How Teachers Evaluate Breaches of Norms in High School Geometry

Nicolas Boileau
University of Michigan
nboilea@umich.edu

Patricio Herbst
University of Michigan
pgherbst@umich.edu

Abstract

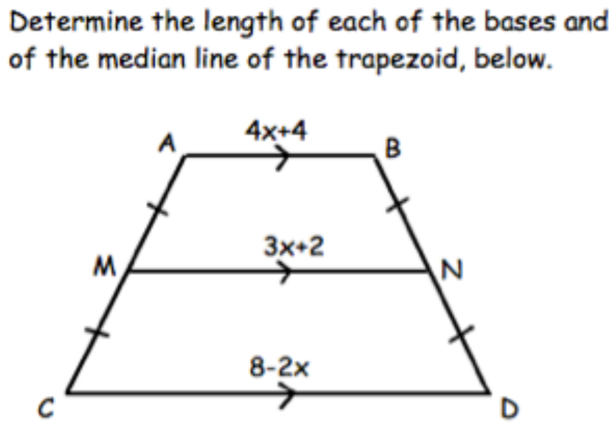
This paper reports on a study of two norms of an instructional situation in high school geometry - Geometric Calculation in Algebra (GCA) - that consisted of confronting practitioners with representations of practice as a way to stimulate them to relay their knowledge of practice. In the situation of GCA, students are provided with a diagram in which some dimensions are represented by algebraic expressions and are tasked with setting up and solving equations, on the basis of properties of the figure, then using the solutions to those equations to calculate one or more of those dimensions. One of the norms that we conjecture regulates work on GCA tasks is that solutions to valid equations will result in valid dimensions (i.e., non-negative side lengths and/or angle measures). The other is that the teacher would not expect that the geometric property used to set up an equation would be documented in writing, although they may ask for it to be stated verbally or may be satisfied with it being implied by the equation. The purpose of the study was to determine whether participants recognize the two hypothesized norms and how those participants evaluate actions that breach them. The focus of this paper is on our use of the Appraisal system (Martin & White, 2005) in Systemic Functional Linguistics to detect evidence of participants' evaluation of actions that breach one of those norms.

Introduction

This paper reports on a study of two hypothesized norms of an instructional situation in high school geometry that Hsu (2010) called Geometric Calculation in Algebra (GCA). The study was a variation of a breaching experiment (Garfinkel, 1963) that consisted of confronting 40 high school mathematics teachers with representations of practice (specifically, a set of cartoon-based storyboard representations of classroom scenarios in which a student is solving a GCA task) as a means of eliciting their tacit knowledge about the situation of GCA (specifically, their recognition of its norms). In order to capture that knowledge, we asked participants to complete an online multimedia questionnaire that consisted of open- and closed-response questions about each storyboard that they saw. The purpose of the study was to determine whether high school mathematics teachers recognize our two hypothesized norms and how they negotiate the meaning of episodes where a norm has been breached; specifically, how they evaluate that breach. The focus of this paper is on our use of the appraisal system (Martin & White, 2005) in Systemic Functional Linguistics (Halliday, 1978; Halliday and Matthiesen, 2004) to detect evidence of participants' evaluation of actions that breach one of those norms in the open-response data.

Theoretical Perspective

The practical rationality of mathematics teaching (Herbst & Chazan, 2003; Herbst & Chazan, 2011) underpins the design of the research instrument presented in this paper (the multimedia questionnaire) and provides the theoretical grounds for investigating norms of instructional situations. The theory contributes to the literature on teachers' decision-making that teachers' actions are motivated not only by their individual beliefs and knowledge, but are also responsive to instructional norms and professional obligations (to the discipline of mathematics, the institutions within which they teach, individual students, and to the class as a social group). It also suggests a focus on instructional situations - "segments of interaction whose goal is to produce mathematical work and exchange it for claims on the knowledge at stake," (Herbst, Nachlieli, & Chazan, 2011, p.225). One example of such an instructional situation is the situation of GCA, described by Hsu (2010) as one in which the teacher provides a geometric figure in which some of the dimensions are represented by algebraic expressions and are tasked with setting up and solving one or more equations on the basis of one or more properties of the figure. What is at stake in this situation is both the knowledge of those geometric properties and of the algebraic skill required to solve the equation(s). The following is an example of a normative GCA task that was included in one of our scenarios:



We hypothesize that, in this instructional situation, it is normative (i.e., tacitly expected by teachers and students) that:

1. When a GCA task is given to students, it is expected that the algebraic expressions associated to the dimensions of the figure are such that, when an equation is set-up on the basis of one or more true geometric properties of the figure, the numerical measures obtained from the solution of such an equation will have interpretable geometric meanings (e.g., side lengths and angle measures will be positive).
2. When a student is tasked with solving a GCA problem at the board, it is expected that the geometric theorem or property that they use to set-up any equation in order to determine one or more dimensions of the figure will either be implied by that equation or that the teacher could request that it be stated verbally (e.g., if the teacher believes that it was not clear).

We refer to the first of these as the GCA Figure (GCAF) norm and to the second as the GCA Theorem (GCAT) norm.

Method

Data Collection

As stated in the introduction, the purpose of the study was to determine whether high school mathematics teachers recognize our two hypothesized norms and how they negotiate the meaning of episodes where a norm has been breached; specifically, how they evaluate that breach. To that end, we adopted a variation on Garfinkel's (1963) breaching experiments, referred to by Dimmel (2015) as a "virtual breaching experiment with controls". This consisted of showing a sample of 40 high school mathematics teachers (from the Heartland) representations of classroom scenarios realized with storyboards of cartoon characters, some of which contained breaches of one or both of the aforementioned hypothesized norms, then asking them a series of open- and closed-response questions about each scenario. After participants viewed a segment of a scenario, they were asked: “what did you see happening in this segment of the scenario” and were given an open-response field to type their response. Then, we asked them to evaluate the teacher’s actions in certain segments of that scenario, using a 6-point Likert-scale, and to justify each of those ratings, in an open-response field.

The sample of teachers was a convenience sample, who volunteered to participate in the study, in response to a recruitment email that was sent to high school mathematics teachers from across the state.

Data Analysis

In Boileau and Herbst (2015), we report on the analysis of the open-response data for evidence of participants’ recognition of the two norms and of their ratings of the teachers’ actions, in order to compare how participants evaluated instances where either of the two hypothesized norms were breached with instances when they were complied with. To complement that analysis, this paper reports on the ways that participants who recognized instances where the teacher in the scenario breaches the GCAT norm evaluated those breaches. The reason for focusing on the GCAT norm is that none of the participants that recognized any of the breaches of the GCAF norm evaluated them.

In order to analyze how participants evaluated breaches of the GCAT norm, we applied two 1/0 codes to all of the open-response data - one indicating whether that response contained a positive appraisal of a breach of the GCAT norm and a second indicating whether the response contained a negative appraisal of a breach of the GCAT norm. In the next section, we provide examples of when this code was applied.

Based on that coding, we assigned the following two dichotomous scores to each participant’s set of responses to a given scenario:

- Positive appraisal of a breach of the GCAT norm (1 if there was evidence in any of their open-responses that the participant positively appraised the GCAT norm, and 0 otherwise).
- Negative appraisal of a breach of the GCAT norm (1 if there was evidence in any of their open-responses that the participant negatively appraised the GCAT norm, and 0 otherwise).

Results

One of the main results of the analysis is that the proportion of participants that recognized the breach of the GCAT norm in a given scenario and negatively appraised it (M=0.30, SD=0.06) was higher than the proportion of participants that recognized the breach of the GCAT norm in a given scenario and positively appraised it (M=0.16, SD=0.05), $t(55)=1.83$, $p=0.07$. The following are two examples of responses that were coded as containing an appraisal of the GCAT norm, from our data:

> The equation to begin is proof enough that the student knows the theorem. I suppose it can be argued that we want OTHERS to be able to follow the written solution and that's valid. Still, minutes are being used while the class watches the kid write what he/she just said out loud. Unnecessary.

> I like the questioning, "What theorem did do you use?" [which the teacher asked after the student set up an equation, based on a property of the figure, in each of the scenarios] I don't think it was necessary to have the student write justifications on the board while the entire class waits.

