2016 NCTM RESEARCH CONFERENCE
April 11-13 • San Francisco

Paper Title: An Early Algebra Intervention's Positive Impact on Arithmetic Comprehension

Author(s): Michael D. Eiland, Maria Blanton, Eric Knuth, Ana Stephens

Session Title: LEAPing into Algebra

Session Type: Brief Report

Presentation Date: April 12, 2016

Presentation Location: San Francisco, California

An Early Algebra Intervention's Positive Impact on Arithmetic Comprehension

Algebraic thinking and reasoning have been identified in recent standards documents (e.g., CCSSI, 2010; NCTM, 2000, 2006) as an important and emerging curricular strand in K-8 education. Recognizing that historical paths to algebra, such as the "arithmetic-then-algebra" approach, have been largely unsuccessful, scholars now advocate that students have long-term algebra experiences, beginning in the elementary grades. Embedding algebraic thinking throughout the K-8 curriculum helps transition students' formal intuitions about structure and relationships into formalized ways of thinking. According to Van deWalle, Karp, and Bay-Williams (2010) "algebraic thinking or algebraic reasoning involves forming generalizations from experiences with number and computation, formalizing these ideas with the use of a meaningful symbol system, and exploring the concepts of pattern and functions (p. 254)." A fundamental tenet of early algebra education is that it will increase children's algebraic understanding as well as improve their success with formal algebra in later grades.

Research Focus

The focus of this brief report is to highlight the measurable positive gains due to participation in a sustained, comprehensive, early algebra instructional intervention by students who are currently experiencing difficulty demonstrating arithmetic competence as measured by a standardized generalized mathematics assessment. Blanton et al. (2015) defines comprehensive early algebra instruction as "an approach that intentionally integrates early algebraic practices into the elementary school curriculum across different conceptual domains that are recognized as important entry points into algebraic thinking (e.g., Carraher & Schliemann, 2007) so that these concepts and practices are broadly addressed in ways that build mathematical connections and are accessible to students at multiple levels of thinking (p. 42)."

Methods

In this report, a longitudinal early algebra intervention was first implemented in grade three classrooms and subsequently followed students to grade five. A pre-intervention algebra pretest in grade three and the end grade posttests were administered to students in participating schools who received the early algebra intervention as well as an academically and demographically comparable group of students who did not receive the intervention.

Participants

The students who participated in this study were all from one school district located in the Northeastern United States. Two academically and demographically comparable elementary schools supplied the grade three to grade five students. The mathematics curriculum for the school district was Pearson's *EnVision*.

There were 130 students for whom we had fourth-grade state standardized mathematics scores, 73 students from six classrooms in one school formed the intervention group (who received the early algebra intervention); the remaining 57 students from four classrooms in the second school did not receive the intervention served as the comparison group. Students from the two groups were comparable in their standardized mathematics performance, $t(128) = 0.62$, $p = .539$. All participants were either in an intervention or in a comparison classroom for all three years of the study. Given the design of school structures, random assignment of students to a condition was not feasible. Teachers at intervention schools volunteered their classrooms for participation. A member of the research team taught the intervention lessons.

Data Collection

The instructional treatment consisted of 20 replacement lessons interspersed throughout the school year. No more than one replacement lesson was taught during the week. Upon the completion of the instructional treatment and state-mandated the standardized test, students in all 10 classrooms completed a written algebra assessment aligned with the five identified conceptual "big ideas" towards the end of the school year. To develop the algebra battery, Blanton et al. (2015) used items similar to those that had performed well psychometrically in previous research.

Data reported in this study consist of students' responses to written assessments completed at the following four time points: the beginning of grade three and the end of grades three to five. Students were given 1 hour to complete the assessment. The assessment was administered to both the early algebra intervention students and the nonintervention comparison students.

The four written assessments were based on the early algebra learning progression discussed in Blanton et al. (2015) and was designed to assess students' understanding of core algebraic concepts and practices within five big ideas: equivalence, expressions, equations, and inequalities (EEEI); generalized arithmetic (GA); functional thinking (FT); variable (V); and proportional reasoning (PR). Each assessment contained approximately 12 multi-part questions representing of the five big ideas. Table 1 describes the number of items representing each big idea on algebra assessments. Most assessment items were open response with a maximum of two multiple choice items per battery. Twenty-two items were featured on all four assessments.

Table 1.
Assessment item correspondence by grade and targeted big idea.

	EEEI	FT	GA	VFE[a]	PR	Total
Grade 3	12	6	4	3	1	26
Grade 4	10	11	4	3	1	29
Grade 5	13	14	4	3	1	35

[a]Note. VFE is a combination of EEEI, FT and V.

Data Analysis

Student responses to assessment items were coded for correctness and strategy use. Information regarding coding, scoring, reliability, and resolving discrepancies can be found in Blanton, et al. (2015). Within treatments, students were further grouped based on their grade four performance on the standardized North Eastern Mathematics Assessment[1] (NEMA). Three performance tiers, detailed in Table 2, designate the level of mastery exhibited on the following learning objectives: operations & algebraic thinking; number & operations in Base Ten; number & operations-fractions; geometry, and measurement & data. Each learning objective represents 15% to 25% of the NEMA battery.

