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# Elementary Students Articulation and Application of Theory of Measure

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## Abstract

This project aimed to identify the different measurement principles elementary students articulate when solving clock problems. Interviews were conducted with elementary students in which they problem solved using the clock as a measurement tool, with the present analysis investigating student thinking and articulation of measurement principles. Data was analyzed using a mixed methods design, in which qualitative data was transformed into quantitative data. It was found that students are able to articulate their theory of measure, but they do so in different ways. Different principles of measurement present themselves different ways as students articulate their thinking. Findings show that how students articulate their measurement is complex, but students are able to articulate and translate between different representational tools.

## Introduction and Brief Theoretical Framework

"Students who grow up recognizing the complexity of measurement may be less likely to accept unquestioningly many of the common mistakes of number and statistics. Learning how to measure is the beginning of numeracy" (Steen, 1990). Children's conceptions of measure unfold over time, and these understandings come to fruition throughout the course of school, and through their everyday experiences with common objects and events. This leads to an understanding of measure using more commonly accepted measurement tools, like rules, to better understand the construction of space (Lehrer, 2003). Unlike length or area measure, measuring and calculating time intervals may be difficult because of its abstract nature (Earnest, 2015; Harris, 2008). The clock system is what we use to measure time and is one of the least studied symbolic systems that confront children (Earnest, 2015; Friedman & Laycock, 1989). Friedman and Laycock (1989) argue that there are three distinct components of clock knowledge: first, the ability to look at the clock and read the time; second, the ability to operate on the clock to extract relationships and; third, the understanding of the meanings of times . This study attempts to examine how students articulate their clock knowledge and their ability to operate on the clock using the measurement principles that Lehrer describes.

Lehrer (2003) lays out eight of the conceptual foundations of measurement that students gradually coordinate into an informal theory of measure:

1) Unit-attribute relations- There needs to be a correspondence between units and the attributes being measured. For example, it would be unwise to measure the distance to the moon in centimeters. One might think that miles might be more appropriate.
2) Iteration- Units can be reused. A unit can be divided and accumulated.

3) Tiling-In order to measure length, one must not leave space between the tiling of the ruler. No cracks are allowed.
4) Identical units- Units should be identical, and if they are not identical then explicitly stated.
5) Standardization- Conventions about units facilitate communication.
6) Proportionality- Different quantities can represent the same measure.
7) Additivity- Units can be decomposed and recomposed so that the total distance is equivalent to the sum of the distances of the segments that subdivide it.
8) Origin (zero-point) - Measurement of Euclidean space conforms to ratios so that the distance between 0 and 10 is the same as the distance between 30 and 40. This implies that any point can serve as the zero point.

Researchers debate the order of the development of the different concepts and the ages at which they are developed, but they do agree that these form the foundation for measurement and should be considered in measurement instruction (Clements & Stephan, 2004) .

Traditionally the goal of measurement instruction has been to help students learn the skills necessary to use a conventional ruler. Recent curriculum advises a sequence of instruction in which students first begin describing and comparing measurable attributes. Earnest (2014) argues that like length measure, time is a measurable quantity for which standard units have been develops, but that standards and curriculum emphasize a procedural approach for reading the clock .

Curriculum standards lay out a sequence of time instruction (National Governors Association Center for Best Practices Council of Chief State School Officers, 2010).. They include teaching time first to the hour and half hour in first grade. Second grade includes teaching time to nearest five minutes and third graders should be able to tell time to the nearest minute and measure time intervals in minutes. By fourth grade, students theoretically have been exposed and should have mastered the teaching of telling time, and using the clock.

## Research Questions

Our research questions emerged from preliminary coding of interview data.

1) *How often do students use proportionality and additivity when solving clock problems, and how often are these used together?*
2) *Do students actively use the clock to solve elapsed time problems?*

Children's conception of measure reflects a collection of emerging concepts whose coordinate unfolds over the course of school. This study provides a glimpse into how students develop and articulate their own theory of measure, how they apply it to clock problems, and how it might change and develop over time.

