2016 NCTM RESEARCH CONFERENCE
April 11-13 • San Francisco

Paper Title: Third Graders' Development of Quantitative Reasoning About Angle Measure

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Session Type: Brief Research Report

Presentation Date: Wednesday, April 13, 2016

Presentation Location: San Francisco, California

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Third Graders' Development of Quantitative Reasoning About Angle Measure

The purpose of this session is to report on the findings of a study investigating the effects of two instructional interventions designed to provide third graders with opportunities to work with dynamic and static models of angles in a dynamic geometry environment. We discuss the effects of the instructional interventions on the children's development of quantitative reasoning about angle measure.

The research on angle measurement has identified several misconceptions that elementary children have, but mathematics education has yet to fully determine how to address them. Children continue to attend to ray lengths (Clements, 2003; Foxman & Ruddock, 1984; Keiser, 2004; Mitchelmore, 1998; Noss, 1987) and wedge areas (cf. Devichi & Munier, 2013; Mitchelmore, 1997) rather than turn. It is this interpretation of angle as a measure of turn that the authors of the Common Core State Standards in Mathematics (CCSSM, National Governor's Association for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010) advocate. They asserted that fourth graders in the United States should understand:

> An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. (p. 31)

To promote this interpretation of angle and extend the literature on angle, we designed two instructional interventions. Both were enacted in a dynamic geometry environment utilizing the computer software, Geometer's Sketchpad, to provide children with opportunities to work with movable angle situations as well as reflect on dynamic (the motion of an angle sweeping open) and static (the resulting image of an angle after sweeping open) angle models. In this presentation, we report on the results from our testing of the two instructional interventions. We posed the research question

> In what ways do instructional interventions focused on linking dynamic and static angle models affect children's understanding of angle measurement?

Theoretical Perspective

We approached this study from a quantitative reasoning approach. According to Thompson (1990), "a quantity is a quality of something that one has conceived as admitting some measurement process. Part of conceiving a quality as a quantity is to explicitly or implicitly conceive of an appropriate unit" (p. 5). In the case of angle measure, quantifying involves reasoning about the unit (i.e., a degree) in terms of a quantitative relationship (i.e., a multiplicative relationship) between a fraction of the circular arc of a circle and the circle's circumference[1].

Method

The data presented in this presentation was collected during the Spring of 2015 with 19 children in Grade 3 (ages 8 and 9). These children were selected from two classes in a Midwestern, suburban public school. Participants were selected based on parental consent, student assent, and teacher input regarding their mathematical abilities in order to get a group of students per grade with a range of ability. The 19 participants were divided between the two instructional interventions: 10 were in Intervention group 1 and nine were in Intervention group

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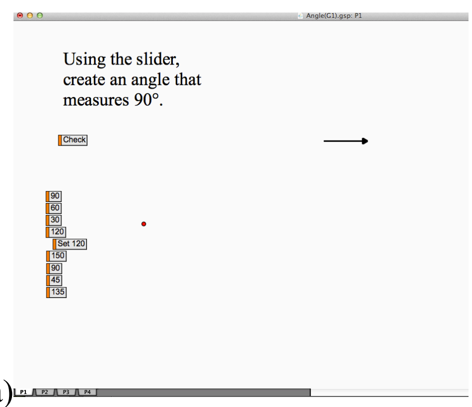
[1] This quantitative reasoning approach is consistent with the authors of the CCSSM's (NGA & CCSSO, 2010) recommendations for how fourth grade children should understand angle measure.

2. An additional eight children served as a control group. Because we wanted to observe and describe the effects of the two instructional interventions on the participants' understanding of angle measurement, we utilized the microgenetic method (Siegler & Svetina, 2006).

We interviewed each student three times using a structured interview protocol. Prior to the first interview, children took a written survey. In this survey we asked the student to give a definition of angle, estimate the measure of three angles, and draw four angles of a specified measure.

During the first interview, we asked the child to use the slider (see Figure 1) to create an angle of a specified measure on eight construct trials. Next, we asked the child to make a good guess about the measure of the angle shown on the screen for three estimate trials. This process was repeated for nine construct and three estimate trials during the second interview and for nine construct and three estimate trials during the third and final interview. After the final interview, we gave the children the written final survey, which had the same items as those on the initial survey.

a) 

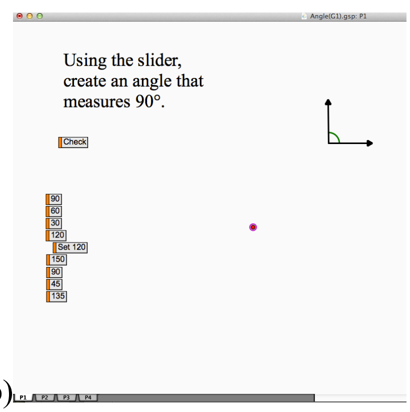
b) 

Figure 1. Screen shots of a) slider in initial position and b) after the slider was dragged left to open the angle.

For the construction of all 26 angles (construct trials), the student started with one initial horizontal ray and one slider. For each angle, the interviewer told the student to use the slider to create an angle of a specified measure. For children in Intervention group 1, during alternating construct trials in Interviews 1 and 2, benchmark rays of 30 degrees would appear as the child used the slider to open the angle (see Figure 2). During the construct trials in Interview 3, the benchmark rays did not appear during any of the trials. When the child indicated that he or she was ready to check, the check button was clicked, and a ray swept from the initial ray of the angle to the terminal ray of the angle the child had been instructed to create. During each check, benchmark rays appeared at each 30-degree interval until the ray stopped at the terminal side of the desired angle. The measure of the angle the child constructed was briefly displayed for the interviewer to record (see Figure 3).

