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Abstract

To improve mathematics achievement, students' errors should be treated as a source to stimulate their understanding of the conceptual and procedural basis of their errors. The study investigated 20 Chinese and 20 U.S. high school teachers' interpretation and response to a student's errors in solving a quadratic equation. The teachers' responses were analyzed quantitatively and qualitatively. Analysis results show that the Chinese teachers provided more negative evaluations toward students' errors and identified more students' errors than the U.S. teachers did. Responding to students' errors, the two groups of teachers highlighted conceptual explanations targeting students' mistakes. The U.S. teachers were more likely to provide general knowledge guidance while the Chinese teachers tended to go back to basic knowledge. Implications of these findings for teachers, teacher educators and researchers are discussed.

Introduction

Algebra has long been regarded as a critical bridge to high school mathematics. NCTM (2000) highlighted the importance of algebra to all students. The content in school algebra mainly covers two major themes: equations and functions (NCTM, 2000; Drijvers, Goddijn & Kindt, 2010). Quadratic equations take on an important role in the high school Algebra I curriculum. From straight lines to curves it is an essential transition that requires students' conceptual understanding and computational proficiency. Prior research reveals that many students are challenged with solving quadratic equations (Vaiyavutjamai, Ellerton, & Clements, 2005; Zaslavsky, 1997). For example, Lim (2000) noticed that it is important for students to write quadratic equations in the standard form before attempting solving them. Didis and his colleagues (2011) found that 10th graders lacked conceptual understanding of the null factor law in solving quadratic equations. Additionally, Clements and Ellerton (2006) noted that when asked to solve a quadratic equation in the form $(x\text{-}a)(x\text{-}b) = 0$, many students who correctly found the solutions mistakenly held the concept that x in $(x\text{-}a)$ was equal to a, and simultaneously the x in $(x\text{-}b)$ was equal to b.

Helping students develop mathematical understanding, teachers' knowledge is an essential factor. In fact, both high school teachers' content knowledge and pedagogical content knowledge affects students' achievement (Goldhaber & Brewer, 2000). NCTM (2000) indicated that teachers should recognize and respond to students' errors appropriately. Students who figured out the misunderstandings under their mistakes can learn what they did not know and what they thought they knew. Rather than avoiding discussing students' errors, teachers are being called to use such errors as catalyst for stimulating reflection and exploration (Ashlock, 2006; Borasi, 1994). Taking good advantage of students' errors initiates the path of developing student understanding of the conceptual and procedural basis of their errors.

In this study, we investigated Chinese and U.S. high school algebra teachers' knowledge of interpreting and responding to students' errors in solving quadratic equations. Given that U.S. students are known for falling behind their counterparts from other economic competing countries in mathematics, particularly, the issue is more and more serious as students move from 8^{th} grade to 12^{th} grade (e.g. TIMSS 2011), it is imperative to explore the similarities and differences of teachers' knowledge. The research questions that guided the study are: (1) How do Chinese and U.S teachers interpret students' errors in solving quadratic equations?; (2) How do Chinese and U.S teachers respond to students' errors in solving quadratic equations?; and (3) What are the similarities and differences between Chinese and U.S. teachers' knowledge of interpreting and responding to students' errors?

Theoretical Framework

Students' Conceptual Obstacles in Solving Quadratic Equations

The methods of solving quadratic equations are introduced through factorization, the quadratic formula, and completing the square by using symbolic algorithms. Of these techniques, Didis (2011) argues that students prefer factorization since it is much faster than the other two methods. This result aligns with that from Eraslan's study (2005). However, while applying factorization to solve quadratic equations students tend to follow the procedural rules without pay attention to the structure and conceptual meaning (Sönnerhed, 2009).

Didis and his colleagues (2011) analyzed challenges faced by two 10th graders in solving quadratic equations in terms of instrumental understanding and relational understanding. Table 1 provides the examples of students' mistakes from both instrumental understanding and relational understanding. To summarize, based on students' explanations in the third mistake, we can discern that the reason why students missed the root $x = 0$ in the first and third problem is not that they solved the problem carelessly. They simply did not understand the underlying reasons why they would miss a root in the simplification process. While attempting to solve quadratic equations presented in a factored form, students tended to expand the two parentheses to get the standard form and then re-factorize. Also, they lacked conceptual understanding of the null factor law in solving quadratic equations. These mistakes align with previous researchers' findings (Vaiyavutjamai & Clements, 2006; Lim, 2000). Additionally, Ellerton and Clements (2011) found that 79% of the 328 preservice middle school teachers in their study did not know that $x^2 + 6 = 0$ has no real-number solutions and many of them thought two x's in $(x\text{-}2)(x + 3) = 0$ hold different values.

