Learning to Teach with Vision:

A Case of Preservice Elementary Mathematics Lesson Study

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1. Introduction

The visions teachers have for their classrooms represent their professional and personal aspirations. They range from those related to the immediate future, such as envisioning the nature of student interactions and conversations in an upcoming lesson, to more long-term visions, such as envisioning students with productive identities as learners. Visions for teaching are crucially important to teachers because they offer a guiding framework by which classroom decisions are made and judged. A teacher who envisions her future teaching uses the vision not only to influence her current classroom practices (Hodge, 2010) and performance goals (Shulman & Shulman, 2004), but to also frame her sense of self as a teacher. In the present investigation, we trace how preservice teachers' (PSTs) visions develop and support their learning to teach mathematics during lesson study.

In the literature review that follows, we first discuss teachers' visions, evolving global and national visions of mathematics teaching, and resulting challenges. We then discuss how teachers prepare to construct aspirational visions, concluding with the practice of lesson study and its unique affordances for PSTs. Before describing our methods and results, we outline the conceptual framework guiding this study.

2. Literature Review

2.1 Teachers' Professional Visions

In her study of novice teachers, Hammerness (2003) draws attention to how the development of a vision for teaching is consequential to teachers' future success. Early in their

professional training, teachers' visions may be vague, with a long distance between their developing visions and perceptions of their own practice. This distance can prompt extensive personal critique, negative judgments of their own professional capacities, and pessimistic thoughts that their visions are not attainable. Such personal critiques, though, are mediated by how teachers feel about the various learning environments they are inducted into for the teaching profession. In Hammerness' study (2003), when a novice teacher felt supported by her colleagues and school community, the teacher found aspects of the immediate setting (e.g., teaching resources) to be consistent with her vision, and was more likely to persevere to try to carry out her vision. In a non-supportive environment, however, another teacher felt her vision was unrealistic, concluding that she and the students were powerless to reach it.

van Es and Sherin (2002, 2008) conceptualize professional visions differently. Based on Goodwin's (1994) original notion of teachers' knowledge in their visions, they define *teacher professional vision* as a teacher's capacity to notice and interpret critical features of classroom situations (van Es & Sherin, 2002, 2008). Simply put, this vision is similar to what one's eyes can see with the recognition that what one sees relates to what is in one's consciousness. What a teacher sees and notices in the classroom is grist for her reasoning and makes it possible to connect local decision making to broader educational issues. Such thinking is crucial to her professional development. With this perspective, professional vision is not stable, but likely to change and develop over the teacher's career. The researchers use "Learning to Notice Framework" (van Es & Sherin, 2010, p. 156) to trace how teachers develop a set of different capacities to notice, reason, and make connections among classroom happenings.

In the current study, we combine these two perspectives on teachers' visions. We will consider that teachers have visions of ideal classrooms and work toward achieving them (Hammerness, 2003), while teachers' professional experiences provide opportunities to challenge, refine, and articulate their visions (Sherin & van Es, 2009). Hammerness (2003) characterizes teachers' visions as having three dimensions: *focus, range,* and *distance*. *Focus* is the vision's center of interest and clarity, from sharp to blurry; *range* is the field of the vision from narrow (e.g., individual classroom) to broad (e.g., ethnic community); and *distance* is how close or far one's vision is from her current teaching. Referring to the Learning to Notice Framework (van Es & Sherin 2010), we also hypothesize that teachers come to notice aspects of classroom practices more as their vision becomes more focused, becoming more adept at reasoning and analyzing different features of the classroom. We investigate this process by following a group of preservice teachers (PSTs) enrolled in a teacher education program. We consider their learning to be significant since it is a formative period of development .

2.2 Novice Teachers and Visions for Teaching

Nearly half of the teaching force in the United States leaves the profession within the first five years of teaching, and the rates are even higher in urban schools or high poverty areas (Alliance for Excellent Education, 2014; Darling-Hammond, 2006; NCES, 2015). Some novice teachers may feel their visions are not attainable, feel lost, or do not know how to make sense of the difference between where they are and what they envision. While professional learning communities (PLCs) are becoming commonplace in schools, teachers often work in isolation (Ingersoll & Strong, 2011), without the optimal degrees of learning opportunities provided by

PLCs (Horn & Little, 2010). Taken together, these factors contribute to how novice teachers burn out quickly and leave the profession early.

Some programs have addressed this issue by helping PSTs reframe this enactment gap as an opportunity to learn to teach by learning from teaching (Darling-Hammond & Hammerness, 2005; Shulman & Shulman, 2004). Development is reconceived as something that is not all or nothing, but incremental. Teacher educators must aim to take advantage of the PSTs' visions to support them in making strides towards ideal classrooms.

2.3 New Vision of Mathematics Teaching

Although a teacher's vision is internal, external information and experiences can influence its development. Recent global efforts have aimed to transform school mathematics by way of aspirational visions (e.g., NRC, 2001; Venkatakrishnan & Graven, 2006). Conway and Sloane (2005) report the global concern for mathematics education in our time, and it is documented that students worldwide are more adept at lower-level mathematics skills (e.g., computing, recall) than higher order skills (e.g., reasoning, application). While mathematics teaching has traditionally focused upon procedural fluency (Hiebert et al., 1996; Mullis, Martin, Foy, & Arora, 2012; Zakaria & Iksan, 2007; Schoenfeld, 1992), there is considerable professional agreement now that teachers need to broaden their focus (e.g., conceptual understanding) if students are to have access to higher-level mathematical understanding (National Council of Teachers of Mathematics, 1991, 2000, 2003; Conway & Sloane, 2005). Mathematical practices (e.g., reasoning and generalizing) should be at the center of instruction, and these practices are, for example, now a part of the *Common Core State Standards in Mathematics* (Common Core State Standards Initiative [CCSSI-M], 2010) in the United States.

The new vision of mathematics classrooms emphasizes these *mathematical practices*[i], which support students to engage with content as they grow in mathematical maturity and expertise (CCSSI-M, 2010).

Although teaching for conceptual understanding has been emphasized in international contexts (Dessimone, Smith, Baker, & Ueno, 2005), historically these practices have not been a significant part of mathematics teaching in the United States, and engaging students in them poses new challenges for many teachers (Ball, 1993; Chazan & Ball, 1999; Lobato, Clarke, & Ellis, 2005). PSTs may feel especially confused if they have not experienced or observed these practices previously. This lack of appropriate models can create distance (e.g., Hammerness, 2003) between the image of the classroom teachers aspire to have and current classrooms. Several scholars describe this gap between teachers' beliefs and practices (e.g., Clerk & Peterson, 1986; Richardson, et al, 1991), suggesting the distance between teachers' current and envisioned practices creates a space where teachers must constantly negotiate their vision and practice. This can contribute to the lack of clarity of the vision if, in fact, PSTs strive to teach according to this new vision without sufficient support.

2.4 Preparing Teachers to Teach Mathematics

The extant research literature identifies how PSTs are not equipped with appropriate content knowledge to teach mathematics well (Ball 1990a/b; Ball, Thames, & Phelps, 2008; Ma, 1999; Shulman 1986) and discusses the challenges generated by the distinction between classroom practices PSTs experienced as students and learning expectations for students in their own classrooms (Brady & Bowd, 2005; Brizman, 1991; Feiman-Nemser & Buckmann, 1986; Lortie, 1975; Zeichner & Gore, 1990). In this web of conflicting images of teaching, and

without strong content knowledge, PSTs could selectively attend to what seems relevant in their experiences in teacher preparation programs and the field, while disregarding other information (Borko, et al., 1992: Holt-Reynolds, 1992). In negotiating different paradigms, however, their beliefs and practices could remain inconsistent (Raymond, 1997), making it more difficult for them to develop practices aligned with one solid vision. In order for PSTs to continuously develop, the PSTs' learning experiences must be meaningfully coordinated.

