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A scaling up study of the SimCalc approach revealed significant learning gains that were robust across demographic and regional variation in teachers and students. In order to determine what might have contributed to these gains, we theorized that students' opportunities to engage with content would be a significant source of information about what and how students learned. We developed a representational tool we call Content Maps that we used to analyze the content of classroom discourse around mathematical tasks. Maps that were generated from three teachers' enactments of three lessons reveal the various ways in which these teachers drew on their mathematical knowledge in whole-class discussion. These maps may therefore prove to be a more useful assessment of their mathematical knowledge as a learning resource than quantitative measures of their mathematical knowledge for teaching.

Background

In this paper we present portions of the data from larger case studies of three teachers selected from a randomized, experimental study that investigated scaling up the SimCalc approach to proportionality (Roschelle et al., 2007). The experimental study revealed significant learning gains in students' knowledge. These gains were robust across demographic and regional variation in teachers and students, and attributable to a combination of potential factors that the experiment did not attempt to disentangle, including professional development, a curriculum replacement unit, and teacher knowledge and practices. Thus, the experimental study also generated new questions about how teachers' knowledge is brought to bear in instruction. Noting that studies of curriculum interventions that found effects rarely studied how the effects were achieved, Stein, Remillard, and Smith (2007) called for more studies "designed to include both large-scale tests of curricular effectiveness and smaller, but embedded, observational studies of instructional practice" (p. 339).

In the spirit of responding to this call, we chose three classrooms in which to document the kinds of opportunities that were created for students to engage with the content. We theorized that students' opportunities to engage with content would be a significant source of information about how teachers' knowledge is brought to bear in instruction. We found in the results of the experimental study (Roschelle et al., 2007) the lack of a consistent relationship between teachers' scores on an assessment of their mathematical knowledge for teaching and their students' pretest-posttest gains (Shechtman, Roschelle, Haertel, & Knudsen, 2008) and were reminded of Hill, Rowan, and Ball's (2005) claim that "effectiveness in teaching resides not simply in the knowledge a teacher has accrued but how this knowledge is used in classrooms" (pp. 375-376). This led us to wonder if an assessment of how a teacher uses what she or he knows about a given topic to direct tasks and interact with students could be a more accurate characterization of teacher knowledge as a learning resource than a teacher's score on a content knowledge assessment. In this proposal, we present some of the findings of our explorations of how teachers' mathematical knowledge generates opportunities for their students to engage with content through whole-class discussion around mathematical tasks.

Theoretical Framework

As part of the field's ongoing efforts to identify links between the qualities of teachers and the achievement of their students, Hill, Schilling, and Ball (2004) have built on the work of Shulman (1986, 1987) and Ball (1990) to develop a measure of *mathematical knowledge for teaching* (MKT), or "the mathematical knowledge used to carry out the work of teaching mathematics" (p. 373). After identifying a positive relationship between MKT and student gains in mathematics achievement, Hill, Rowan, and Ball (2005) called for research that investigates how teachers' mathematical knowledge affects their instructional decisions. We draw on Hill and colleagues' conception of mathematical knowledge for teaching and situate this work within a research agenda that seeks to understand the relationship between teacher knowledge and student learning.

Next, we use *connectedness* as a lens through which to examine the nature of mathematics content as it was presented and developed during instruction. Connectedness is associated with learning with understanding and the coherence of instruction (Hiebert & Carpenter, 1992; National Research Council, 2001). Cohen and colleagues emphasized that "coordinating instruction … depends on making connections among teachers' and students' ideas, among students' ideas, among both over time, and between both and elements in the environment" (Cohen, Raudenbush, & Ball, 2003, p. 126). Similarly, Ma suggested that a teacher who possesses a "profound understanding of fundamental mathematics" has the ability draw on the connectedness of the domain in instruction to "connect the current topic under discussion with concepts from the field with more conceptual power (depth), as well as those concepts that are of equal or lesser power (breadth) while being able to weave them together (thoroughness)" (1999, p. 120). Consistent with these claims, Hiebert and Grouws (2007) synthesized evidence from a variety of studies to argue that teaching for understanding is associated with explicit attention by students and teachers to making "connections among ideas, facts, and procedures" (p. 391). The connectedness of classroom discourse can therefore serve as a measure of students' opportunities to engage with mathematics, in particular, the mathematics that supports learning with understanding.