Table 1 *Students' mistakes in solving quadratic equations (Didis, et al., 2011)*

<table>
<tr><th>Types of mistakes</th><th>Problems</th><th>Students' sample mistakes</th></tr>
<tr><td rowspan="3">Procedural mistakes</td><td rowspan="2">1.Find the solution set of the equation
x^2-$2x = 0$</td><td>$x^2 = 2x$
$x = 2$</td></tr>
<tr><td>x^2-$2x = 0$
$(x$-$2)(x + 1) = 0$
$x = \{-2, 1\}$</td></tr>
<tr><td>2.Find the solution set of the equation
x^2-$x = 12$</td><td>$x(x$-$1) = 12$
$4(4 - 1) = 12$
$4\times3 = 12$
$x = 4$</td></tr>
<tr><td rowspan="3">Conceptual mistakes</td><td>3.To solve the equation $(x$-$3)(x$-$2) = 0$ for real numbers, Ali answered in a single line that "$x = 3$ *or* $x = 2$" Is this answer correct? If it is correct, how can you show its correctness?</td><td>The answer is right. Since I wrote $(x$-$3)(x$-$2) = 0$ as x^2-$5x + 6 = 0$ and factorize to find roots of it. From $(x$-$3) = 0$ and $(x$-$2) = 0$ "$x = 3$ *and* $x = 2$".</td></tr>
<tr><td>4. A student hands in the following work for the following problem.
Solve x^2-$14x + 24 = 3$
$(x$-$12)(x$-$2) = 3$
$(x$-$12)(x$-$2) = 3\times1$
x-$12 = 3$ x-$2 = 1$
$x = 15$ $x = 3$
$x = \{3, 15\}$
Is this answer correct? If it is correct, how can you show its correctness?</td><td>The answer is wrong. Since the equations are separated as (3,1) there is no error when $(x$-$12) = 3$ However, there is error when$(x$-$2) = 1$. It must be $(x$-$2) = 3$ then, x=5. Therefore, the solution will be {5, 15} rather than {3, 15}</td></tr>
<tr><td>5.The solution of the quadratic equation "$2x^2 = 3x$" is given in the following; According to you, is this solution correct or not? Explain your answer with its reasons?
Solution:
I. step $2x^2 = 3x$
II. step $2\,x\,x = 3\,x$
III. step $2x = 3$
IV. step $x = \frac{3}{2}$
$x = \{\frac{3}{2}\}$</td><td>The answer is right.
$2x^2 = 3x$ and $x^2$is opened.
$2\,x\,x = 3\,x$
Then the x is simplified.
$2x = 3$ so $x = \frac{3}{2}$.</td></tr>
</table>

Analytical Framework of Teachers' Knowledge of Students' Errors

Peng and Luo (2009) developed a framework to analyze teachers' knowledge of students' mathematical errors (see Table 2). They identified four analytical categories for the dimension of phrases of error analysis, namely, identify, interpret, evaluate, and remediate. The levels within each dimension of teacher knowledge of students' mathematical errors are sequential and hierarchical, with progress from one level to the next, and the different levels of analysis support and complement one another by giving a holistic and structured picture of teacher knowledge of students' mathematical errors.