Teacher educators have an immediate challenge to prepare teachers to teach mathematics in ways aligned with the new vision of mathematics classrooms. If prepared well, the new generation of PSTs can be the change agents in creating classrooms that emphasize ambitious mathematical practices, even when their current classrooms differ from their visions. To make that a reality, it is imperative to support PSTs to navigate the space where their initial teaching practices/skills and knowledge required for teaching may not be in sync. Professional learning experiences are key for supporting teachers to make sense of this process. In the following section, we describe how PSTs participated in lesson study—a form of a PLC—integrated into their mathematics teaching methods course to provide them multiple powerful learning opportunities.

2.5 Lesson Study

Lesson study is a professional development effort originating in Japan over 100 years ago, aimed at helping teachers explore and implement effective teaching practices (Author, 2006, 2011; Chokshi & Fernandez, 2004; Lewis, Perry, & Hurd, 2004). Throughout the 1980s and 1990s, lesson study supported a shift from didactic to student-centered teaching practices among Japanese teachers (Author, 2002, 2006, 2011; Lewis & Tsuchida, 1998), and over the last 15

years, it has attracted global attention (Author, 2011; Fujita, Hashimoto, & Hodgson, 2004; Lo, 2003). As a professional development model, lesson study provides a context for teachers to expand their knowledge of teaching (Fernandez, 2005; Yoshida, 2008) through a cycle of (a) goal setting, (b) curriculum analysis, (c) lesson planning, (d) teaching a lesson while being observed (research lesson), and (e) debriefing and reflecting, in an open and collaborative setting (Fernandez, 2005; Yoshida, 2008). Student thinking drives this process, as teachers are guided to make connections between each step of lesson study and student learning. In lesson study, teachers not only grow as professionals, but also enhance the professional community's body of knowledge about teaching (Fernandez, 2005; Lewis, 2009; Stigler & Hiebert, 1999).

Lesson study has been used primarily for inservice teacher professional development outside of Japan, but there are increasing examples of effective uses of lesson study in teacher preparation programs internationally (Author, 2011; Amador & Weiland, 2015; Bjuland & Mosvold, 2015; Fernandez, 2005, 2010; Helgevold, Nashelm-Bjorkvik, & Ostrem, 2015; Parks, 2008; Sims & Walsh, 2009). As teacher education communities place more emphasis on implementing the vision of ambitious math instruction as discussed above, lesson study presents a new possibility to facilitate PSTs learning. The cycle of teaching embedded in lesson study may help PSTs make connections between different parts of classroom practices while developing a deeper understanding of each practice (Author, 2011). Other scholars point out that lesson study can provide an integrated way for PSTs to engage in learning to teach (Parks, 2008), help PSTs question their own practice, and view feedback as a problem-solving approach rather than being solely evaluative (Sims & Walsh, 2009).

In order to help PSTs become agents of change in their future classrooms and school communities as discussed, it is essential to involve them in a professional community (Britt, Irwin, & Ritchie, 2001; Clark, 2001; NCTM, 2015), like those created in lesson study. In PLCs, teachers have opportunities to share and develop knowledge and expertise together, grounded in own local practices (Nelson & Slavit, 2008). The teaching vision developed in the PLC is likely to be more sustainable, as it is strengthened through the collaborative analyses and reflection based on shared professional knowledge of teaching. In the current study, we examine how visions of teaching develop when a group of PSTs work collaboratively rather than individually, and use lesson study as a research context to investigate PSTs' sense-making of classroom practices in relation to their developing visions of teaching.

3. Conceptual Framework

Through participation in teacher preparation programs and field practicum, PSTs not only develop new teaching knowledge and practices, but also begin to make sense of their new role as teachers. For some teachers, this may mean that they come to feel closer to their vision (i.e., that they have greater clarity and focus), and are more prepared to teach (Hammerness, 2003). Teachers may also begin to notice different aspects of classrooms and refine and change their visions (Sherin & van Es, 2009) to guide their professional learning.

The conceptual framework used to guide this investigation illustrates the relationship between the PSTs and their vision in Figure 1. Without professional experiences, the PSTs' vision for teaching may be loosely structured, unfocused, and vague. When the *distance* between PSTs and their visions becomes shorter, and the picture of ideal classroom becomes clearer, the teachers come to *focus* better on the vision, allowing to see themselves capable of

creating the classrooms in the vision[ii]. Different factors influence the development and articulation of PSTs' visions during their learning process, including an external reform agenda, local field practicum, and course content. PSTs may notice different aspects of classrooms along the way and reason about classroom practices as they move closer to their vision. Since the distance is a multi-dimensional phenomenon, and PSTs are constantly exposed to different aspects of classrooms, the *distance* may feel shorter or longer, depending on what they notice.

-- Insert Figure 1 here --

In this framework, we considered the learning of the group of PSTs as a whole, rather than that of an individual teacher. We value collaboration-based professional development for teachers and wanted to understand how the group context supported the development of shared understanding rather than focusing on an individual teacher's learning. Our thinking stemmed from sociocultural perspectives (Vygotsky, 1978) that focus on learning in social interactions, considering the meanings people shared in the group to be the main unit of analysis instead of an individual speaker's ideas (Roth & Hsu, 2010). We aimed to capture the essence of such processes.

Furthermore, we use lesson study as a research context because the collaborative process illuminates changes in teachers' thinking through discussions and interactions, in addition to identifying aspects of the teaching process that support the occurrence of these changes. In a professional education setting, a lesson study sequence can be further decomposed such that PSTs' interactions with key elements of the process receive particular attention. We leveraged this idea in designing the sequence of activities we later describe. We ask the following research questions in this paper:

How do the PSTs' visions for teaching guide their learning?

How does the focus of the PSTs talk change as they learn to teach mathematics?

How does lesson study support their learning?

4. Methods

4.1 Participants and Settings

We collected data from a cohort of 19 PSTs who attended a graduate-level teacher education program (multiple subject teaching credential plus master's degree) at a research university in an urban area in Northern California, United States. Their ages ranged between 23 and 45 (median age: 28), and they were placed in local elementary school classrooms during the 15-month program (academic year and summer sessions). Figure 2 provides the overview of the program structure, with course requirements and field placement schedule.

-- Insert Figure 2 here --

The study was conducted during the spring semester of the program year. In the fall semester, the PSTs took the required Cognitive Development course (Figure 2), which addressed some topics of mathematics teaching and learning. The teaching methods course in the spring semester provided the initial formal experiences for teaching mathematics. The PSTs participated in lesson study as part of the course, grouped by their placement grade-levels, for a total of five groups (three to five teachers each): kindergarten, first grade, second/third-grade (combined), fourth grade, and fifth grade. Lesson study was adapted to work with field placement arrangements and programmatic time constraints, while focusing on each step of lesson study cycle. Each lesson study group met together once a week for approximately one hour over six weeks to collaboratively develop a geometry lesson (content area chosen by course

instructor, the primary author of this paper), which was taught by one of the group members; the agenda for each meeting is listed in the lesson study calendar, Table 1.

-- Insert Table 1 here --

4.2 Data Sources and Analysis

In this section, each subsection begins with a description of the data, followed by collection and analysis processes.