Methods

The content of the replacement unit used by teachers in the experimental study was based on SimCalc MathWorlds® (Kaput, 1997) and used computer simulations of motions to teach the big ideas of rate and proportionality with a focus on linear function. To explore students' opportunities to engage with the content around mathematical tasks in the replacement unit, we focused on the development of concepts across lessons and the quantity and quality of connections among ideas that were made by teachers and students within whole-class discussion. Teachers' MKT was not brought to bear on the assembly of curriculum, because the same curriculum was provided to all of them. Therefore, enactments of that curriculum in whole-class discussion provided for a more viable resource for information (Hill et al., 2008) about how teachers' knowledge is leveraged in instruction.

We transcribed all lessons and segmented the transcripts into episodes. Then we created *Content Maps* to represent the content and connectedness of classroom talk in these episodes. Like concept maps (Novak, 1990), Content Maps are composed of links and nodes. Consistent with the ways that Hiebert and Carpenter have described networks of knowledge (1992, p. 67), nodes are external representations of pieces of information including mathematical ideas, assertions, propositions, procedures, predictions, reasons, and story lines. Links between nodes represent connections or relationships that are explicitly stated or implied in what participants say or do. Nodes and links are organized into clusters that correspond to instructional episodes focusing on a task.

Unlike concept maps, which are intended to represent the knowledge structures of an individual, Content Maps are based on classroom interactions and are meant to represent the *enacted content*—the substance of what teachers and students talked about and otherwise attended to. We draw on Herbel-Eisenmann and Otten (2011) who demonstrated that mathematical meanings are indeed construed in discourse. Then we use content maps as a means to operationalize that construal and display it in a form that illustrates the meanings that are made available to students through that discourse.

In deciding what to include in our maps, we looked for content that was valued or emphasized by the teacher or a student. For example, in the following exchange in which the class is discussing the graph that appears in Figure 1, students are oriented to the content shown in bold (the nodes of the map) by a variety of talk moves that alert students to its value. Table 1 provides the corresponding discourse moves for each node of the map.

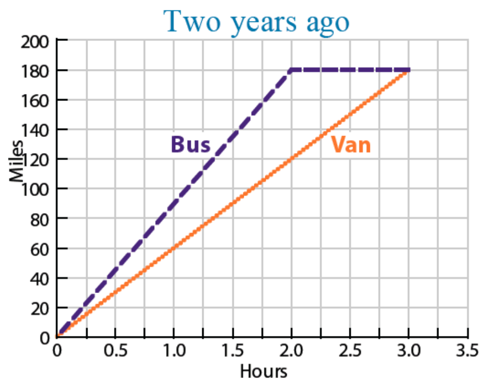

Figure 1. The graphical component of a task in which the bus stopped after two hours.

Teacher: I already heard Mara say that the van traveled at this **constant rate of speed**. Now I heard several people say that. …. And she said that the bus did what?
….
Abel: It would have been going, like, you said a **straight line** (using pencil to point to horizontal graph on the computer to teacher) is like this.
Teacher: Ah, remember if we have a **horizontal**—remember **they're all straight.**
Student: Maybe it slowed down to wait for cars.
Teacher: If we have a **horizontal line**, that's **when they're stopped**. What do you think happened? At what time did that happen?
Neil: At 2 hours.
Teacher: At 2 hours, what happened?
Neil: The bus, uh, uh.
Abel: **Speed declined.**
Teacher: The **speed declined**. I love that.
Zane: I wish we could get there in 2 seconds.
Teacher: If we could get there in 2 seconds?
Zane: Or 3.
Teacher: It feels like it takes forever, particularly, I tell you, from Abilene to Dallas does feel that way.