Table 2 *Framework for phrases of error analysis* (Peng &Luo, 2009)

Dimension	Analytical categorization	Description
Phrases of error analysis	Identify	Knowing the existence of mathematical error
	Interpret	Interpreting the underlying rationality of mathematical error
	Evaluate	Evaluating students' levels of performance according to mathematical error
	Remediate	Presenting teaching strategy to eliminate mathematical error

Analytical Framework of Teachers' Responses to Students' Errors

Son (2013) analyzed elementary and secondary preservice teachers' interpretations and responses to students' error of proportional reasoning in similar rectangles. In this study she presented an analytical framework to analyze PST's responses to students' mistakes (See Table 3). According to Son (2013), conceptual knowledge is defined as the explicit or implicit understanding of the principles that govern a domain and the interrelations between pieces of knowledge in a domain. Procedural knowledge is defined as the action sequences for solving problems. Form of address signifies whether teachers deliver verbal or non-verbal information for students to hear and see (this kind of responses usually uses the very words "show" or "tell") or for students to do something and to answer questions (this kind of responses usually uses the very words "give" and "ask"). Act of communication barrier refers to the difficulties students and teachers have in communicating about student errors. In the over-generalization category, teachers tend to provide too general an intervention that doesn't directly address students' misunderstandings. By using a Plato-and-the-slave-boy approach, teachers assume that students actually know how to solve the problem correctly but simply have forgotten. Therefore, teachers plan to ask students questions in helping them to remember the math facts and procedures to solve problems. Returning to the basics means simply leading students to return to underlying principle. This method is regarded as either introducing more problems for students or making students forget the original problem.

Table 3 *Analytical framework for PST's responses to students' mistakes*

Aspect	Categories
1 Mathematical/ instructional focus	Conceptual vs. procedural
2 Form of address	Show-tell vs. give-ask
3 Pedagogical action(s)	Re-explains, suggests cognitive conflict, probes student thinking, etc.
4 Degree of student error use	Active, intermediate, or rare
5 Act of communication barrier	Over-generalization, a Plato-and-the-slave-boy approach, or a return to the basics

Analytical Framework of the Study

Comparing table 2 with table 3, it is clear that Son's framework of analyzing teachers' responses to students' mistakes is the "remediate" phrase of error analysis in Peng and Luo's framework. In addition to "remediate" Peng and Luo's framework also focused on identifying students' mathematical errors, interpreting underlying rationality of students' errors and evaluating students' levels of performance according to mathematical error. In fact, to understand teachers' knowledge of students' error, it is necessary to investigate both how they analyze students' errors and how they respond to students' errors. Relating the current study to these existing frameworks, we apply the adapted version of Peng and Luo's framework (see Table 4) to explore how teachers analyze students' errors and then use Son's framework to analyze how teachers respond to students' errors. In particular, in the designed problem scenario we first ask teachers to identify, interpret and evaluate students' errors and then ask the teachers to respond to the students' errors. As for the "identify" and "interpret" phrases, we examine whether teachers are able to identify all the students' errors and to discover all the underlying principles of the students' errors. "Evaluate" phrase is quite subjective since different teachers may obtain different evaluation ideas according to students' mistakes. Son's framework suits well with the analysis of "remediate" phrase.

Table 4 *Framework of analyzing teachers' analysis of students' errors*

Sub-domain	Analysis aspects
Identify	The number of students' errors
Interpret	The underlying knowledge of students' errors (number; concept vs. procedure-oriented)
Evaluate	Nature of students' levels of performance

Methods

Twenty Chinese teachers and twenty U.S. teachers who have taught Algebra I before or are currently teaching Algebra I participated in this study. While most of the U.S. teachers hold Master degrees most of the Chinese teachers have bachelor degrees. The group of Chinese teachers is more experienced than the group of U.S. teachers. However, the U.S. teachers took more college level math courses than the Chinese teachers. In terms of the time that students spent on learning Algebra, it seems that Chinese students do not take as many classes as U.S. students do, but Chinese students spend more than twice of the time that U.S. students spent in doing homework. All the participants are currently teaching at high schools that have characteristics typical of each nation's public schools with respect to the students' ethnic, economic, and cultural diversity,

Figure 1 shows the main task used for this study. This problem was developed by Ellerton and Clements (2011) to test teachers' knowledge of quadratic equations. The participants were asked to interpret and respond to Amy's errors. We were curious to know what errors our participant identified from Amy's response. The participants were given a score from 0 to 4 based on the number of students' errors that they identified.

In addition to the number of errors identified, teachers' written responses were analyzed according analytical frameworks shown in Tables 3 and 4. Simultaneously, we expected new categories to come out of the participants' responses, which would contribute to the existing frameworks. We first coded the participants' evaluations of the student's performance on the math topic. Then, we examined whether the participants discovered all the student's mistakes presented in the question scenario. Furthermore, we checked whether the participants identified all the underlying mathematical concepts and principles of the student's errors.

