of addition & subtraction story problems from NGACBP (2010, p. 88)
Two	Multi-digit addition & subtraction problems levels of sophistication: 1. Direct Modeling 2. Counting 3. Number Fact Strategies: making a ten, decomposition, creating equivalent but easier problems—all draw on knowledge of place value, properties of operations and/or relationship between addition and subtraction *Adapted from Carpenter et al. (1998); Fuson (2003) & NRC (2001)
Three	Developing questioning techniques based on student's mathematical thinking: Probing vs. Extending Questions from (Jacobs & Ambrose, 2008)

This paper addresses the following research question: How does the development of MKT through experience with a set of guided PLTs focused on analysis of student multi-digit addition and subtraction work relate to the elementary student teachers' professional noticing?

Data Sources & Methodology

Primary data sources were the student teachers' student work samples and transcriptions of the six PLT sessions. The student teachers' discussions during the PLT sessions were divided into "distinct shifts in focus or change in topic" known as idea units (Jacobs, Yoshida, Stigler & Fernandez, 1997, p. 13). The choice was made to code discourse idea units rather than individual talk turns because collective analysis had emerged as an important aspect of what occurred during the sessions. The student teachers' comments often shifted the opinions of each other and moved the analysis forward; it was not possible to claim levels of noticing for each individual talk turn. Thus, idea units allowed for the consideration of many talk turns, by various preservice teachers in which they collectively discussed one topic. All of the idea units, when applicable, were coded for each of the noticing components. As a means of reliability, one of the college supervisors was asked to apply the professional noticing codes to the idea units related to student work samples; on the second iteration of analysis, 88% reliability was reached and thereafter, I continued to code alone. After each idea unit was coded for the four noticing components, each component was assigned a level within that component (0: Lacking, 1: Limited, 2:Robust).

All idea units, when applicable, received a level for attend (A), interpret (R), identify (I) and/or decide (D). To illustrate the manner in which coding occurred, idea units 13 & 14 from first-grade PLT #2 will be discussed in detail. Idea unit 13 began with a discussion about a student work sample. As the group of student teachers worked to interpret the students' understanding, one of the student teachers drew on evidence from the same students' work on another problem to claim that the student "understood tens and ones," but approached the two problems differently because of the number involved. A discussion followed related to this idea;

during the discussion one student teacher mentioned the strategy level of sophistication, but without details. Their entire conversation was considered part of the same idea unit. The student teachers' talk eventually shifted away from interpreting the students' understanding and towards the wording of the problem. At this point the conversation became labeled as idea unit 14 which now focused on the problem and not the student. Idea unit 13 was coded as A:2, R:2, I:1; there was no code for decide because the idea unit did not contain discussion around instructional decisions. Idea unit 14 was coded as A:2 because in the exchange the student teachers were attending to mathematically significant details in general but they were not discussing any particular student work samples. Therefore, they did not interpret, identify or decide. Note: It was also possible for an idea unit to receive no codes relating to the professional noticing components. For example, idea unit 11 from second-grade PLT #2 contained pedagogical discourse about students using the standard algorithms for multi-digit addition and subtraction. Their discussion was not related to students' mathematical thinking, but to pedagogy; therefore, the idea unit did not receive any codes for noticing.

In addition to the leveled coding, for each of the four noticing components, within each of the three levels of coding, the designation "with evidence of SCK," was used to classify instances where the idea unit provided explicit evidence of the student teachers applying their SCK to any of the components of professional noticing. The integrated SCK and professional noticing framework presented in Table 2 was used as a guide while coding each of the professional noticing components for evidence of SCK. Coding for "evidence of SCK" required explicit evidence that the student teachers were drawing on SCK while noticing their students' mathematical thinking. There most likely were instances where the student teachers drew on SCK but did not explicitly talk about it in the discourse exchanges, but for the purposes of coding, explicit evidence was required. For example, in the exchange presented below, the student teachers interpreted a students' counting up strategy (See Figure 2). Their discussion begins with one of the student teachers connecting the strategy to the concept of decomposition. Decomposition was one of the non-standard strategies discussed during PLT #2. As the exchanged continued (not shown), the student's strategy was compared with skip counting by tens and ones on a number line. Because the student teachers explicitly connected their interpretation of the students' mathematical thinking to SCK regarding decomposition, a non-standard method, the interpret noticing component of this idea unit was coded as "with evidence of SCK."