As we created Content Maps, we examined our transcripts to determine what content was expressed, valued, and reinforced (Mercer, 1990) through discourse and then created nodes to represent that content. We accounted for the distinction between students' and teacher's contributions by coloring nodes in our Content Maps red if they refer to ideas contributed by students and blue if they refer to ideas contributed by the teacher. The thickness of a link is proportional to the frequency with which that link was reiterated. If two nodes are not linked, then there were no connections made between them in classroom talk.

Table 1: Corresponding discourse move for each node

Line #	Node	Supporting talk move
1-2	Constant rate of speed	Revoice: "I heard Mara say… Now I heard several people say that"
5-9	Straight line, they're all straight	Draw attention: "Ah, remember…"
11-12	Horizontal line means stopped	Monitor content and remind of correct claim in response to student's ambiguous claim: "If we have a horizontal line … stopped"
16-17	Speed declined	Confirm and reinforce: "…I love that"

To test reliability of our maps, each member of the research team created a Content Map of a lesson previously mapped by another team member. We compared the maps by identifying primary and secondary nodes for each cluster, according to the number of links to these nodes. In cases of ties, we listed both (or all) nodes as primary or secondary. We then counted the number of matching nodes, aiming to achieve 80% or higher in matches. In all cases, the percentage matched was 86% or higher.

Results

Teachers who were selected to implement the SimCalc unit participated in a summer workshop where appropriate pedagogy for teaching with the unit was modeled. An assessment of MKT was administered to all teachers at the conclusion of that workshop. The assessment was constructed based on both the common content knowledge of the key curricular concepts and the specialized mathematical knowledge necessary to support teaching and learning, include the knowledge to help students make connections across concepts. Table 2 includes MKT scores for our three case-study teachers as well as their student gains. Ms. Garfield and Mr. Simmons, in particular, present a surprising combination of features. Ms. Garfield's MKT score was significantly below average, yet her mean class gain was above average for both years. In contrast, Mr. Simmons's MKT score was among the highest of all the treatment teachers, yet his mean class gain was consistently below average. Further, the standard deviation of students' achievement gains in his class was the largest among our cases and significantly above the average standard deviation of the treatment sample, suggesting differences in students' learning were more pronounced in this classroom.

Table 2: Information about the three case-study teachers.

		Student Achievement			
Teacher	MKT*	Mean Pretest**	SD Pretest	Mean Gain	SD Gain
All 48 teachers		13.57	5.60	5.24	3.87
Driver	16	19.42	3.27	5.21	2.82
Garfield	7	13.36	6.00	6.45	2.94
Simmons	18	8.05	3.34	3.90	4.79

*Total possible was 24. Average score for entire Treatment group was 13.1.
**Total possible was 30.

Coherence and Connectedness: The Case of Ms. Driver

Ms. Driver was a 28-year veteran mathematics teacher who held a certificate to teach grades 4-8. Ms. Driver's MKT score—16 out of 24—was above the average of 13.1 for the treatment sample, suggesting relatively strong content knowledge. Her students' average gain was about average. She taught in a high-ability classroom, which was reflected in her students' pretest scores.

Whole-group discussion in Ms. Driver's classroom provided students with multiple opportunities to make connections involving the big ideas of the unit. Content Maps of Ms. Driver's lesson showed how rate in particular was linked to related concepts, examples and counter-examples, narratives, and procedures for calculating speed and slope. To illustrate, whole-group discussion in the "On the Road" lesson exemplified the coherence of instruction. Ms. Driver framed speed in relationship to the constructs of rate and slope, providing students with opportunities to deepen their understanding by interconnecting speed with more sophisticated concepts. Specifically, she used the phrase "unit rate" to refer to speed and explicitly linked the concept to "slope" (Figure 2). The link between unit rate and ratio was articulated several times in definitional statements—such as speed is "distance in time"—and in characterizing unit rates as a type of "per change" unit. Discussion included a constellation of procedures, category exemplars, and interpretations, such as motion, consumption, and cost.

A second feature of Ms. Driver's instruction that provided opportunities for students to expend effort and make connections was the practice of pressing for reasons for students' claims. The quality of these elicitations distinguished her from the other case-study teachers and provided students with opportunities to make and deepen connections within a single content strand.