4.2.1 Pre- and post-interview video and transcripts. Each grade-level lesson study group was interviewed by trained research assistants at two time points, before and after the lesson study cycle (Table 1). Both the pre- and post-interview protocols (see Appendix) were designed in order to gain insight into the PSTs' general sense about teaching and to provide a context to discuss specific teaching practices.[iii] Interviews were video-recorded and transcribed.

Data analysis began with iterative examinations of the transcripts to identify main themes based on the specific interview questions about PSTs' general experiences. This portion of analysis was conducted collaboratively in research meetings with all authors present, and discussion continued until we came to 100% agreement on each item (Miles & Huberman, 1994). We initially identified categories that appeared important to framing PSTs' very broad visions (e.g., *Bringing Community Knowledge to Classrooms, Professional Support, Communication with Parents,* and *Classroom Community*). Because we wanted to understand how the PSTs envisioned their ideal classrooms, we then specifically focused on interview sections when the PSTs discussed their teaching practices. In doing so, we attempted to identify concrete descriptions of such practices that if an experienced teacher reads it in the lesson plan, she would be able to implement it. We utilized Hammerness' (2003) conceptualization of

teacher visions and hypothesized that as novice teachers' visions become clearer, their descriptions of classrooms and teaching practices would become more detailed (Figure 1). While there were many such instances in the interviews, we began by identifying two main dimensions of teaching practices the PSTs appeared to value: *Accessible Classroom Practices (ACP)* and *Supporting Student Collaboration (SSC)*. Within these dimensions, there were sub-dimensions (e.g., SSC included student discussion and student collaboration, ACP included materials, settings, and content), but maintaining larger dimensions helped present a clearer picture of the PSTs' vision development described through practices[iv].

At this point, we also recognized that sometimes these practices were discussed in relationship to students' mathematical thinking. Therefore, we also considered that *Understanding Student Thinking (UST)* is an important and distinct dimension. Descriptions and examples of the dimensions are found in Table 2.

-- Insert Table 2 here --

4.2.2 Lesson study meeting video transcripts. Lesson study meetings were video-recorded for three of the five lesson study groups[v]. The meeting data of the kindergarten group were chosen for analysis because they provided the richest data with careful description of their learning process.

The transcripts of the kindergarten group's six lesson study meetings (Table 1) were analyzed using the dimension codes in Table 2. In order to develop inter-coder reliability, two trained researchers met and coded one meeting transcript together while discussing and solidifying their understanding of the dimension. They then independently coded half of the second meeting transcript and discussed their coding results and resolved discrepancies (initial

coding produced 64% agreement, and 100% agreement was reached after discussion). Because the initial level of agreement was rather low, they again independently coded the other half of the second meeting transcript (this time, the initial round produced 90% agreement, 100% agreement was reached after discussion). The remaining transcripts were divided between the two coders for independent coding.

After the transcripts were coded, we computed the PSTs' utterances across different dimensions, to allow for a frequency analysis of dimensions. One utterance was typically one turn of talk made by a person. In some cases, an extended utterance including more than one idea was coded as two chunks. These were then separately analyzed as "integrated talk" (double coded) to see how the PSTs were making connections among different dimensions in their discussions (we will discuss this later).

4.2.3 Research lesson video. A video recording of the kindergarten group's research lesson was coded using *StudioCode* software. First, two trained researchers reviewed the video and coded for teacher and student actions. The coders then labeled different teacher and student actions to better understand the lesson sequence, producing lesson segments. Finally, the coders reviewed the video again, focusing on each lesson segment and coded teacher actions for their alignment with our category dimensions in Table 2.

4.2.4 Research lesson materials. Lesson materials (e.g., revised lesson plans) and student work (e.g., completed worksheets) were collected and analyzed to add details to the research lesson analysis.

5. Results and Discussion

In this section, we will first describe the visions the cohort of PSTs expressed in this study. We then use examples from the kindergarten lesson study group to carefully outline how their vision of teaching guided the learning process through lesson study and how their vision was in turn shaped. We will give examples of their teaching of the research lesson, and how the vision was reflected in the teaching.

5.1 Vision of Teaching: Pre- and Post-Interviews of Whole Cohort of PSTs

Pre- and post-interview data analysis revealed that the PSTs shared a strong desire to teach mathematics in student-centered and collaborative ways similar to the vision of new mathematics classrooms presented in international and national reports (e.g., CCSSI-M, NAEd, NCCA, TIMSS). In this section, we describe how the entire cohort shared this vision to ground the detailed account of the kindergarten group's learning that will follow.

The PSTs envisioned classrooms where instruction was driven by student collaboration, which was reflected in their descriptions of specific classroom practices. In a pre-interview, a fourth-grade PST described his approach to lesson planning:

> "[in my lesson] I would plan a lot [of] opportunities for students to talk to one another and to share out as a whole class their thinking, their understanding. I think that equal voices is an important part of the classroom… the students speak as much as I do, so that I'm not just saying content, but that we're having a conversation."

This focus on *all* students engaging in discussions is closely aligned with the visions presented in international and national documents for mathematical practices, and had an undertone of equitable classrooms. This could be partially because the PSTs were exposed to these visions through course materials with detailed discussion of the reform movement[vi].

A first-grade PST described the motivation behind her decision to include opportunities for deep collaboration in her math lessons:

> "…all of our students have math partners, but we've thought a lot about the pairing of those, so that they're beneficial to each other … everyone's gonna learn differently, and maybe how I explain things isn't as helpful as their peer who explains it. And I think that's almost another level of math is not only being able to do it yourself but then assist someone else in their learning."

A second-grade PST discussed a lesson she had taught,

> "… so even in just a small way trying to relate or link it to their interest, it's like, "Oh, we play half-court basketball, how can we compare [to the whole court]?" So, just a small thing. Those boys then kind of shifted and showed signs of interest."

These excerpts from the pre-interviews illustrate how the PSTs' visions of classrooms are those in which students' voices, interests, and thinking are central to the instruction in collaborative contexts. In our analysis, we noted similar descriptions of this idea in every interview. Even early in their teaching career, they held strong images of classrooms that engage all students in successfully *doing* mathematics.

We were interested in finding out how this vision guided their learning to teach mathematics through lesson study. In the following sections, we will now zoom in on the kindergarten lesson study group to describe how envisioning student collaboration supported the group's learning process in lesson study.

5.2 PSTs' Learning Process: Kindergarten Lesson Study

The kindergarten lesson study group consisted of three PSTs (Stevie, Cole, and Tina[vii]) who began the teacher education program with prior experiences working with children (e.g., camp counselor, after-school art instructor), but without formal classroom teaching. They were

placed in kindergarten classrooms to align with their desire to work with young children in lower-elementary grades upon graduation.

In starting to code the entire set of kindergarten lesson study meeting transcripts, we anticipated that the specificity of the PSTs' talk describing teaching practices might increase through the lesson study process (as informed by Hammerness' original framework and suggested by the patterns seen in interview analysis across grade-level groups). The results of the data analysis did not support our prediction entirely. Overall, the ratio of talk regarding specific classroom practices to all talk changed only slightly across lesson study meetings, and without a clear increasing trend (meetings 1-6 contained respectively 34%, 38%, 48%, 47%, 35%, and 54% of talk related to specific practices). This might have been driven by the lesson study structure that guided the PSTs to discus more than specific teaching practices and focus on particular aspects of teaching in certain meetings (e.g., curriculum, content).