## Data and Methods

As part of the larger study, where the data for this study was taken, a paper and pencil assessment study with students across Grades 1-5 captured common responses to problems involving the clock and time. In order to reveal the character of student thinking, a videotaped interview study was conducted in which Grade 2 and 4 student engaged in problem solving using three different clock manipulative tools (Earnest, 2015). These interviews were standardized open interviews in which all interviewees were asked the same questions (Teddlie & Tashakkori, 2009, p. 229). These problems ask students to find certain times on the clock, as well as elapsed times. Students were assigned to one of three condition groups. Tasks featured one of four possible starting times: time on the hour (x:00), time on the half hour (x:30), time on the first half of the clock (x:10 or x:20), and time on the second half of the clock (x:40 or x:50). Second, four elapsed durations were provided: an hour, a half hour, an hour and a half, and three hours. Third, unit types were either hour units (e.g., an hour and a half) or minute units (e.g., 90 minutes). The interviewer asks students to articulate their thinking as they solve the clock problems by asking "Can you tell me how you're thinking about that?" and "How do you know?" (see Figure 1). There were thirty-nine time problems that children were asked to solve. Identification of time and elapsed time problems made up about half of the questions (For more information about the larger project, see Earnest (2016).

For this project, we chose to analyze the interviews of nine students from the same school. Four students are in grade 2, and five students are in grade 4. All students were in the same condition group (Condition group A), and used the coordinated hands clock. "Analyzing interview data is a multi-step sense making endeavor" (DeCuir-Gunby, Marshall, & McCulloch, 2011). To investigate these questions, we transcribed and analyzed nine video interviews. Due to issues in audio quality, one grade two interview was not analyzed. The interview lengths varied from about 20 minutes to one hour.

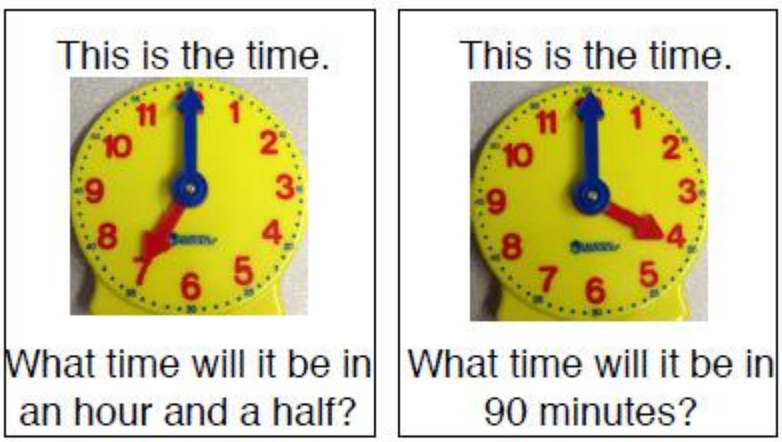


*Figure 1.* An example of an elapsed time problem

Mixed method designs require the researcher to be flexible, and their research design and analysis plan may change over the course of a study (Creswell, 2009; Teddlie & Tashakkori, 2009). While watching the interviews, we realized that development of a codebook in order to analyze the interview data was the first step in answering my research question. This development was the majority of this project, and is still in progress

## Codes

The development of theory driven codes requires constant revisiting of theory, as well as repeated examination of the data. This is an iterative process. The codebook is a set of codes, definitions and examples used to help the researcher analyze interview data (DeCuir-Gunby et al., 2011; Galman, 2013). Although there is a theory of measure that we all develop over time (Lehrer, 2003), there was a need to find a way to apply this theory to the data that we collected from the interviews.

Watching the interviews, we noticed that there was a need to determine the different kinds of categories we wanted to extract from the interview data. Coding allows researchers to distinguish between codes and determine non examples of those individual codes (DeCuir-Gunby et al., 2011). The first step in developing theory driven codes is to create the codes, second, revise and review the code in context of the data, and then determine the reliability of the coders and the code.

We created code labels and definitions and then looked through the data to find examples that best illustrated each code (DeCuir-Gunby et al., 2011).. For the purpose of this project, we developed two codes in order to analyze the data. These include using proportionality and additivity correctly. We have other theory driven codes that we want to review and revise later and then apply to the data. Other codes were also created based on the different type of task that students solved, as well as whether or not they solved the problem correctly.

### Proportionality

Proportionality includes when students articulate that different quantities can represent the same measure. Students seem to articulate proportionality in different ways, but the understanding of sameness stays intact. Students say things like “ninety minutes, it was an hour and then thirty minutes" or "its seven o'clock which means that we have to go an hour, which means that we have to go on the twelve again. And that gets us exactly on the eight, which makes it eight o'clock. And then we have to add thirty minutes, and if you start at zero and you add thirty minutes you get to the six. And that's eight thirty." The previous examples show that proportionality may be wrapped up in other measurement principles, or it may come through simply and succinctly. This type of thinking may be an indication that children are developing logical mathematical knowledge, the ability to translate their knowledge into new situations (Kamii & Russell, 2012) .

