

Paper Title: Designing Professional Development to Support Learning Trajectory-Based Instruction

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Abstract

Learning trajectories (LT) in mathematics have great promise for helping teachers organize their instructional and assessment practices around students' ways of thinking; however, the design and features of professional development that can best support teachers have not been firmly established. We worked closely with six third-grade teachers in the development and delivery of learning trajectory-based professional development (LTPD) designed to support their understanding of a LT on area measurement, their ability to use classroom activities and assessments to understand students' mastery of the LT stages, and their ability to tailor instructional responses based on the stages individual students still need to reach. The LTPD sessions included a variety of tasks and activities to engage teachers in discussions to promote their understanding of the LT and become comfortable using the LT as a framework for interpreting student ideas. Data were collected and analyzed to examine the extent to which the LTPD facilitated teachers' use of an LT for formative assessment and instruction. The findings support the use of LTPD to enhance teachers' formative assessment practices and identify specific types of activities that can best support learning trajectory-based instruction.

Keywords: design experiments; formative assessment; learning trajectory; measurement; teacher education; inservice/professional development

Designing Professional Development to Support Learning Trajectory-Based Instruction

Schools across the United States are facing the persistent challenge of increasing mathematics learning for all students, so they are college and career ready upon high school graduation. In order to enable students to meet rigorous standards, mathematics teachers need to develop expertise in planning and implementing effective instruction, which requires them to clearly and deeply understand learning targets and the pathways students can take to reach them. In recent years, learning progressions (LPs) and learning trajectories (LTs) have been widely touted as tools to help teachers implement ambitious practices and help all students meet rigorous standards. A LT is “a researcher-conjectured, empirically-supported description of the ordered network of constructs a student encounters through instruction (i.e., activities, tasks, tools, and forms of interaction), in order to move from informal ideas, through successive refinements of representation, articulation, and reflection, towards increasingly complex concepts over time” (Confrey et al. 2009, p. 346).

There are several examples of PD efforts aimed at helping teachers understand and use LTs. (We refer to these efforts as Learning Trajectory-Based Professional Development [LTPD]). These efforts (e.g., Carpenter, Fennema, Franke, Levi, & Empson 2000; Mojica & Confrey, 2009; Furtak et al., 2012; Petit 2011; Wilson, 2014) have shown some promising results, but have also encountered key challenges in preparing and supporting teachers to use LTs as the basis of instruction and assessment. This paper describes a research study designed to understand the features of LTPD which help teachers use a LT for formative assessment and provide differentiated instruction for individual students. In contrast to other LTPD efforts, ours was designed to be part of an integrated learning system tied together by the LT. The learning system consists of a digital game, online performance assessments, non-digital classroom

activities, and professional learning all aligned to a common LT on geometric measurement of area for grades K-4. Lai, Kobrin, Nichols, and Holland (2015) describe the development and content of the LT and the components of the learning system. Our two primary research questions were:

1. What LTPD components can credibly support teachers' use of a LT for formative assessment and instruction?
2. In what ways and to what extent did the LTPD support teachers' ability to use a LT to identify and interpret student thinking and devise instructional responses to enhance their formative assessment practices?

Method

Sample

Six third grade teachers from a public elementary school in the Midwest participated in the project which included the LTPD experiences. The teachers are all female and have an average of 7.2 (ranging from 2 to 14) years of teaching experience.

LTPD Design

The LTPD was designed using features and tasks described in the literature as successful in assisting teachers to learn about a LT, interpret students' thinking, develop their mathematics content knowledge, and use formative assessment (Mojica & Confrey, 2009; Wilson, 2014; Wilson, Mojica, and Confrey, 2013). The LTPD consisted of three main components - an introductory webinar, four face-to-face sessions and virtual forum discussions - which were delivered over 20 hours during the period October 2014-February 2015. Each session focused on one component of the larger project: the LT for geometric measurement of area (Session 1), online performance assessments (Session 2), classroom activities (Session 3) and student profile

(Session 4). Between face-to-face sessions 3 and 4, the teachers piloted the entire learning system in their classrooms.