There was a change in the foci of the PSTs' talk, however. Table 3 shows the frequencies of talk of specific classroom practices and student thinking over six meetings. Overall, the PSTs discussed student thinking increasingly over the lesson study process (7% to 39%), and the talk about accessible classroom practices stayed more or less constant. However, this pattern did not hold in Meeting 5, as the PSTs talked less about student thinking and more about accessible classroom practices. More careful examination illuminated how the PSTs' discussions in Meeting 5 were qualitatively different compared to other meetings. This was the final meeting before the research lesson where PSTs finalized their lesson plan and logistical details.

Discussion of *how* to support student collaboration, a central part of their visions, remained fairly constant over time. While the frequencies were not high, we hypothesized that the constant presence of this dimension was an important and consistent theme of their discussions. Overall, the increase in the PSTs' discussion around student understanding is the most notable trend in this portion of data analysis.

-- Insert Table 3 here --

Further analysis revealed that not only did the foci of the discussions change over time, but there was also a shift in the way the PSTs' talk integrated the three central dimensions: *Understanding Student Thinking, Accessible Classroom Practices, and Supporting Student Collaboration*. For example, in Meeting 6, when discussing ideas for their research lesson, the PSTs' discussion was more integrated. In the following excerpt, they anticipated how students would match and sort four-sided shapes (paper cut-outs):

> Stevie: And you have to match the same shapes up like this [ACP].
> Others: oh, cool!
> Stevie: because these students kind of talk about, like …
> Cole: … the orientations [UST]
> Stevie: Yeah, right? And then they'll kind of realize that they can manipulate the orientation of shapes (to match) [ACP, UST].
> Cole: Mm-hm.
> Stevie: To make them match something … so that would be like position [ACP].
> Tina: Right, or even, in terms of like conversations, too, like begin able to communicate what you're thinking (while working together) … [SSC, UST]

In comparison to Meeting 1, in which individual utterances or parts of conversations tended to focus on a single dimension, in Meeting 6, the PSTs' talk often integrated the dimensions together. This happened in two ways: sometimes, one PST's utterance included more than one dimension, while at other times, more than one PST discussed the same topic in a short

conversation in ways that combined different dimensions (example above). The bottom row of Table 3 shows the increasing frequencies of integrated talk in the meeting data.

By combining these dimensions when discussing an idea, the PSTs were making sense of how student understanding of mathematics could inform (accessible) classroom practices, which often included student collaboration. A more integrated way of talking reflects a more integrated set of ideas to create their envisioned classroom, not merely noting features but conceiving of the more complex, interdependent components of teaching. In their study of inservice lesson study, Author (2012) found that teachers made connections among student learning, teaching, and content over time, shown by the ways their talk changed. Integrated talk could also support the further development of the PSTs' visions (student collaboration), because understanding student thinking is a foundation for initial planning for student collaboration, and accessible classroom practices (based on that understanding of student thinking) is necessary in order to support meaningful collaboration among all students.

Considering all of the analyses discussed above, we developed a hypothetical model of the PSTs learning through lesson study (see Figure 3). The PSTs' learning process in this model included four phases: First, PSTs tried to identify collaborative classroom practices (Phase 1). Next, they realized they needed to know how students thought about mathematics in order to predict students' engagement and collaboration in activities, which prompted careful discussion about student thinking of mathematics (Phase 2). The developing understanding of student thinking then helped to inform lesson decisions for collaborative classroom practices to make the content accessible (Phase 3). This entire process led to a point in which PSTs enacted the vision in the lesson (Phase 4).

-- Insert Figure 3 here --

In the following section, we will use excerpts of the meeting discussions to illustrate how the nature of PSTs' talk changed through the lesson study process, based on these four learning phases.

5.3 Kindergarten PSTs' Learning Process through Lesson Study

5.3.1 Phase 1: Identifying collaborative practices. At the beginning of the lesson study process, the kindergarten PSTs primarily focused on planning a lesson to encourage student collaboration. Their discussions illuminated the strong desire for meaningful collaboration among students, but lacked detailed descriptions of practices or student thinking.

In Meeting 1, the PSTs discussed potential goals for the lesson with each other:

Cole: Do we want to come up with some kind of social and affective goals, and then maybe come up later with our activity? *[looking from one to the other]*
Stevie: Yeah … I think that'll be good.
Tina: … So, like, whether it's partner work or table group work or whole class …
Cole: Oh, that's really good point. 'cuz I know [in] your class, you said that they work mostly in …
Tina: … in pairs. Always in pairs.
Stevie: … they can share their thoughts with each other, before sharing out full group, y'know?
Cole: … well, do we think the partner will be beneficial for them in this [lesson]?
Tina: Well, I feel like it's so closely linked to what the lesson [would be] …
Stevie: Or, we could also do a combination of, like, several of those [partner, whole group, small groups] things.
Tina: well, yeah, like, in my lessons, I always start whole class … and then they branch out into partners or table groups, and then we come back whole class …
Stevie: So, we have to think about … I think we should think about the activities we want to do …

Attempting to create a shared image of the lesson they would teach, the PSTs' attention was on the structural aspects of the lesson—rather than conceptual—which is typical of novice teachers'

lesson planning. It was apparent that the PSTs wanted students to work together, and from there, they began brainstorming possible activities and discussed the following:

> Tina: But what would the lesson actually be?
> Stevie: I think, classifying, maybe? Classifying shapes and coming up with, like commonalities and differences?
> Tina: Hmmm …
> Stevie: Making a chart, like a compare and contrast type thing?
> Tina: Like, *"go out with your partner. Look around the room!"*
> Stevie: Uh-huh. Yeeeah, that would be cool.

In the excerpt above, Tina and Stevie tried to think of a lesson activity, and became excited when they thought of a way to help students interact as partners in the activity. Their planning discussion, while meaningful to the PSTs, was not purposefully guided by a conceptual focus. They considered possible lesson activity ideas and their desire for student collaboration and engagement, but students' mathematical thinking was missing from the conversations. This discussion lasted about 45 minutes, with the PSTs discussing different collaborative lesson structures and brief moments when one PST brought the group back to the goal of the lesson, which was yet to be determined.

5.3.2 Phase 2: Realizing the importance of students' mathematical thinking. In Meeting 2, the PSTs created a pre-assessment to gain a sense of their students' current understanding of mathematics before planning the lesson[viii]. During the meeting, the PSTs discussed how they needed to consider students' prior mathematics knowledge as a part of their assessment. For example, while they created the pre-assessment, Stevie and Tina had a conversation anticipating how their kindergarten students would respond to the assessment activity:

Stevie: What if our kids know what rectangles are? Should we talk about other shapes with them to find out, what shape should we bring to the lesson? Or what do you guys think?
Cole: Even if they can name it, that may not mean that they recognize it in the way that we're asking them to – if it's bigger, or if it's smaller …
Stevie: Yeah, right, right …
Cole: We may want to just have like … kind of like your idea of having a jar of different shapes, and then just saying *"can you pull out all the rectangles?"* Just to see if they can …
Others : Right.
Cole: and … "why did you pick these?" Or "how do you know?" … and so then we can kind of see like where their thinking is? That's definitely a good thing.

In this conversation, the PSTs were not only starting to consider student thinking in relationship to the activity, but were also presenting a stance as inquiry-oriented teachers wanting to learn more about their students' mathematics understanding. The PSTs discussed interactions between activities and student thinking as an essential part of planning. Compared to Phase 1, they were realizing (in Meeting 2) how student thinking is an essential component of planning an engaging lesson, yet showed no direct connection between that and their vision of teaching, which will become more solid in latter phases.