Both of these responses contains a positive appraisal of the GCAT norm (“The equation to begin is proof enough...”; “I like the questioning, ‘What theorem did do you use?’”), but a strong negative appraisal of a breach of it (“the class watches the kid write what he/she just said out loud. Unnecessary”; “I don't think it was necessary to have the student write justifications on the board”). The negative appraisals are consistent with Garfinkel’s (1963) report on some of his breaching experiments, in which participants who experienced a breach of a norm “vigorously sought to make the strange actions intelligible and to restore the situation to normal appearances." (Garfinkel, 1963, p. 232). The positive appraisals can be interpreted as attempts to “restore the situation”, by suggesting how they expected it to unfold.

Educational Importance of the Research

Investigations of norms of instructional situations are important as they improve our understanding of why mathematics is taught and learned in the ways that it is in the course(s) in which the situations occur. In terms of the specific instructional situation of GCA, “research on geometric calculations has… received little attention” (Hsu, 2007), despite the fact GCA problems “promote reasoning” and require students to “use and connect mathematical representations” – mathematical practices supported by NCTM (2014) - Principles to Actions.

Scientific Importance of the Research

This research is also important as tacit knowledge (e.g., norms) has been widely acknowledged as difficult to elicit and measure (Goffin et al., 2010; Lee, 2000; Murray & Hanlon, 2010; Szulanski, 2000), although also influential on the ways humans interact in social situations (e.g., students and teachers in instructional situations). In this paper, however, we have demonstrated how breaching experiments and linguistic theory can be paired to tackle this challenge.

References

Dimmel, J. K. (2015). How do teachers expect students to represent mathematical work? A study of teachers’ recognition of routine ways that proofs are presented and checked in high school geometry (Unpublished doctoral dissertation). The University of Michigan, Ann Arbor, MI.

Garfinkel, H. (1963). A conception of and experiments with" trust" as a condition of concerted stable actions. In O.J. Harvey (ed.), Motivation of Social Interaction (381-392). New York: The Ronald Press Company.

Goffin, K., Baxter, D., Szwejczewski, M., Cousens, A., & van der Hoven, C. (2010). Tacit knowledge development in a new product development teaching simulation.

Halliday, M. A. K. (1978). *Language as social semiotic* (p. 136). Arnold: London.

Halliday, M.A.K. and C.M.I.M. Matthiessen (2004) Introduction to Functional Grammar. 3rd Edition, London: Edward Arnold.

Harel, G., & Sowder, L. (1998). Students' proof schemes: Results from exploratory studies. CBMS Issues in Mathematics Education, 7, 234-283.

Herbst, P. & Chazan, D. (2011). Research on practical rationality: studying the justification of actions in mathematics teaching. The Mathematics Enthusiast, 8(3), 405-462.

Herbst, P., Nachlieli, T., & Chazan, D. (2011). Studying the practical rationality of mathematics teaching: What goes into “installing” a theorem in geometry?. *Cognition and Instruction*, *29*(2), 218-255.

Hsu, H-Y. (2007). Geometric calculations are more than just the application of procedural knowledge. In J. H. Woo, H. C. Lew, K. S. Park, & D. Y. Seo (Eds.), Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education (vol. 3, pp. 57- 64). Seoul, Korea.

Hsu, H. (2010). The study of Taiwanese students' experiences with geometric calculation with number (GCN) and their performance on GCN and geometric proof (GP). Unpublished doctoral dissertation, The University of Michigan, Ann Arbor, MI.

Lee, L. L. (2000). Knowledge sharing metrics for large organizations. Knowledge management–Classic and Contemporary Works, 403-419.

Martin, J. R., & White, P. R. R.. (2005). The Language of Evaluation: Appraisal in English. Palgrave, London.

Murray, A., & Hanlon, P. (2010). An Investigation into the Stickiness of Tacit Knowledge Transfer. Paper presented at the 13th. Annual Conference of the Irish Academy of Management, Cork Institute of Technology

National Council of Teachers of Mathematics (NCTM). (2014). Principles to Actions: Ensuring Mathematical Success for All. Reston, VA: Author.

Szulanski, G. (2000). The process of knowledge transfer: A diachronic analysis of stickiness. Organizational behavior and human decision processes, 82(1), 9-27.