Introductory webinar. The teachers attended a one-hour introductory webinar where they were introduced to the concept of LTs and how they relate to and are different from educational standards. The webinar focused on discussing the purpose and goals of the integrated learning system and described how its various components are connected to assist teachers with the teaching of area measurement including their formative assessment practices.

Session 1: Introducing the LT. The aim of Session 1 was for the teachers to learn how to elicit and interpret student thinking based on the LT. To meet this goal, we used a variety of hands-on activities, described in Table 1.

Name of activity	Description
<i>“Predict the strategy”</i>	Teachers were given a task asking them to write their predictions of how their students might approach the task. These tasks were open-ended so teachers could comment on a variety of strategies at different levels of sophistication.
<i>“Rank the strategy”</i>	Teachers were given student strategies on particular tasks and were asked to rank them from least to most sophisticated. These included strategies the teachers anticipated as part of the “predict the strategy” activities but also strategies that we created for them.
<i>“Match the strategy to the LT”</i>	Teachers were given multiple student strategies for one task and were asked to identify the corresponding LT stages. In another variation, teachers were given a task and the LT stages and were asked to describe strategies that students at those stages may use. Some of these activities included watching videos of students working on tasks.
<i>“Rank the tasks”</i>	Teachers were given multiple tasks on area measurement and were asked to rank them based on the level of difficulty or sophistication required to solve the tasks by referring to the LT stages.

Session 2: Online performance assessments. The aim of Session 2 was for the teachers to learn how to assess student thinking based on the LT. Therefore, this session was designed to familiarize teachers with the online performance assessments that were part of the integrated learning system. The performance assessments captured students’ strategies as they completed

each task, such as dragging unit squares to cover a shape, cutting and rotating shapes to fit into other shapes, and entering numbers into an online calculator. We used “match the strategy to the LT” activities where we showed the different tasks to the teachers and the stages of the LT that were intended to be assessed, and asked them to describe what a student strategy would look like at that stage. We also used a “build a model of student thinking” activity, which engaged teachers in examining the same student’s strategy to several different tasks. The teachers watched a series of videos of one student solving paper versions of one set of performance tasks and were asked to gather information about the student’s strategies in solving those tasks (including misconceptions) in order to create a model of the student’s thinking about area and make a conjecture about what stages the student had mastered and not yet mastered. In this session, we also started discussing what feedback or instructional activities could help the student master more sophisticated stages of the LT.

Session 3: Classroom activities. The aim of Session 3 was to engage teachers in the process of formative assessment during classroom instruction. To meet this goal, this session focused on the classroom activities from the integrated learning system and engaged teachers in a mock lesson designed to elicit student thinking and determine student status with regard to the LT stages. One of the researchers acted as the teacher and her role was to illustrate how formative assessment can be used in the act of teaching. We used “freeze the lesson” prompts to pause the mock lesson at different points and asked the teachers “What are some ways that students could respond to these tasks?” We also asked the teachers to discuss how they would respond with follow-up questions and feedback and additionally how they would modify tasks to evaluate students’ reasoning of other stages of the LT. The aim of the latter was to initiate a

discussion around the design of the classroom activities, emphasizing that these are just sample activities that can be modified according to the needs of their students.

Session 4: Student profile. After Session 3, teachers piloted the learning system in its entirety over a period of two weeks during their planned instructional unit on area. Student data collected in the piloting were used to create student profiles, which reported aggregated results from the performance assessments, game, and checks for understanding. Subsequently, the aim of Session 4 was to engage teachers in activities that would help them make instructional decisions targeted to specific LT-based goals using the student profile data.