5.3.3 Phase 3: Using assessment and research to inform lesson decisions. In Meetings 3 - 5, the PSTs analyzed pre-assessment data, and used that information to inform their planning of the research lesson. They also made sense of different curricular sequences and typical patterns of student thinking by studying different lesson materials and research literature provided in their course. Lesson planning was the longest phase of the lesson study process, as the PSTs considered how to take student thinking into account while planning the activities.

In this phase, the PSTs' vision of collaborative classrooms remained strong. In Meeting 3 during lesson planning, Cole considered the possibility of differentiating the activity to support different levels of thinking while students collaborate:

Cole: Can we give them a (work) sheet that they work together on that has Ricky the Rectangle[ix] in different environments? And that's where our differentiation can come in, like we have more rectangles of different orientations …

In Meeting 4, Tina and Cole also discussed:

Tina: If we wanted to have a collaborative element, maybe each student would do like one page on something, and we could staple it into a book?
Cole: [in agreement]… not each person is experimenting with different rectangles. But they sort of see it coming together in class …

In the two excerpts above, the PSTs' discussion of student collaboration included student understanding of mathematics. Now, collaborative activities are serving multiple purposes– creating a space for differentiation and teacher assessment. The PSTs consider how to balance individual work and collaborative work in the lesson by providing students different sets of rectangles, and emphasize how the group context would help individual students to experience more than each could do separately.

In the following excerpt from meeting 4, the PSTs discuss connections among how students think mathematically, a lesson activity to make content accessible, and student collaboration:

Tina: … even though we haven't really had these, like, critical thinking class discussions, I feel like, because of their age, they get really into it – because they get to express what they see and what they think, and like about them … *it's so exciting to learn how to like bridge their conversations, like help them connect to each other*, y' know? (emphasis added)
Stevie: … vocabulary that students need to be able to use and understand in order to really explain positions and locations … I feel my kids know how to do that, right? … they would say like, *"next to" "de lado de"*. … I've heard my kids use that language …
Cole: Yeah! My students have definitely used [the term].
Tina: … for them to know it and use the word, and then also to put those two things together and, like, spatially use them and know where things are?
Stevie: … I think this is pretty interesting [turns the student textbook to show a particular page to the others] … and you have to match the same shapes up like this.
Cole: Oh, coooool! I love this!

Stevie: Because these students kind of talk about like …
Cole: … the orientations.
Stevie: Yeah, right? And then they'll kind of realize that they can manipulate the orientation of shapes and to make them match something.

In this instance, the more the PSTs discussed lesson details and student thinking, the more concrete their descriptions of student collaboration (vision) became. The collaborative setting pushed the PSTs to articulate how they envisioned their classrooms individually, to create shared vision as different possibilities were discussed and evaluated.

Conducting the pre-assessment in their field placement classrooms, analyzing the data with van Hiele's geometric thinking framework, and using the results to inform their emerging lesson ideas helped the PSTs situate student thinking in the center of their planning process. The PSTs were somewhat surprised at the results of the pre-assessment and made several adjustments to their lesson plans accordingly. The lesson study process made this possible by helping the PSTs use student data meaningfully to inform their practices.

5.3.4 Phase 4: Classroom practices to support the vision of collaborative classrooms. Planning and teaching the research lesson was a way for the PSTs to instantiate their vision. Cole taught the lesson, while Stevie and Tina assisted. Cole's cooperating teacher, the course instructor, and a research assistant observed and collected student-learning data during the lesson, and afterwards everyone debriefed.

After teaching the lesson, all three PSTs discussed their lesson observations. In research lesson debriefing (Meeting 6) the PSTs were asked to utilize the actual student data collected and reflect upon 1) lesson implementation, 2) surprising moments in the lesson, and 3) what they would do differently if they taught the lesson again.

Stevie discussed the lesson by first noting student thinking observed in the lesson, the collaborative aspect that supported their learning, and a lesson direction that was not taken in the lesson implementation (to show the integration of dimensions in teacher talk, we added the different dimensions coded in the quotes).

> Stevie: … [the] majority of students were able to identify these two characteristics [of a rectangle: four sides and four corners] *(UST),* and they were able to then give us a reason. The generation of the rubric [written list of properties of rectangle, created collaboratively with students] *(SSC)* was extremely helpful in that way *(ACP)* …I thought there would be more talk about square, that student[s] would be interchanging those a bit more *(UST).* Well, then that could have totally taken a different turn in the lesson, and we would've talked about the length *(ACP),* which would've been okay …

Stevie's talk was integrated with different dimensions, indicating how she was making connections among them. The vision of student collaboration is now substantiated by accessible classroom practices, based on student thinking of mathematics.

In talking about how they might teach the lesson differently in the future, the PSTs discussed different possibilities:

> Cole: … maybe just have "a" rectangle … something where they can manipulate – whether it be on geoboards that they're building rectangles *(ACP).* That'll be kind of interesting. It could be a good for [the/a] group to see if the one they're building, 'is yours the same as mine?" *(SSC)* so they can kind of manipulate that, and then they'll get into some kind of a sort …
>
> Stevie: Using shapes in the environment … that would have been cool. Because they [students] were just super interested *(UST)!*

The PSTs could now talk about different aspects of the lesson, potential lessons, and student learning, demonstrating a significant amount of learning since the beginning of the lesson study, when the primary concern was lesson structure and surface features of collaboration. In the following section, we will now illustrate how the kindergarten PSTs' vision of a collaborative classroom was enacted in the research lesson.

5.4 The Research Lesson

The research lesson is a context in which teachers can demonstrate their learning through lesson study (Author, 2011), and the lesson the kindergarten group taught was no exception. In this section, we use examples to discuss how different dimensions were integrated in the group's actual teaching of the research lesson.

The lesson focused on student exploration of rectangles, and was taught in Cole's classroom as the first lesson in the geometry unit. In the PSTs' written lesson plan, the learning goal was identified as correctly recognizing a rectangle regardless of orientation and size (CCSS, Kindergarten Geometry, 4).

There were four main segments to the actual lesson, for a total duration of slightly over an hour. The first segment began with a whole group discussion, where Cole used a big book the team created, titled *Ricky the Rectangle*, in which rectangles are oriented in various ways (e.g., horizontally, vertically, diagonally). Cole led the students in a discussion of the characteristics of Ricky the Rectangle as Ricky changed his orientation. For the second segment of the lesson, Cole elicited responses from students to generate a list of the characteristics of rectangles while Tina charted students' responses. Students came up with following characteristics: "*[oriented] up," "four sharp tips [corners]," "four outside lines," and "marks"*[x].

For the third segment, Cole led the students in an activity to determine if all of the shapes in the book were, in fact, rectangles based on the student-generated list of characteristics. After this brief discussion, Cole explained the sorting sheet students would use at their desks to sort shapes into rectangles or non-rectangles. While students worked with each other on this task,

Stevie, Cole, and Tina circulated the room to provide assistance as needed. For the last segment of the lesson, students returned to the whole group discussion, where Cole debriefed the activity.