The participants' responses in helping students to correct their errors were analyzed in terms of five aspects as elaborated in Table 3. The conceptual versus procedural distinction was utilized first, followed by the identification of pedagogical actions. After addressing these global oriented characteristics of the teachers' responses, more detailed analysis was conducted with respect to teaching approaches: form of address, pedagogical action, use of student error and communication barriers. Each participant's response might be assigned more than one code within each category.

Figure 1: Main task for the study

Students were asked to solve $(x+2)(2x+5)=0$, then to check their answer. One student,

Amy, wrote the following (line numbers have been added):

$(x+2)(2x+5)=0$	*Line* 1
$\therefore 2x^2+5x+4x+10=0$	*Line*2
$\therefore 2x^2+9x+10=0$	*Line*3
$\therefore (2x+5)(x+2)=0$	*Line* 4
$\therefore (2x+5)=0 \ and \ (x+2)=0$	*Line* 5
$\therefore 2x=-5 \ and \ x=-2$	*Line* 6
$\therefore x=-\frac{5}{2} \ and \ x=-2$	*Line* 7

Check: Put $x=-5/2$ in $(2x+5)$, and put $x=-2$ in $(x+2)$.

Thus, when $x=-5/2$ and $x=-2$, $(2x+5)(x+2)$ is equal to 0×0 which is equal to 0. Since 0 is on the right-hand side of the original equation, it follows that $x=-5/2$ and $x=-2$ are the correct solutions.

Results

Amy in Figure 1 did not have a clear understanding of the following four pieces of mathematical concepts and principles: (1) Rationale of the method of factorization; (2) Zero product property; (3) Difference between "and" and "or"; and (4) Meaning of solutions for quadratic equations. We were curious to know what errors our participants identified from Amy's response. The participants were given a score from 0 to 4 based on the number of students' errors below that they identified.

- Mistake 1: Lines 2, 3, and 4 were unnecessary, since the left-side is already factored in Line 1.
- Mistake 2: In Lines 5 through 7, the word "or", and not "and", should have been used.
- Mistake 3: For the check, each solution should have been substituted into both parentheses in the initial equation.

Interpreting Errors

In evaluating Amy's performance, 90% of the Chinese teachers condemned Amy's performance whereas only 10% gave what may be considered a half and half comment that suggested that Amy did something correct in solving the equation but she also made mistakes. No Chinese teacher provided positive evaluations. Different from the Chinese teachers, 30% of the U.S. teachers did not evaluate Amy's overall performance. Almost half of the U.S. teachers gave half and half evaluations, whereas 15% of the teachers were positive about Amy's performance. None of the U.S. teacher gave negative evaluations. Table 5 presents the distribution of US and Chinese teachers in identifying Amy's mistake.

Table 5 Identifications *of Amy's mistakes on solving the quadratic equation*

Categories	Chinese (n=20)	U.S. (n=20)
Mistake 1	14(70%)	14(70%)
Mistake 2	17(85%)	8(40%)
Mistake 3	12(60%)	11(55%)
No mistake	1(5%)	2(10%)
One mistake	3(15%)	6(30%)

Two mistakes	8(40%)	9(45%)
Three mistakes	8(40%)	3(15%)

Most of the Chinese and the U.S. teachers identified that Amy did some unproductive work. Also around half of the Chinese and U.S. teachers noticed that Amy mistakenly checked the solutions together. Notice that the number of the Chinese teachers who found the second mistake Amy made was twice as many as that of the U.S. teachers. In other words, while 80% of the Chinese teachers recognized Amy used "and" to combine the two solutions only 40% of the U.S. teachers recognized this. In addition, The Chinese teachers identified more of Amy's errors in solving a quadratic equation than US teachers. Yet, most of the Chinese teachers and the U.S. teachers did not interpret Amy's knowledge deficiencies.

Responses to the Student's Errors

Around 50% of the Chinese teachers did not specifically address any mistake. 20% of the Chinese teachers demonstrated one and three mistakes respectively. Within the group of teachers who addressed the mistakes while responding to Amy, 45% of them addressed the second mistake, that is Amy used "and" to connect the two solutions. Twenty-five percent of the teachers explained the first mistake that Amy multiplied out the product of binomials (which does not advance the problem at all). Twenty percent of the teachers identified the third mistake that Amy checked the two solutions by substituting them into the equation simultaneously.