T1: He used addition so he separated the tens and ones and did a tens stick and one circles and just counted up. So, I guess it's still…it's kind of counting but it's also decomposing in a way.
T2: When I looked at this, I thought it was interesting that he…So he did the 34 here. He knew that he had to get to 65 so instead of counting up to 34 to 50, he knew…the way he did it was interesting, like he went in and did tens first.
T3: So he just held 34 in his head and did…Like 34, 54, 64.

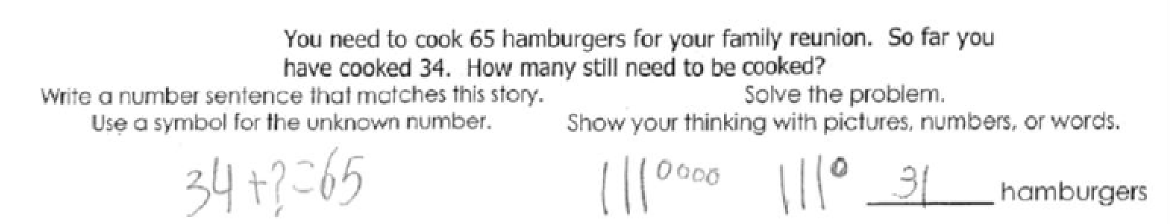

Figure 2. First Grade PLT session 3, idea unit 22 transcript with accompanying student work

Results & Conclusion

Table 4 contains the overall coding results for each group of student teachers for the idea units dealing specifically with student work samples. When analyzing the tables that include all of the idea units, very similar results were found. From the leveled coding that occurred, it is evident that for the most part, the student teachers increased their engagement with professional noticing throughout the three PLT sessions; their growth can be seen in two directions in Table 5. The horizontal line represents the change from level 0: limited evidence toward level 2: robust evidence for each of the professional noticing components which is visible in all four rows of the table. The diagonal line represents the student teachers' development in their overall engagement with professional noticing: moving from attend toward decide.

The following two sections will discuss both the KCS and SCK measurement results in detail. First the interpret professional noticing component based on the development of content specific KCS and added to the professional noticing framework will be presented followed by analysis of the SCK mapped coding within each of the levels of professional noticing.

Table 4. Professional noticing codes for idea units dealing with student work throughout the PLT sessions—first grade & second grade.

1st Grade	Session1									Session 2												Session3							
Idea Unit	1	6	8	9	10	11	12	13	19	1	2	7	8	9	10	11	12	13	15	16	17	12	13	14	16	17	18	19	20
Attend	0	2	1	1	1	1	2		1	2	2	2	2	2	1	2	2	2	1	2	2	2	2	1	1	2	2	2	2
Interpret		2	1	0	1	1	1		0	2	2	2	2	1	0	1	2	2	1	1	1	1	2	1	1	2	2	2	1
Identify		1				0	0	2		1	0	1	0	0		1	1	1	0	0	0	2	2	0	1	1	0	1	0
Decide							2															2	1	1	2	2	1	0	1

2nd Grade	Session1																	Session 2									Session3										
Idea Unit	3	4	5	6	7	9	10	12	14	15	16	21	22	23	24	25	26	6	8	9	11	14	15	16	20	21	5	6	7	9	11	12	14	15	16	17	18
Attend	1	2	1	1	2	1	1		1	2	2	2	1	1	2	1		2	2	2	1	2	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Interpret	0	1	0	1	2	1	1			1	2	1	1	1	2	1		0	2	2		2	0	2	1	2	2	2	2	1	2	2	1	2	2		
Identify		1	1	1				0		0	1		1	0	0		1									2	1	2	2	1	2	0	2	2			
Decide				1			2			1	2	2			1											1						2			2	2	2

Measurement of KCS

By design, the PLTs focused on developing SCK and KCS regarding non-standard methods and the level of sophistication of different strategies. During PLT #1 & #2, the student teachers' identification of levels of sophistication of strategies was mostly I:0 or I:1, which meant that the student teachers missed opportunities to identify levels of sophistication, discussed sophistication without providing evidence, or drew on evidence but incorrectly labeled levels of sophistication. As the PLT sessions progressed, the student teachers began to draw upon evidence to correctly identify levels of sophistication of strategies. While few, this robust identification, I:2, mostly occurred during PLT #3, after the student teachers had exposure to and time to work with identifying strategies' levels of sophistication.