**Additivity**

Additivity arises differently and in different contexts with different students. Similar to the concept of proportionality, when students show that they understand something about additivity, they might not show it in the same way. However, additivity is much easier to spot in the data and students often attach phrases "add" or "plus" when they discuss thinking that includes additivity. For example, when asked about their thinking, students might say things like, "because it's thirty minutes, so you're just adding thirty minutes, and sixty minutes is thirty plus thirty"

Students can also use the clock to help them with their understanding of additivity, like in the example below where the student used the clock to help her count (Table 1). She knew that three hours could be decomposed into three separate one hour chunks and recompose them to make the new unit of three hours. She said, "Three hours, twelve o'clock is one hour, one o'clock is two hours, and three o'clock is three hours

Table 1

*Example of a student applying additivity and proportionality to problem solving.*

| | Definition | Example |
|---|---|---|
| Additivity | Students realize that units can be decomposed and recomposed. Thirty minutes can be decomposed into ten minutes and twenty minutes, which can be recomposed into thirty minutes. You can also break thirty minutes into fifteen minutes and fifteen minutes, which if recomposed will be thirty minutes. | “Because it’s thirty minutes, so you’re just adding thirty minutes and sixty minutes is thirty plus thirty” |
| Proportionality | Different quantities can represent the same measure. For example: one hour can be represented by sixty minutes. | “Sixty minutes is an hour”<br>“Half an hour is thirty minutes” |

## Results

### How often do students use additivity and proportionality?

Using Nvivo 10 to help me with my conversion of data (Teddlie & Tashakkori, 2009) from interview data to categorical codes that could be analyzed numerically. Students solved 39 problems, but of those 39 problems, 7 were considered “show” problems, in which students had to use the clock to show a time. Because we were interested in the “tell” conditions, where students articulated their answers and how they got there, we ended up with 32 items in which students did not have to use the clock to answer the question. We created this table below that offers an insight of the overlap of measurement principles that students use, as well as when they use them correctly. This data is presented below in Table 1. Overall, there were 351 items students answered. After removing the show conditions, we found 288 eligible to be analyzed. Of those 288 items, students answered correctly 75.3% of the time. Students expressed additivity principles in 24 correct items, and proportionality in 17 correct items, which translate to 11 %

and 8% of total correct responses respectively. Students also use the principles of additivity and proportionality together in 11 of their responses. You can see from the table below that students use principles of proportionality correctly in 37 of the total items, and additivity in 39 of the total item responses of all students. The additional cases of additivity and proportionality may overlap with items that were answered incorrectly, or in the "show" condition, but for the purposes of this paper, those were not explored. There does not seem to be any strong distinction between grade levels at this point regarding correct or incorrect solutions (see Figure 2).

Table 2
*Frequency counts, and overlaps between the codes additivity and proportionality*

| | Additivity | Proportionality | Correct |
|---|---|---|---|
| Additivity | 37 | 11 | 28 |
| Proportionality | 11 | 36 | 31 |
| Correct | 28 | 31 | 217 |