During Session 4, the teachers were asked to view the student profiles for their classes and use the information to identify the stages of the LT their students had mastered and not yet mastered. In addition, the teachers discussed what they would do next in their instruction based on the student profile results. The student profile includes “watch outs” which flag individual students demonstrating common misconceptions or less sophisticated strategies in area measurement represented by particular LT stages. Teachers were asked to identify and discuss instructional strategies that would address each of these watch outs.

Virtual forum. An online virtual forum was established to engage the teachers in conversations about the LT in between face-to-face sessions. A “question of the week” was posted with additional examples of student work for teachers to analyze and discuss, similar to the “predict the strategy” and “match the strategy” activities provided during the face-to-face LTPD sessions.

Data sources and analyses

To inform the design cycles and provide evidence to address our research questions, we collected and analyzed data from various sources and engaged in both an ongoing and

retrospective analysis. As part of an ongoing analysis, we video-recorded the LTPD sessions and collected teachers' work and their responses to the questions of the week on the virtual forum. In our retrospective analysis we coded the video transcripts and virtual forum messages to identify instances where teachers: provided examples consistent with the LT; correctly or incorrectly identified stages of the LT; asked questions about the LT; gave general comments about the LT concept; and made relevant or vague instructional suggestions. We also identified places where we provided review of specific LT stages. Both authors coded the transcripts, and discrepancies were discussed until full consensus was reached. The frequency of the codes was examined over the course of the four LTPD sessions to determine if the teachers' discussions and interactions were consistent with our expectations based on how we designed the sessions.

Additionally, at the end of Session 4 we gathered retrospective testimonial data by asking the teachers to reflect on their experiences in the LTPD. Teachers discussed the LTPD aspects that were most helpful to understand the LT, the extent to which the LTPD impacted their instruction, whether LTs are useful and practical for teachers, and several other related questions. To analyze the testimonial data, we used thematic analysis (Braun & Clarke 2006) to examine themes and patterns across the responses.

Results

Teachers' responses to LTPD activities

The teachers' responses to the LTPD activities over the course of the four sessions show evidence of development in their ability to use a LT to identify student thinking and devise instructional responses. Towards the end of Session 1, after the teachers were introduced to the LT, they began identifying students' strategies by referring to the LT stages. By Session 2, teachers were able to predict students' strategies on the performance assessment tasks based on

specific LT stages, but it was clear that the teachers needed additional practice using the LT to devise instructional strategies to support students' progress. Consequently, in Session 3 we began to ask more questions and initiate discussions around instructional suggestions. Although initially teachers' responses were vague such as "I think I'll have to tell my kiddos to use the manipulatives," gradually they became more focused and aligned with the LT.

In Session 4, we gave the teachers a series of typical student misconceptions ("watch outs") and asked them to come up with specific instructional tasks that they would use to address each. Table 2 presents some examples provided by the teachers for assisting students with specific misconceptions and developing their reasoning about area measurement. These ideas are more targeted and specific than those offered during previous LTPD sessions for helping students address common misconceptions and master specific stages of the LT, showing development in teachers' adoption of the LT as a framework for assessment and instruction.

Table 2. Examples of teachers' instructional responses

Student Strategy	Instructional suggestions made by teachers
While using tiles to measure, a student leaves spaces between squares.	"Highlight the area they left out and remove the rest so they can see the area they left out." "Give real-world examples, e.g., if you were going to carpet a room, you couldn't do part of it, since you would have spaces..."
The student does not combine single units into rows and columns to measure.	"Don't give them enough blocks to cover the space - reduce the number of blocks so they would have to do a row or a column"
The student does not add columns and rows to compute area	"Ask the student, how can we find a faster way to do this, rather than counting them all. How can we use addition or skip counting?"

Teachers' interactions during the LTPD sessions

The analysis of the codes of the video transcripts and virtual forum provided additional evidence of teachers' progress in understanding the LT and using it as a basis for formative

assessment and instructional decisions. Based on our design of the LTPD sessions, we expected that teachers' interactions during the first two sessions would center mostly around the LT itself, and would shift more to instructional decisions in the third and fourth sessions. Figure 1 summarizes the teachers' interactions during each face-to-face session, based on the percentage of statements or comments coded in each category in a given session.