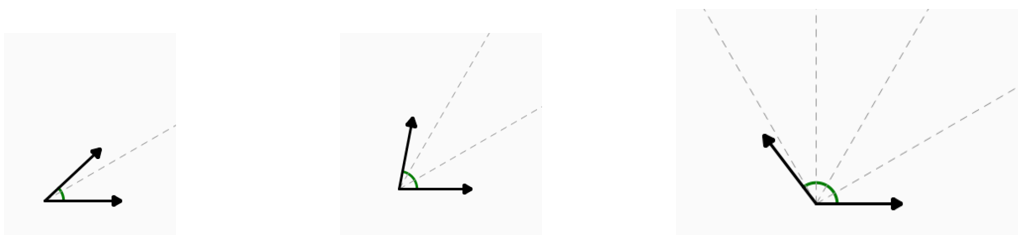


Figure 2. A sequence of screen shots displaying the 30-degree benchmark rays appearing for children in Intervention group 1 as the terminal ray of the angle sweeps open.

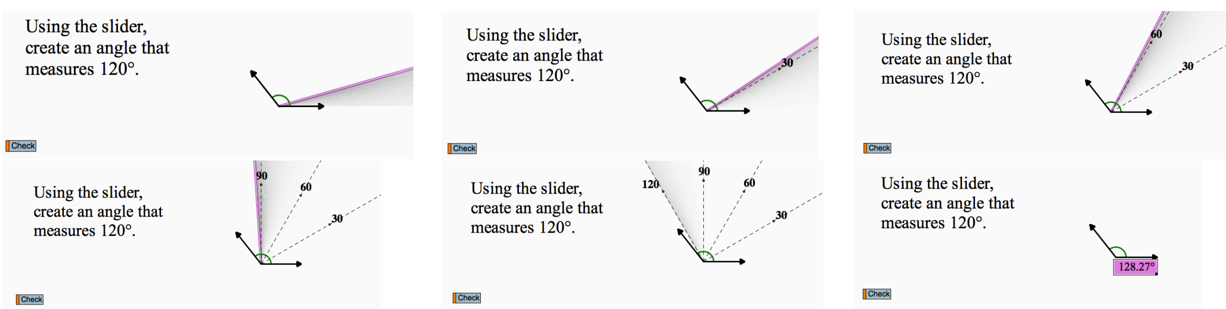


Figure 3. A sequence of screen shots displaying the labeled benchmark rays (e.g., 30, 60, 90) appearing as the child in Intervention group 1 checks her attempt at a 120-degree angle.

For children in Intervention group 2, benchmark rays neither appeared during construction nor check. Instead, when the check button was clicked, only the measure of the angle the child constructed was displayed.

For the nine estimation trials, the child started with one initial horizontal ray. When the interviewer clicked a lettered button, a ray swept from the initial ray of the angle to the terminal ray of the desired angle without benchmark rays. Then the interviewer asked the child to make a good guess about the measure of the angle shown on the screen. After the child reported an estimate, the check button was clicked, and the measure of the angle was displayed.

In the design of Intervention 1, we privileged 30-degree benchmarks by providing unlabeled benchmark rays at 30-degree intervals on alternating trials. Additionally, the check on every trial displayed 30-degree benchmark rays with labels (e.g., 30, 60, 90). Our purpose for including these supports was to help children improve their quantitative reasoning. Multiple trials were designed to be within 10 degrees of a 30-degree benchmark angle (i.e., 40, 70, 80, 110) in an attempt to provide children with experiences that would support their development of a sense of 10 degrees.

In the design of Intervention 2, we privileged feedback. Upon clicking the check button, the measure of the angle the child constructed on a given trial was displayed. This provided the children with the opportunity to reflect upon how the measure of the angle they constructed compared to the desired angle measure (cf. Jaehnig & Miller, 2007).

Summary of Findings

Based on our review of the related literature and our work with children on an earlier version of the interventions, we conjectured that both instructional interventions would more fully support the participants' learning of angle measurement than children in the control group. During this presentation, we will highlight case studies of four individual children, two per instructional intervention group, to illustrate how children's explanations and reasoning developed over the course of the trials. These children progressed from describing how they were reasoning about a little more or a little less than benchmark angles or the desired angle (e.g., too big, a little further, needed to be closer) to reasoning about specific numeric relationships (e.g., the difference between the measure of a benchmark angle or constructed angle and the measure of the desired angle) to showing flexible thinking about a unit of units (e.g., 30-degree angle as 10-degrees and 20-degrees and then a 10-degree angle as half of a 20-degree angle—10 more than 90 degrees).

Educational Importance of the Research

The experimental instructional interventions enacted in a dynamic geometry environment were designed to provide opportunities for children to engage with dynamic and static angle models while also providing information about third grade children's understanding of angle measurement. The results of this study extend the previous literature on children's understanding of angle measurement and suggest important instructional implications. Specifically, our results suggest providing children with opportunities to reason about angles as multiples of 30 (e.g., a 120-degrees angle as four 30-degree angles) and as partitionings of 30s (e.g., a 40-degree angle as a 30-degree angle plus one-third of another 30-degree angle) has the potential to support children's development of quantitative reasoning about angle measure.

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