One student, Jaime, shared his thinking about how and why he sorted the shapes; he had labeled a rectangle, oriented diagonally, as a non-example. Cole wanted to reinforce the idea that the properties of rectangles held true regardless of orientation and asked,

Cole: Why was that *not* a rectangle?
Jaime: Because it's skinny … and it's facing that way [pointing to its diagonal lines]
Cole: Let's ask ourselves those two questions [point to the list]
Ss: How many sharp tips does it have?
Cole: Let's count them.
Ss: 1, 2, 3 … 4!
Cole: So we do have 4 sharp tips. What's the next thing that we have to ask?
Ss: Does it have 4 lines?
Cole: Let's check. Does it have 4 lines?
Ss: 1, 2, 3, 4! (undistinguishable murmurs, some students say 'it's a rectangle!')
Cole: What are you thinking now? … tell your friends why you think it's a rectangle? (Students talk in pairs simultaneously, while Cole listens to their individual discussion carefully.)
Cole: … So this shape has 4 sides and 4 corners. It doesn't matter if it's a bit skinny. It's a rectangle.

Using the list of characteristics generated on their own, students analyzed the shape and agreed that it was a rectangle[xi]. The sequence of the activities and the discussions in this lesson were all targeted to the lesson goal of recognizing shapes regardless of orientation, which some students struggled with (e.g., Jaime thought that if oriented diagonally, it was not a rectangle). Cole involved the whole class to review the criteria on the list, check the shape, then engage in partner talk and share why they thought the shape was a rectangle (or not). In multiple ways, the lesson reflected the PSTs' vision for student collaboration while simultaneously focusing student thinking on the content goal of the lesson.

6. Summary of the PSTs Learning in Lesson Study

In synthesizing our analyses, Figure 4 illustrates the PSTs' learning phases over six lesson study meetings in the study. The bar graphs show the frequencies of their talks over time per different dimensions (changing focus), and the line graph shows the increasing integrated talk in each meeting when more cohesive connections among the dimensions were made. The structure of lesson study (Figure 4 includes meeting descriptions) supported the PSTs learning process by helping them articulate a primary vision (student collaboration), focus on student thinking of mathematics, and evaluate accessible classroom practices to enact the vision while keeping the practices close to students' mathematical thinking.

-- Insert Figure 4 here --

The visions of classrooms the PSTs brought to the setting was supported and refined through the lesson study process. Table 4 summarizes how lesson study supported the PSTs learning in this study.

-- Insert Table 4 here --

By recurrently bringing the PSTs attention to student learning, lesson study encouraged PSTs to make connections among different parts of teaching (e.g., goal setting, assessment, planning, teaching, reflecting), with their visions in mind. Student thinking is the core driving force of the process, and the PSTs developed stronger and more connected visions in relationship to student thinking. The collaborative nature of lesson study also helped the PSTs to openly discuss their visions, ask questions, examine and evaluate different ideas together, and try ambitious teaching approaches based on student exploration and discussion (Author, 2011). The supportive nature of the group process made it possible for the PSTs to incorporate open-ended questions and facilitate

students' conceptual understanding in the lesson. Our analysis of post-interviews indicates that the PSTs felt more prepared to teach in ambitious ways aligned with their visions and to venture into the profession with bolstered confidence in their abilities as teachers. Without this focused structure of lesson study, the PSTs' learning could have taken longer, and essential aspects of their learning may have never come together, thus leaving possibility for confusion and disconnections in their development processes. Lesson study did not force the PSTs to do what that they might perceive irrelevant to their own learning, instead guided the process by making close connections to natural components of classroom practices.

7. Conclusion and Implications

As illustrated above, the PSTs in this study learned to teach in the process of enacting their vision of teaching. The vision of student collaboration guided the PSTs' learning process, and at the same time, they developed better understandings of how the accessibility of the content, addressing appropriate levels of student thinking, and relevance of the activities can increase student engagement. This was evident in the way the focus of their talk shifted over time to support the development of these classroom practices and the different dimensions of their learning became further integrated.

All teachers have visions of teaching, even PSTs. With lack of experience, however, PSTs may have little ideas about how to make their vision come to life in the classrooms; their image of an ideal classroom is vague and unfocused. They may think of a lesson activity that ostensibly connects to the vision, but without deeply considering student understanding, the activity may not be engaging or accessible to the students. This may give PSTs the impression that successful teaching largely depends on chance, and the important relationships between

teaching and learning will remain unexplored. When PSTs have only experienced traditional and procedural teaching as young students, it makes it even harder for them to make sense of different lesson ideas for conceptual and student-centered teaching. PSTs may become confused and end up teaching in uncertainty, which may in turn disengage their students. When the learning of PSTs is meaningfully aligned with their vision, they can feel empowered and supported, and the path from where they are to their vision becomes clearer (Figure 1). A novice teacher may have a vision of an ideal classroom or ideal student learning, but in each of these visions, the teacher needs to develop her agency in order to feel prepared to teach.

Research has identified that it is critical for teachers to understand the importance of students' mathematical thinking (e.g., Carpenter, et al., 1996; Franke, et al., 2001), and the findings of this study suggests one effective way to help PSTs learn this and feel prepared is by connecting it to their developing visions and classroom practices. If it is taught separately, student mathematical thinking can remain an abstract idea with little relevance to actual classroom teaching for PSTs. Experiencing student thinking as a crucial lever for enacting one's vision creates a professional commitment, and seeing how classroom practices can be used to enact this vision can help develop a sense of efficacy for teaching.

The study extends the previous research on teacher vision by illustrating how the novice teachers came to notice different aspects of classroom practices in more integrated ways, made the aspects work together concretely, and attained more stable professional vision. Their learning resulted in not only noticing different dimensions of classrooms but also understanding connections among what they noticed. Their vision was nurtured through the lesson study

process, from being a one-dimensional vision of collaboration focusing on the structure of activities, to a mutli-dimensional and integrated vision of collaboration.

The two frameworks of teacher professional visions – guiding vision (Hammerness, 2003) and functional vision (van Es and Sherin, 2010) – were essential in understanding this teacher learning process in the study. As the PSTs' visions further developed, the minimizing distance illustrated in Figure 1 no longer accurately portrayed their learning. The clarity of the vision was achieved not by simply reaching it but by being able to *notice* different dimensions of classroom practices and accepting that it would be an ongoing process of negotiation to make the vision possible. Several PSTs mentioned, in the post-interview, how they had come to realize there was much more to learn, while they were more confident about how they might approach classroom challenges in the learning process. This could imply that the PSTs came to accept the existence of the space between the envisioned and current classrooms, while feeling more equipped to navigate the space in between.. Developing a complex vision, paradoxically, helps teachers see more clearly about what the vision is, along with more nuanced sense of what the work of teaching involves. For novice teachers, in simply thinking they know something without understanding the complexity of the idea can lead to confusion. Experiencing small successes while gaining the understanding of the complexity surrounding one's vision should allow the development of efficacy. The vision of teaching is not something to reach and finish, rather, something to strive for while noticing additional factors that influence classroom practices and continuously making adjustments for increased student learning.