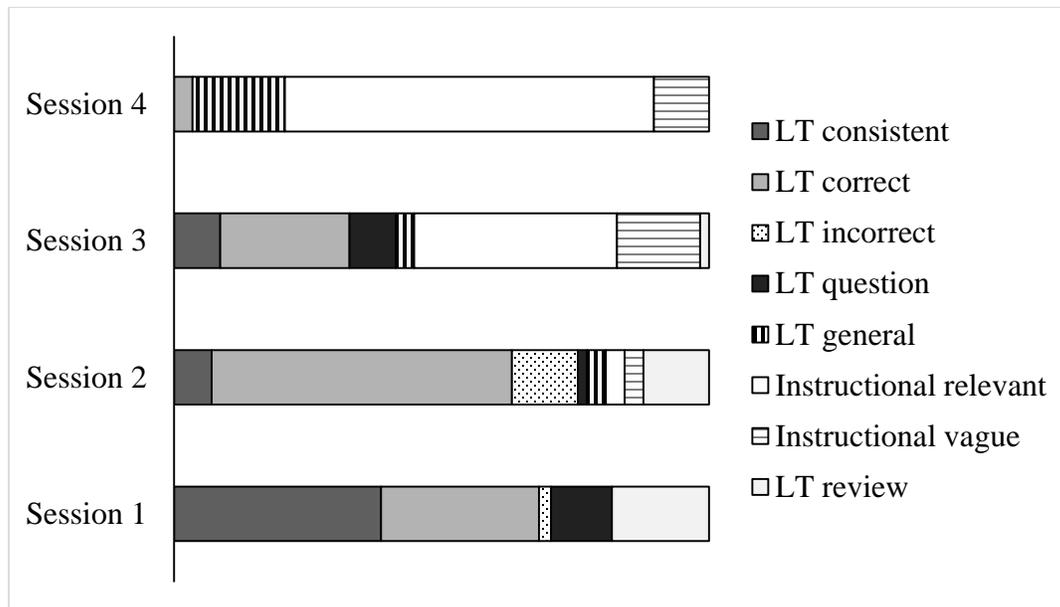


Fig. 1 Teachers' interactions during the LTPD sessions

As expected, during Session 1, teachers spent most time providing examples consistent with the LT and correctly identifying the LT stages in response to the questions and activities presented to them. There were some instances during the session where the authors reviewed LT stages to clear up uncertainty or confusion. During Session 2 the teachers began to get very familiar with the LT and most of their discussion included correct references to the LT stages when questioned or prompted. During Session 3, the discussion began to include instructional responses, and teachers' comments were evenly split between correct references to the LT and relevant instructional suggestions. Yet, there were also a number of vague instructional responses in Session 3. In Session 4 the vast majority of teachers' statements were relevant instructional

suggestions reflecting their understanding of how to help their students master particular stages of the LT, which was a primary goal of the LTPD.

Teachers' testimonials on the LTPD

The teachers' reflections on their experiences in the LTPD support the findings from our ongoing analyses. The teachers' comments suggested that the LTPD assisted them in better identifying student thinking to determine instructional next steps. For example, "It was kind of fun to see them struggle with something and be like, oh, I know what they were doing . . ." and ". . . my big takeaway from it is that I think when I do instruct that I am so much more knowledgeable now about what my kids need to know in order to, to do this."

Initially, the teachers thought instruction on area measurement was simply teaching the "length \times width" formula, but through the LTPD they became aware of how students gain a full conceptual understanding leading up to their use of the area formula. For example, "I think before I probably would have just been more leading them to using the formula, and just saying, you know, when you need to find the area this is how you do it. And let's practice." The teachers indicated that the LTPD helped them to break down the goals that they have for their students in order to help them gain full conceptual understanding of the area formula. For example, "I think I have a much deeper understanding of why the child is doing what they are, and how to help them more so because I understand the, the breakdown of [the stages that students move through]." Similarly, another teacher stated:

Yeah, I definitely think it gives you a deeper understanding. I mean, cause I would have gone through area at a much quicker pace probably and not, you know, there would have been a lot of gaps in what my kids knew. Because you

know, now looking back in the past at how I've taught area, it's like, okay, this definitely changes the way I'm gonna look at it.