Around one fourth of the U.S. teachers did not respond to Amy's mistakes. While almost fifty percent of the U.S. teachers addressed one mistake, a few teachers responded to two or three mistakes. Among the teachers who responded to Amy's mistakes, more than half of them responded to the solution-checking mistake. The first mistake also attracted the U.S. teachers' attention while the second mistake was overlooked.

We found that the U.S. teachers differed from the Chinese teachers in terms of the number of teachers who addressed Amy's mistakes. The same number of Chinese teachers and U.S. teachers responded to two or three mistakes. In terms of Amy's three mistakes, the Chinese teachers highlighted using "or" but not "and" to connect the two solutions while the U.S. teachers emphasized how to check the solutions. Furthermore, it was found that Chinese teachers tended to address Amy's errors conceptually while the U.S. teachers favor conceptual and procedural explanations equally. Since some teachers addressed more than one piece of conceptual knowledge, the percentage for each knowledge category in Table 6 was calculated out of 100%.

Table 6 *Mistakes addressed by the teachers*

Category	Chinese (n=20)	U.S. (n=19)	Total (n=39)
Mistake 1	5(25%)	7(36.8%)	12(30.8%)
Mistake 2	9(45%)	3(15.8%)	12(30.8%)
Mistake 3	4(20%)	11(57.9%)	15(38.5%)
No mistake	11(55%)	5(26.3%)	16(41.0%)
One mistake	4(20%)	9(47.4%)	13(33.3%)
Two mistakes	1(5%)	3(15.8%)	4(10.3%)
Three mistakes	4(20%)	2(10.5%)	6(15.4%)

As for the four pieces of mathematical knowledge which have been identified as the reasons for Amy's mistakes, most of the Chinese teachers addressed the zero-product property and around half of the Chinese teachers explained the difference between "and" and "or" and the meaning of solutions of quadratic functions. Only one Chinese teacher explained that the rationale of the factoring method was the zero-product property. Also, one U.S. teacher addressed this rationale. While all the U.S. teachers elaborated the zero-product property, the other three pieces of knowledge were overlooked by them. To conclude, the Chinese teachers outperformed the U.S. teachers in both the variety and the quantity of the addressed conceptual knowledge.

Table 7 *Mathematical knowledge addressed by the teachers*

Category	Chinese (n=17)	U.S. (n=10)	Total (n=27)
Rationale of the factoring method	1(5.9%)	1(10%)	2(7.4%)
Zero-product property	13(76.5%)	10(100%)	23(85.2%)
Difference between "and" and "or"	7(41.2%)	1(10%)	8(29.6%)
Meaning of solutions of quadratic functions	9(53.0%)	1(10%)	10(37.0%)
One piece of knowledge	6(35.3%)	7(70%)	13(48.2%)
Two pieces of knowledge	9(52.9%)	3(30%)	12(44.4%)
Three pieces of knowledge	2(11.8%)	0(0%)	2(7.4%)

Table 8 summarizes the local characteristics of the teachers' responses to Amy's errors. The Chinese teachers all applied a "show and tell" strategy to teach Amy while some of them simultaneously asked Amy questions to likely include her in the teaching and learning process. Almost half of the Chinese teachers did not employ Amy's mistakes in their responses while the number of the Chinese teachers who actively addressed Amy's errors and intermediately used Amy's errors are equally distributed.

Similar to the Chinese teachers, the U.S. teachers also emphasized a "show and tell" approach when responding to Amy. In terms of the "use of student error," the number of the U.S. teachers who intermediately employed Amy's errors is similar to that of the Chinese teachers. However, more of the U.S. teachers than the Chinese teachers actively responded to Amy's errors.