(8933 Words)

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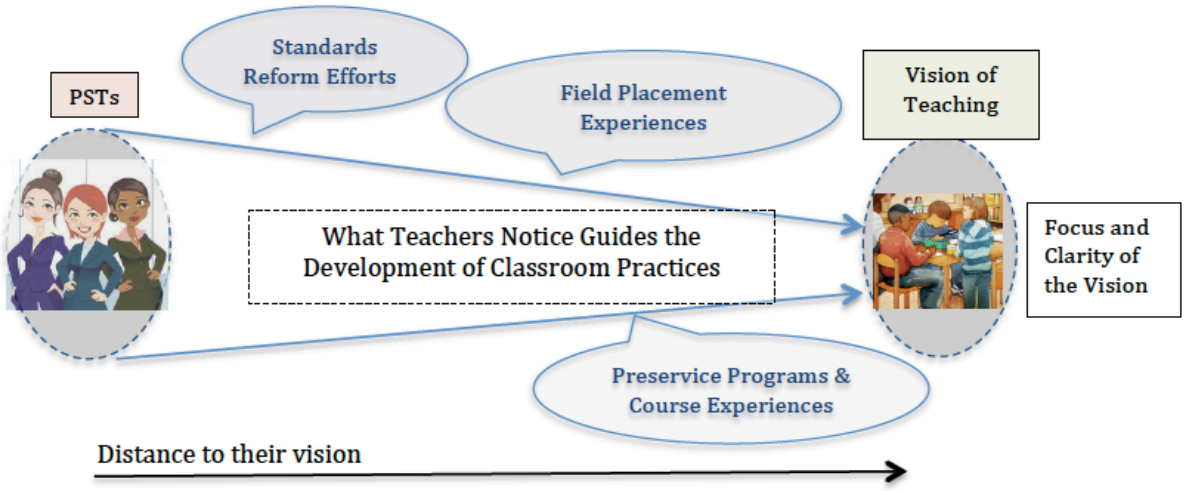

Figure 1. PSTs and Vision for Teaching.

Timeline (15 months)	Summer Session (June – August)	Fall Semester (August – December)	Spring Semester (January – May)	Summer Session (June – August)
Courses	- Social Development - Language Arts Methods - Science Methods - Arts - Technology	- Cognitive Development - Urban Education - Reading Methods - Special Education	- Math methods - Social studies methods - ELD methods	- Masters of Arts Thesis Seminar
Field Placement	Summer School (2 weeks as interns)	First Placement (2 full days or 3 mornings per week)	Second Placement (3 full days or 5 mornings per week)	Summer School (2 weeks as lead teachers)

Figure 2. The teacher-education program structure.

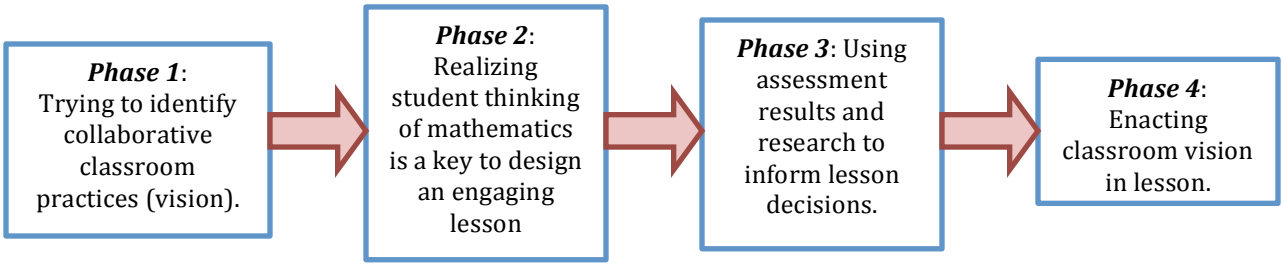

Figure 3. Four-phase model of PSTs learning to teach

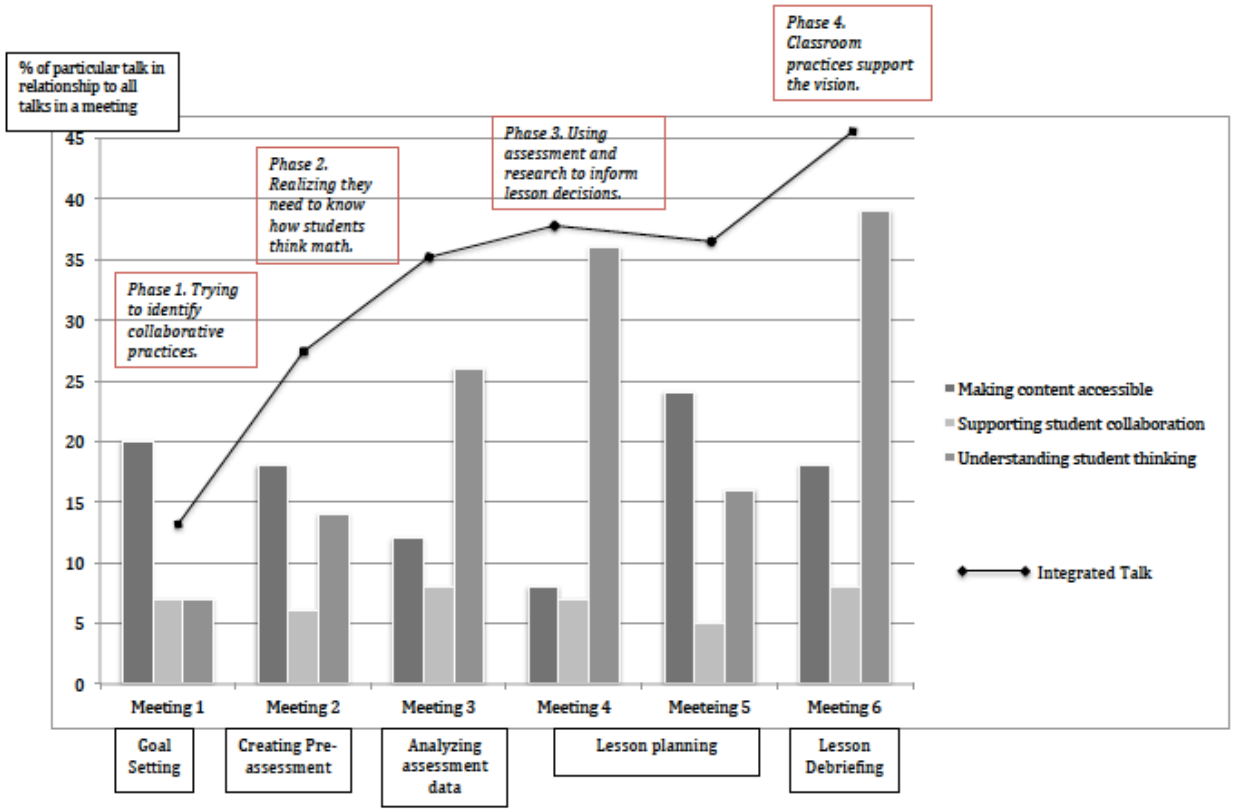

Figure 4. The PSTs' learning phases over six lesson study meetings

Table 1.
Lesson study calendar.

Meetings	Lesson study schedule and agenda
(Before meeting 1)	Introduction to lesson study (pre-interview)
1	Set goals for lesson and examine CCSSM standards
2	Create pre-assessment tailored to explore student understanding of the lesson content
(Between 2 and 3)	Conduct pre-assessment with small samples of students in placement classrooms
3	Analyze students' pre-assessment data
4	Plan unit and lesson (general)
5	Plan unit and lesson (specific)
(Before meeting 6)	Teach the research lesson in one PST's classroom, with all members observing
6	Debrief the research lesson (immediately afterwards)
(After meeting 6)	Group reflection (post-interview)

Table 2.
Dimensions for analysis.

Dimension		Description	Examples
UST	Understanding (anticipating) student thinking	Discussing how students think mathematically & how they reason. Description of student thinking processes.	*Students recognized same length sides or different length sides, or four right angles.*
ACP	Accessible Classroom Practices	Description of how certain activities and teaching will give students better access to the content.	*Giving students different shapes and asking them to create a rule with the shapes.*
SSC	Supporting student collaboration	Description of how certain activities and teaching will support students to work together.	*When a kid is sharing, we can have another kid in the audience explain how his friend solved the problem.*

Table 3.
Frequencies of PSTs' talk dimensions over six lesson study meetings.