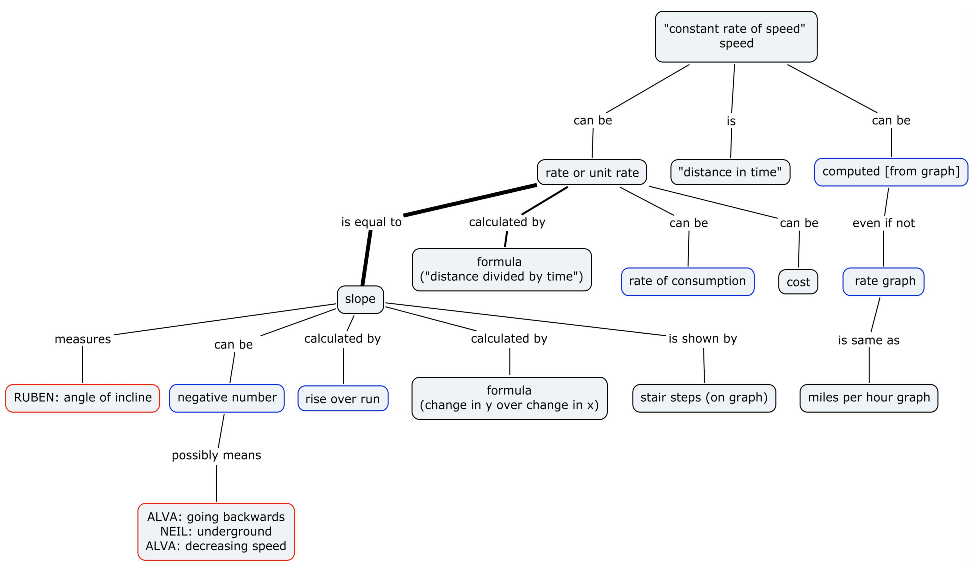

Figure 2. Cluster from Ms. Driver's Content Map for discussion of constant rate of speed

Procedural Explanations: The Case of Ms. Garfield

Ms. Garfield had been teaching for 18 years and had an elementary generalist certification for grades 1-8. She had taught mathematics every year. Ms. Garfield's MKT score—7 out of 24—was significantly lower than the average MKT score in the treatment sample and the lowest score of our case study teachers. Yet Ms. Garfield's students had above average gains in both years of the study.

Figure 3 is typical of Ms. Garfield's Content Maps, which revealed a focus on declarative knowledge and procedures. Classroom talk was teacher-driven and many of the critical connections were made by the teacher rather than by students. In contrast to Ms. Driver, speed and motion were discussed in terms of how to calculate them, with few connections to the big idea of rate. Rather than making conceptual connections, Ms. Garfield focused on the mechanics of computations.

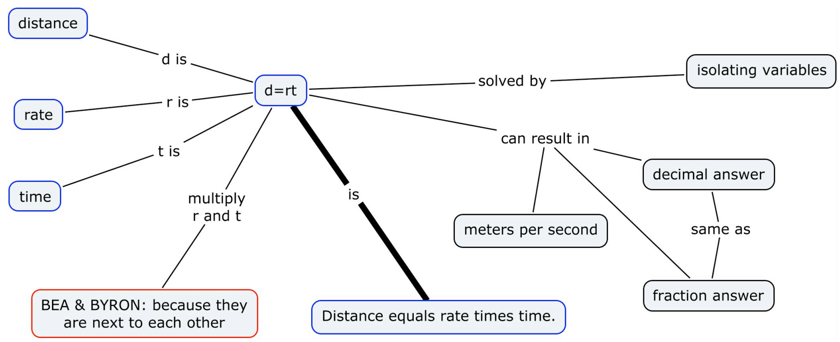

Figure 3. Clusters from Ms. Garfield's Content Map for a discussion of the rate formula

For example, in "A Race Day," the distance formula, $d = rt$, was by far the most talked about content followed by "speed." A good deal of this talk focused on how to apply the distance formula and syntactic aspects of the coordinate plane. Students were asked to chorally repeat the distance formula several times. Speed was defined rather opaquely as "moving [while] time is passing."