Effectiveness of LTPD components

Teachers were also asked to provide feedback regarding the LTPD components that were most effective in helping them understand the LT and how to use it as a formative assessment tool. Regarding the main components of the LTPD (introductory webinar, face-to-face sessions, and virtual forum) the teachers said that they greatly valued the face-to-face sessions but got less out of the introductory webinar and the virtual forum. The teachers felt the introductory webinar was somewhat impersonal, and they would have preferred a recording so that they could go back and listen on their own time.

The virtual forum was designed to foster collaboration and discussion among the teachers so they could share their experiences and learn from one another. However, over the course of the LTPD, the teachers did not participate fully in the virtual forum as intended. Because the teachers were at the same school and interacted on a daily basis, they indicated that it was “just easier to talk in person.” The teachers did believe that the virtual forum would be helpful for teachers if they did not have colleagues participating in the LTPD. Several teachers commented on the importance of the collaborative nature of the LTPD:

Well, and I think the collaborative piece, like one thing that was really nice is when we were all here together; we were looking at the stuff together. And we could collaborate on it, and say, okay, you know, for this, if we're trying to get kids from here, what would you do? And we would write for a second, and then talk about it. Or we talked about it together. Like I think that's where you want the virtual forum to go.

With regard to the activities that were presented during the four face-to-face sessions, the teachers most valued the videos of students working on assessment tasks, which allowed them to “see exactly what they are doing instead of what they’re not doing.” In the first session we presented teachers with a table that summarized the stages of the LT with informal descriptions of student behavior at each stage. During the session, the teachers kept flipping back to the slides which showed pictures of student responses to assessment tasks which helped them visualize what a student response at a particular LT stage would look like. For the next session we added small pictures of example student responses to the table along with the descriptions. The teachers found the table extremely helpful and they referenced it frequently during the LTPD as they became familiar with the LT and examined student work to look for mastery of the LT stages.

Based on feedback from the teachers, we modified the LTPD for the next project iteration. In their testimonials, the teachers gave feedback that they would have liked a “frame of reference” as they were participating in the LTPD; that is, they would have liked more of an integration of the LTPD with classroom practice rather than having all of the sessions occur before implementing the learning system in their classrooms. As stated by one of the teachers,

I think having all the PD upfront and then doing all the stuff with my kids, I feel like when we would sit and talk, I had no frame of reference. Like all these concepts and things that kids would do, like I don’t know what *my* kids would do. And then after all that’s done, then we sit down, and I’m like, oh, look, they’re doing that thing that we talked about . . .oh, yeah. What was I supposed to remember about that? And I almost wish I could have had a little PD, do a little bit with my kids, a little bit. Because it was, I don’t know. I just, I felt at a disadvantage kind of.

In response to this comment, the LTPD was revised and now includes an initial task that teachers give their students and observe responses first-hand. They then reflect on their own students' responses as they are introduced to the stages of the LT.

Discussion: Reflecting upon the LTPD design and its outcomes

The aim of this project was to design and implement a LTPD based on a LT of geometric measurement of area. Stzajn (in Confrey et al. 2010) recommended that LTPD should not only help teachers understand the LTs themselves and how to interpret and use LT-based assessment results, but should also help teachers acquire productive classroom discourse strategies, and amass an array of appropriate instructional tasks. Our research findings provide evidence that our LTPD which was designed around this premise was generally successful in reaching its goals.