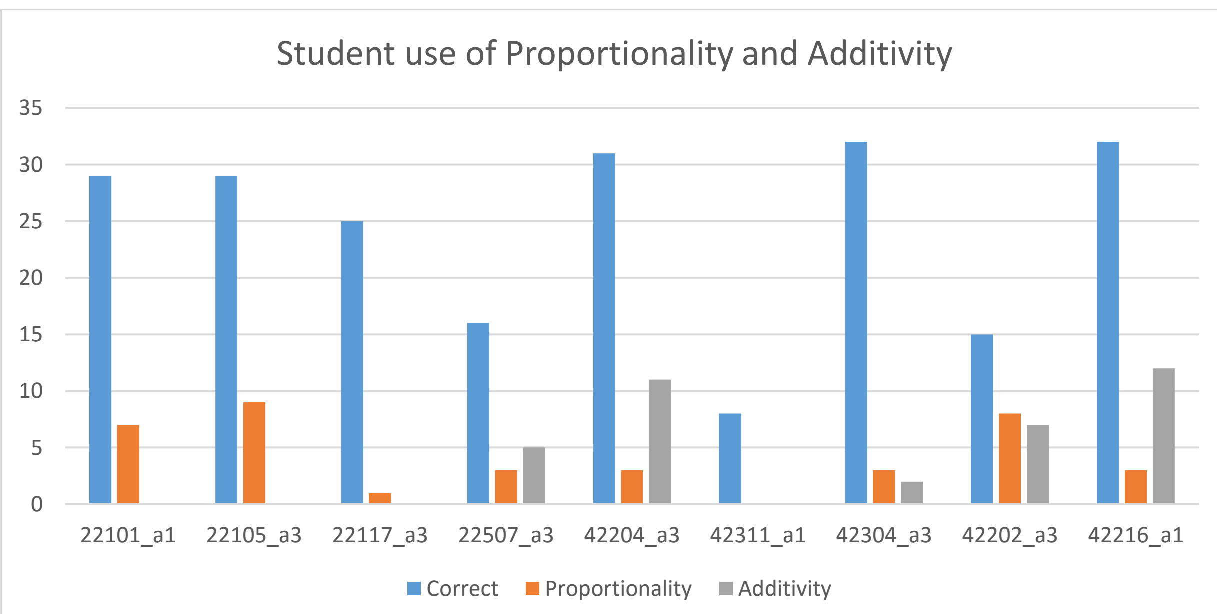


*Figure 2.* A graph of items answered correctly with proportionality and additivity by student respondent (first item number represents grade)

## Data Driven Results

After combing through my data with the codes developed (proportionality, additivity) we also found interesting data that was unable to be coded using my existing theory. We used a grounded theory approach to find patterns and themes that emerged from the data that were not determined by my theoretical framework (Charmaz, 2014). We noticed that some students used the clock as a way to help them organize their explanations, or their thinking.

## Using the clock

The ways in which students solve time problems varies, but we noticed that some students were more prone to using the clock as a measurement tool, while others seemed to work out the problems in their heads and then used the clock in an ancillary way. The examples below illustrate two different solutions to the same problem. These different solution strategies were present throughout the data. Some students used the clock as a primary tool and as a secondary tool, while others used one more prominently than the other.

Table 2
*Examples of the use of clock as ancillary or as a measurement tool.*

| Ancillary Use of Clock | Clock as a Measurement Tool |
|---|---|
| Researcher: What time will it be in ninety minutes?<br>Sixty, sixty-five, seventy, seventy-five, eighty, eighty-five, ninety [moves hands to 5:30] I moved this one an hour and I counted by fives until I got to ninety.<br>Researcher: What does this new time show?<br>Five thirty-three.<br>Researcher: And how do you know?<br>This one is above the five, and this is where thirty is so five thirty-three. | Researcher: And what time will it be in ninety minutes.<br>And that would be five thirty [moves the clock to five thirty]<br>Researcher: And tell me how you solved that.<br>Well I know ninety is just thirty plus thirty plus thirty or sixty plus thirty. So I do a whole entire hour and then half a rotation. |

## Conclusions and Implications

Telling time and reading the clock is not a distinct area removed from other mathematical understandings that students have. Although students do not always use the principles of measurement in ways that lead to correct solutions, this study gave a small glimpse into the idea that students do in fact are able to translate between both internal and external representations and they can articulate their theory of measure when they solve clock problems. This shows that students are able to problem solve, and think mathematically.

Understanding how students develop a theory of measure, or use that theory of measure to solve different types of problems is not easy. It requires asking the right types of questions, and collecting the right type of data. However, understanding how students reason about and use mathematical objects in the elementary grades gives insight into how students think about mathematical ideas. Gaining a glimpse into how students articulate their ideas regarding solution paths will help those who work with students draw out those understandings even further.

We looked for themes in responses that fit principles set forth by Lehrer. For example, in beginning coding, one student said in response to a question about how they solved an elapsed time problem that "sixty minutes is an hour, and sixty plus thirty equals ninety, so you have thirty more minutes left”. This would indicate that the student has some understanding of unit iteration, tiling, and additivity. There is still work to do to analyze these data further, in order to see other measurement principles students are using while solving clock problems.

Other next steps include, combing through the data so see when students are using the clock as a measurement tool. Questions to investigate may include: Do students use the clock differently based on condition, or grade level? Are some questions better aligned to some measurement principles than others?

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