Speed was addressed descriptively and procedurally in later lessons, as well. When varying rates including slowing, resting, and returning and their associated piecewise graphs were introduced in "On the Road," Ms. Garfield focused on relationships of association, such as a "line bending" means the bus "slowed down." The horizontal line segment in Figure 1 was interpreted as "waiting," because the bus "got more time but didn't go further." There was no further discussion of either of these new situations.

Connections as Associations: The Case of Mr. Simmons

Mr. Simmons was in his fourth year of teaching during the study. Before participating in a one-year alternative credentialing program to teach mathematics in grades 4-8, Mr. Simmons was a technical professional. Like Ms. Driver, Mr. Simmons's MKT score—18 out of 24—was above average and suggestive of strong content knowledge. In contrast, his students' average gain of 3.55 was well below the average gain of 5.31 for the treatment sample and the lowest within our set of case-study teachers.

Participation in whole-group discussion in Mr. Simmons's class tended to be limited to short responses to closed questions. As a result, relationships between ideas in his classroom were better characterized as associations than as well-developed conceptual relationships. Simple connections among ideas typify these relationships. In "On the Road," once Mr. Simmons established slope as synonymous with speed (Figure 4), the discussion became about speed and a formula for calculating it. The "spine" in the map that extends the length of the cluster is darkened six times to indicate the number of times in the lesson that Mr. Simmons presented slope as “distance divided by time” and then used the “endpoint technique” to calculate the slope of a segment of a piecewise linear graph using its endpoints.

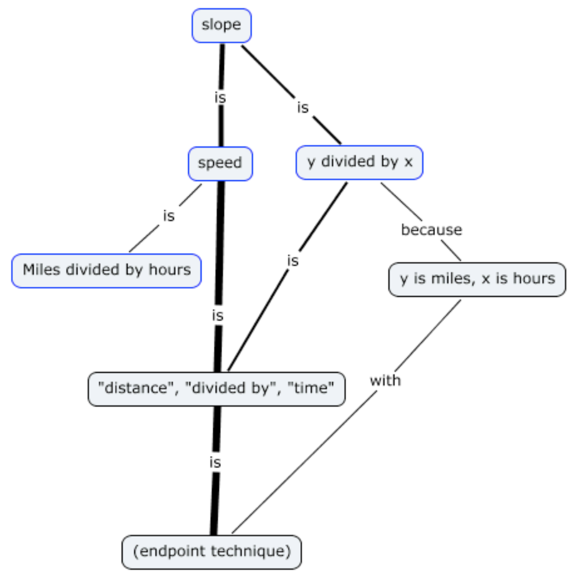

Figure 4. Cluster from Mr. Simmons’s Content Map for a discussion of speed

In contrast to Ms. Driver’s treatment of speed and slope, Mr. Simmons’s treatment was limited to procedural applications, thereby providing fewer opportunities for students to make connections and develop their understanding.

Discussion and Conclusion

Research finds a complex and highly mediated relationship between teachers’ MKT scores and their students’ achievement (Shechtman et al., 2008), and also between those scores and the mathematical quality of their instruction (Hill et al., 2008). Thus, our goal in this proposal was to better understand how students’ learning might be attributed to their teacher’s knowledge by studying three classrooms selected from the larger sample in the SimCalc experimental study. We focused on students’ engagement with the content in classroom discourse, represented using a tool we created and called Content Maps, because, we argued, this engagement afforded opportunities to learn. Representations of knowledge as connections in a content map allowed for a level of analysis that “makes contact with both theoretical cognitive issues and practical educational issues” (Hiebert & Carpenter, 1992, p. 67), namely the mathematical knowledge held by teachers and the mathematical meanings made available to students. One notable aspect of the contrast in these cases that our maps revealed—and that MKT scores could not—concerns the differences in how teachers expressed MKT in instructional talk and in the kinds of content connections that they emphasized. This finding suggests that content maps are a powerful and accessible analytical tool for revealing qualities of instruction that may be linked to student achievement, and in particular for representing students’ opportunities to learn from teachers’ knowledge in action.

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