In contrast to previous efforts, the LTPD was part of an integrated learning system designed to align assessment, instruction, and reporting to the LT. The integration provided coherence to help teachers see connections between the LT and all of the other components of the learning system. The assessments and instructional activities were specifically designed to elicit responses that allowed teachers to determine whether students had mastered the stages of the LT. Providing LTPD without such integration could be quite challenging for teachers if their extant curriculum and assessments are not in alignment (Petit 2011).

Our LTPD purposely included a variety of activities delivered in different modalities so that we could gauge which were most effective and which were most preferred. Similar to prior LTPD efforts (Mojica & Confrey 2009; Wilson, Mojica, and Confrey 2013; Wilson 2014), we found that activities engaging teachers in deep analysis of student work were successful in helping them use a LT as a framework for formative assessment. By reflecting on the many different ways that students can approach a task, and using the LT to interpret those approaches,

we observed teachers making a shift from a focus on performance (i.e., whether the student is correct or incorrect) to a focus on student thinking. We also saw a shift in teachers' ability to make targeted instructional suggestions to help students master the stages of the LT. As they became familiar with the stages of the LT, their instructional suggestions changed from vague ones aimed at promoting mastery of the area formula (e.g., reviewing or re-teaching content), to specific ones aimed at helping students master a particular stage of the LT so that they can take one more step towards full conceptual understanding.

While all of the activities in the LTPD were successful to the extent that they generated reflection and discussion about student thinking, teachers most preferred watching videos of students completing tasks which gave them clear insight into students' cognitive processes and solution strategies, which they could relate to their own students. Moreover, the face-to-face sessions were clearly more successful than the virtual components for fostering rich interaction and collaboration among the teachers as they made meaning of the LT and used it as a framework for interpreting student ideas. While virtual PD has several benefits -- it is cost effective, is often more convenient for teachers, can leverage resources not available locally, and is potentially more scalable (Dede et al. 2006) -- researchers have found that online PD with asynchronous communication is not as effective as face-to-face meetings for creating and maintaining professional learning communities where teachers interact, share, and learn from one another to advance their collective knowledge (Scott & Scott 2010; Yang & Liu 2004). In our project, the fact that the teachers were colleagues at the same school and interacted on a daily basis made the virtual forum superfluous.

We learned that the way the LT is represented to teachers makes a difference in the way it is used and interpreted. We introduced the LT to the teachers using two representations -- a

poster-sized version which displayed all of the stages and their connections; and a set of tables which included for each LT stage a “teacher-friendly” description, observable student behaviors, and pictures illustrating a typical student strategies or responses illustrative of the stage. Throughout the LTPD sessions, teachers continually referenced the LT tables as they were engaged in activities and discussions. They referenced the poster far less frequently, and when they did so, it was usually to examine how the different strands of the LT (i.e., length measurement and area measurement) were connected. We believe that both representations were useful, but the LT tables were a crucial tool for teachers to use as a framework for formative assessment.

An important by-product of the LTPD was a development in teachers’ mathematics content knowledge on area measurement. Initially the teachers believed that area measurement is all about “length times width”; this perception was further enriched by the end of the LTPD to include all the foundational and intermediate stages that embrace and conceptually develop the concept of area. While during the LTPD, teachers showed evidence of increased content knowledge, and the ability to elicit and interpret student thinking and use assessment results to make instructional decisions, we were unable to observe whether they used that knowledge to make actual changes in their classroom practice. Because LTs require a reconceptualization of existing practice for most teachers, ongoing and sustained professional support is necessary for teachers to develop and use LTs (Furtak et al. 2010; Heritage 2008; Sztajn, Wilson, Edgington Myers & Dick 2013).

Although the LTPD in this study was conducted in one small group of third grade teachers, we identify the potential of its design for use in various communities, including, among others, pre-service and in-service teachers, teachers from different countries, teachers that teach

different grades and teachers with different personal learning goals. The feedback we gathered from this study was used to shape the next iteration of the LTPD, which was delivered virtually to a new group of teachers from two different continents.

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