The student teachers did not automatically identify levels of sophistication; they had to be prompted to do so. Participation in the PLTs was not sufficient for developing these skills. Also, it is of note that it was initially hypothesized that drawing on KCS to correctly identify the levels of sophistication of strategy would assist the student teachers as they engaged with the decide component of professional noticing and lead them toward higher levels of decide. While hoped for, there was only one instance where an I:2 corresponded with a D:2 code. This shows that the student teachers needed more exposure with connecting their interpretation and identification of level of sophistication of strategy of the student work samples to making evidenced based instructional decisions.

Measurement of SCK

The PLTs were designed with the goal of developing specific areas of SCK for multi-digit addition and subtraction. "With evidence of SCK" was coded whenever the student teachers exhibited explicit evidence of drawing on SCK when professionally noticing student work samples. Table 5 contains all of the idea units that exhibited explicit evidence of SCK. Both groups of student teachers' showed increased evidence of SCK as the PLT sessions progressed. In PLT #1, only the first-grade group showed evidence of SCK. In PLT #2, the first-grade group showed evidence of SCK in four idea unit exchanges and the second-grade group in two. By PLT #3, the first-grade group showed evidence of SCK in 10 exchanges which was just under half of the 23 total idea units. In contrast, the second-grade group showed evidence of SCK in 5 idea unit exchanges. It is worth noting that PLT #3 was the first session that the second-grade student teachers had brought in non-standard algorithm samples to analyze; the first two sessions many of their students used the standard algorithm (despite directions not to) and therefore there weren't non-standard strategies for them to analyze or interpret. By PLT #3, the second-grade student teachers were beginning to recognize the importance of developing their own SCK regarding different strategy types. At no point during the three PLTs did not have as high level of sophistication of strategies to analyze as did the first-grade student teachers. Thus, it is worth considering the groups of student teachers separately. In doing so, it is evident that SCK increased over time.

In considering the groups as a whole and looking at the relationship between SCK and levels of professional noticing (see Table 5), the student teachers' SCK assisted them throughout all four components of the professional noticing framework. For the idea units where the student teachers exhibited SCK, their SCK led to greater levels of professional noticing: either their attention to their students' mathematical thinking, their interpretations of their students' mathematical thinking, their identification of levels of sophistication of strategies, and/or their decisions as to which questions to ask their students as a next step in instruction. Of the 34 documented instances showing evidence of SCK, 27 of them were for the highest level of professional noticing. Thus, when the student teachers drew on SCK when professionally noticing, there was an increase in the student teachers' levels of professional noticing.

Table 5. Professional noticing codes for idea units that contain evidence of SCK throughout the PLT sessions.

	PLT Session #1		PLT Session #2						PLT Session #3									
	1st Grade		1st Grade				2nd Grade		1st Grade						2nd Grade			
Idea Unit	2	12	1	7	12	13	18	21	1	2	12	13	17	22	4	7	16	17
Attend	1*	2	2*	2	2	2*	2	2	2*	2*	2*	2*	2*	2	2*	2	2	2
Interpret	1*	1*	2*	2	2*	2*	2*	2*	1	1*	1	2*	2*	2*		2*	2*	
Identify		0*	1	1*	1*	1	1	2*	2	2*	2	2	1	2		2		
Decide		2						1			2*	1	2*	0			2*	2*

Implications

This study provided examples of two methods for measuring teachers' professional noticing of children's mathematical thinking and MKT simultaneously within a content and context specific situation. The first method involved considering content specific aspects of MKT that is needed for interpreting children's mathematical thinking—for this study, the additional Identify professional noticing component. The second method used Ball et al.'s (2008) table of mathematical tasks for teaching requiring SCK (Table 1) as a guide for mapping onto the professional noticing framework—for this study, see Table 2. The analysis showed the student teachers' development of MKT was related to their professional noticing, however, three sessions was not enough time to develop all the content specific MKT the student teachers needed to robustly analyze the student work.

Because professional noticing of children's mathematical thinking depends on the mathematical content of focus and/or the teaching practice context being studied (student work, video, group work, etc.), professional noticing can and should be measured differently depending on the situation. To develop measures, the integrated MKT and professional noticing framework presented in this paper can be adapted to other content and contexts. Researchers should consider developing additional methods of simultaneously studying MKT and professional noticing.

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