	Meeting 1 *(goal setting)*	**Meeting 2** *(creating assessment)*	**Meeting 3** *(assessment analysis)*	**Meeting 4** *(planning—general)*	Meeting 5 *(planning—specific)*	**Meeting 6** *(debrief)*
Understanding Student Thinking	**7%**	**14%**	**26%**	**36%**	16%	**39%**
Accessible Classroom Practices	**20%**	**18%**	**12%**	**8%**	24%	**18%**
Supporting Student Collaboration	**7%**	**6%**	**8%**	**7%**	5%	**8%**
Integrated Talk	**14%**	**29%**	**36%**	**39%**	36%	**46%**

Table 4.
Lesson Study and the PSTs Learning.

Lesson study starts with goal setting	It pushed the PSTs to articulate the important aspects of the teaching-learning process, thus clarifying aspects of their classroom visions.
Lesson study requires teachers to know students' current thinking	As the kindergarten PSTs created pre-assessments and collected data on their own students' understanding of geometry, they had opportunities to make connections between students' prior content knowledge and the collaborative lessons they envisioned initially.
Lesson study provides focused planning time	As they planned the research lesson, all three focal dimensions (Understanding Student Thinking, Accessible Classroom Practices, Supporting Student Collaboration) came together, creating a concrete image of their classroom. The lesson study process was purposefully designed to provide structure for continuous analysis of their own teaching practices in relation to student thinking. By spending multiple hours planning, the PSTs could think deeply about different aspects of the lesson, and make informed decisions about how to teach the lesson.
Lesson study has a research lesson (live observation of the lesson) and debrief	The PSTs experienced their vision of classroom concretely in the research lesson. By debriefing, they reflected on what went well in the lesson, what was surprising, and how they may extend the student learning they had observed in the lesson in the future. The collaborative support and shared learning process helped the PSTs feel successful and confident to teach in a way that they envisioned their future classrooms to be.

Appendix. Interview Protocols

Pre Interview

1. What has motivated you to choose teaching as a career?
2. On the basis of what you now know about teaching or what you have already experienced, what do you think will be the most interesting, stimulating and rewarding aspects of your work?
3. What do you think will be most challenging?
4. How well prepared do you feel to teach?
5. How are you feeling about teaching math?
6. What experiences have contributed to your becoming a successful teacher? (e.g., course work, field experiences)
7. Is there anything you feel unsure about teaching math at this point?
8. What aspects of teaching are you looking forward to most?

The PSTs were given the following case written as a handout:

> Lisa is a new 3rd grade math teacher. What concerns her most is how to provide all her students the opportunity to learn as much as possible.
> *Lisa: The students in my class have very different attitudes about school in general and math in particular, ranging from those who are highly motivated to learn to those who don't seem to care in the slightest. Of course, this doesn't just affect their grades but also levels of focus and the learning environment generally. Some of them are so interested in math that they do extra math at home. These students get good support and follow-up from their parents. Others don't seem to do any math work at all, either at school or at home. Part of what is especially difficult is that many of my students do not seem to be prepared to learn the 3rd grade material. I also have six students who have reading difficulties and five whose first language is not English and who have only been living in the US for a short time. How can I help my students learn?*

1. What are your own feelings about this situation? Does anything strike you as especially difficult?
2. What kinds of issues do you think Lisa will encounter when she teaches a class of children with many different needs?
3. What do you think Lisa means when she talks about students who are not prepared to learn 3rd grade material?
4. What kinds of question do you think Lisa should be asking herself?
5. How would you plan this lesson?
6. What do you think would be a reasonable learning objective for the first lesson?
7. What would you keep in mind when deciding how to teach this topic?
8. What would you expect to be especially difficult for third grade students learning multiplication for the first time?

9. How will you assess whether the students have achieved your learning objectives?

Post Interview

1. How is your field experience going? If they say "we enjoy it," follow this up by asking them what parts they are enjoying most of all and least of all. Get the candidates to elaborate on the adjectives they use and to be more specific about why they feel that way.
2. How does it feel to be a part of the school community? Do you interact with other staff members?
3. How was the lesson study experience for you?
4. What did you enjoy the most from this process?
5. What was challenging for you?
6. How did you come to a consensus about which topic to teach? How did you use the ideas from your cooperating teachers? How did you decide on social goals?
7. How did you create the student preassessments? If they don't mention materials: Did you use any materials? Why did you use those materials? If they don't mention van Hiele's levels of geometric learning: How did you incorporate van Hiele's levels of geometric learning?
8. How did you go about making sense of the student preassessment data? What was your experience like finding common patterns, what students did well, and their challenges?
9. In planning lessons, did you have several lesson ideas? If so, please describe them. Why did you choose the lesson that you chose? Where did the ideas come from and how did you decide which one to use for the research lesson? How did you use the information gained from student preassessment to inform lesson planning?
10. Did you find it challenging to anticipate student responses to each question/problem you posed when writing the lesson plan? Please give one example from your group discussions and describe how you came to the several anticipated responses written in your lesson plan.
11. How did you determine the instructor of the lesson and in which classroom to teach the lesson?
12. What was the experience like being observed by your peers? How was it different from your normal teaching? Why was it different from being observed by your cooperating teachers or supervisors?
13. How did the debriefing of the lesson go? What did you learn from your peers' observation (if you taught the lesson) or by observing your peer teach the lesson you had planned together?
14. From what part of the lesson study experience did you learn the most?

15. On the basis of what you now know about teaching or what you have already experienced, what do you think will be the most interesting, stimulating and rewarding aspects of your work?
16. What do you think will be the most challenging aspect of teaching when you have your own classroom in September? What are your next steps to work toward overcoming these challenges?
17. How well prepared do you feel to teach?
18. How are you feeling about teaching math now?
19. What experiences have contributed to your feelings about being a teacher?

20. Is there anything you feel unsure about teaching math at this point? (e.g., students, teachers at the school, own subject knowledge, own skills)
21. What aspects of teaching are you looking forward to most?

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[i] The Common Core State Standards (2010) identifies eight mathematical practices: 1) make sense of problems and persevere in solving them, 2) reason abstractly and quantitatively, 3) construct viable arguments and critique the reasoning of others, 4) model with mathematics, 5) use appropriate tools strategically, 6) attend to precision, 7) look for and make use of structure, and 8) look for and express regularity in repeated reasoning.

[ii] In this study, "range" of the vision, as originally discussed in Hammerness' (2003) framework, is not included as the PSTs focused on the setting (range) of their individual classrooms.

[iii] In the pre-interview, PSTs discussed a hypothetical classroom vignette. In the post-interview, PSTs reflected on their lesson study lesson.

[iv] Practices were coded without regard to tense; that is, PSTs were coded as describing a practice whether it was something they had seen before or something they anticipated in the future.

[v] Limited research assistant availability precluded recording of all five groups.

[vi] Lesson study was conducted in the latter half of the course, whereas the PSTs learned about and discussed student-centered teaching and learning during the first half of the course.

[vii] Pseudonyms are throughout the paper.

[viii] Pre-assessments were administered to a sample of students from each classroom.

[ix] The group named the main character of the story, an anthropomorphic rectangle, Ricky the Rectangle.

[x] Students noted that to draw the lines of the rectangle, they would need to make four marks.

[xi] This is a student-generated and limited list of the attributes of a rectangle.