Table 8 *Categories for describing three aspects of pedagogical strategies to student error*

Aspect	Categories	Chinese (n=20)	U.S. (n=19)
Form of address	1. Show and tell	20(100%)	15(78.9%)
	2. Give and ask	7(35%)	6(31.6%)
Use of student error	1. Active use	4(20%)	7(36.8%)
	2. Intermediate use	5(25%)	4(21.1%)
	3. Rare use	11(55%)	8(42.1%)
With/Without Communicative barrier	1. Over-generalization approach	7(35%)	5(26.3%)
	2. Plato-and-the-slave-boy approach	1(5%)	4(21.1%)
	3. Return to the basics approach	8(40%)	5(26.3%)
	4. Specific to student error approach	7(35%)	6(31.6%)

Discussion and Implications

We found that the Chinese teachers were more inclined to identify Amy's mistakes than the U.S. teachers did. There was a large gap between the number of identified errors and the number of addressed errors for both groups of teachers. Interestingly, both the Chinese teachers and the U.S. teachers intended to use teacher-centered pedagogical actions that highlighted "show and tell." However, more U.S. teachers than Chinese teachers seemed to believe that Amy simply needed help to recall all the needed mathematical knowledge so they actively used Amy's mistakes to deduce her lapses in knowledge about solving quadratic equations. This study has implications to teachers, teacher educators and researchers in both U.S. and China.

First, the U.S. teachers' emphasis on providing general knowledge that may not lead students to correct their errors points to the need for teachers to specifically address students' errors and provide corresponding instructions on basic knowledge. Related to the findings from the Chinese teachers, it is reasonable to set up different expectations for different students, and teachers should expect students to have learning goals that are slightly above their abilities so students can be scaffolded in reaching these goals. Of relevance, teacher educators may consider to integrate teachers' knowledge and skills to address students' errors into their teacher preparation programs to help preservice teachers become sufficient in dealing with students' errors and supporting students in becoming mathematically competent. To explore algebraic thinking of high school teachers, this study focused on quadratic equations, which is a fundamental algebraic topic in high schools. Future researchers may consider investigating teachers' content knowledge and pedagogical content knowledge in a systematic way that includes a series of algebraic topics that challenge both high school students and teachers. In addition, future researchers, if possible, may employ classroom observations and face-to-face interviews with teachers. In this way, I believe more information could be obtained in understanding teachers' knowledge and students' achievements.

References

Didiş, M., Baş, S., & Erbaş, A. (2011). Students' reasoning in quadratic equations with one unknown. In Proceedings of the 7th Congress of the European Society for Research in Mathematics Education. Rzeszów, Poland.

Drijvers, P., Goddijn, A., & Kindt, M. (2010). Algebra education: exploring topics and themes. In Secondary algebra education (pp. 5-26). Sense Publishers.

Ellerton, N. F., & Clements, M. K. (2011). Prospective Middle-School Mathematics Teachers' Knowledge of Equations and Inequalities. In *Early Algebraization* (pp.379-408). Springer Berlin Heidelberg.

Eraslan, Ali, "A Qualitative Study: Algebra Honor Students' Cognitive Obstacles as They Explore Concepts of Quadratic Functions"(2005). *Electronic Theses, Treatises and Dissertations.* Paper 557.

Goldhaber, D. D., & Brewer, D. J. (2000). Does teacher certification matter? High school teacher certification status and student achievement. *Educational evaluation and policy analysis*, *22*(2), 129-145.

Lim, T. H. (2000). The teaching and learning of algebraic equations and factorisation in O-level Mathematics: A case study. Unpublished M.Ed dissertation, Universiti Brunei Darussalam.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics* (Vol. 1). National Council of Teachers of Mathematics.

Peng, A., & Luo, Z. (2009). A framework for examining mathematics teacher knowledge as used in error analysis. *For the learning of mathematics, 29*(3), 22-25.

Sönnerhed, W. W. (2009). Alternative approaches of solving quadratic equations in mathematics teaching: An empirical study of mathematics textbooks and teaching material or Swedish Upper-secondary school. Retrieved April 5, 2010, from http://www.ipd.gu.se/digitalAssets/1272/1272539_plansem_wei.pdf

Son, J. W. (2013). How preservice teachers interpret and respond to student errors: ratio and proportion in similar rectangles. *Educational Studies in Mathematics*, 1-22.

Vaiyavutjamai, P., Ellerton, N. F., & Clements, M. A. (2005). Students' attempts to solve two elementary quadratic equations: A study in three nations. *Building connections: Research, theory and practice, 735-742.*

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