[1] North Eastern Mathematics Assessment (NEMA) is a pseudonym. Tier 3 is a combination of the lowest two designations on the NEMA.

Table 2.
NEMA performance expectations by tier.

Tier	Performance Expectation
1	**Exceeds**: demonstrates a comprehensive and in-depth understanding of rigorous subject matter, and provide sophisticated solutions to complex problems.
2	**Meets**: demonstrates a solid understanding of challenging subject matter and solves a wide variety of problems.
3	**Does not meet**: demonstrates minimal or partial understanding of subject matter and solves some simple problems.

The sample includes 73 intervention students (Tier 1 $n = 16$, Tier 2 $n = 20$, Tier 3 $n = 37$) and 57 comparison students (Tier 1 $n = 8$, Tier 2 $n = 30$, Tier 3 $n = 19$). Traditionally, algebra and algebraic reasoning were relegated to a stand-alone course introduced as early as eighth grade but usually reserved for high school. The prevailing criterion for assigning students to an algebra course was high general mathematics standardized test scores; therefore, students most likely to be placed in and allowed to benefit from algebra instruction would be those in performance Tier 1 and Tier 2 because they possess a level of arithmetic mastery that usually lends itself to reaching competency in structured algebra courses. Tier 3 students were often relegated to preparatory algebra courses such as pre-algebra. If it can be shown that Tier 3 intervention students statistically outperform their Tier 1 and Tier 2 comparison counterparts on portions of the algebra battery, then it would legitimize the importance of offering all students opportunities to learn algebra throughout the K-8 experience.

Results

What follows is a comparison of the 37 Tier 3 intervention students' performance against the 38 Tier 1 & Tier 2 comparison students' performance on the algebra assessment at the four time points. Table 3 outlines the student performance by NEMA score at each test administration on grouped on the five big ideas.

[Insert Table 3 about here]

Performance comparisons between the Tier 3 intervention students and the Tier 1 & Tier 2 were evaluated using a mixed ANOVA design and Bonferroni adjustment with a significance level of 0.0125 ($0.05 \div 4$) to account for the four assessments.

In the following analyses, *intervention students* refers solely to those classified within Tier 3, while *comparison students* refer to those classified in Tiers 1 & 2. On equivalence, expressions, equations, and inequalities items, *intervention students* statistically outperformed the *comparison students* on the grade 3 posttest, $F(1, 73) = 33.57$, $p < .001$, and grade 5 posttest, $F(1, 73) = 7.27$, $p = .009$, while the *comparison students* were significantly higher prior to the intervention on the grade 3 pretest, $F(1, 73) = 6.38$, $p = .014$. For problems involving functional thinking, *intervention students* statistically outperformed the *comparison students* on the grade 3 posttest, $F(1, 73) = 36.42$, $p < .001$, while the *comparison students* were significantly higher prior to the intervention on the grade 3 pretest, $F(1, 73) = 12.76$, $p < .001$. For items related to generalized arithmetic, *intervention students* statistically outperformed the *comparison student*s on the grade 3 posttest, $F(1, 73) = 13.47$, $p < .001$, and grade 5 posttest, $F(1, 73) = 9.44$, $p =$

.003. There were no performance differences at any time point for items involving proportional reasoning or equality, expressions, equations, and inequalities, functional thinking & variable.

To better illustrate an instance where Tier 3 intervention students were better able to generalize a situation using variables to represent the fundamental property of additive commutativity, first discussed in grade one of the Common Core State Standards for Mathematics, is depicted in Figure 1. For Item 3b, Tier 3 intervention students had a mean success rate of 74.3% compared to 37.8% for Tier 1 & Tier 2 comparison students, χ^2 (N = 72) = 9.68, p = .002. Similarly, on Item 3c, Tier 3 intervention students had a mean success rate of 94.3% compared to 70.3% for Tier 1 & Tier 2 comparison students, χ^2 (N = 72) = 7.01, p = .013.

<table>
<tr><td colspan="3">Marcy's teacher asks her to solve "23 + 15." She adds the two numbers and gets 38. The teacher then asks her to solve "15 + 23." Marcy already knows the answer without adding.

Item 3b) Write an equation using variables (letters) to represent the idea that you can add two numbers in any order and get the same result.</td></tr>
<tr><th>Strategy Code</th><th>Description</th><th>Example</th></tr>
<tr><td>Equation(s) using variables</td><td>Student writes a correct equation using variables or shows it using more than one equation.</td><td>$a + b = b + a$

$a + b = c$ and $b + a = c$</td></tr>
<tr><td colspan="3">Item 3c) Will Marcy's idea always work? Explain why.</td></tr>
<tr><th>Strategy Code</th><th>Description</th><th>Example</th></tr>
<tr><td>Property</td><td>Student names "Commutative Property of Addition" or turn-around fact.</td><td>Marcy knows the sum because of the Commutative Property of Addition.
Marcy knows because this is a turn-around fact.
It will work for all numbers because it is a fact family.</td></tr>
<tr><td>Property in words</td><td>Student states the Commutative Property of Addition in words.
(A number of other suitable terms may be used to convey the property.)</td><td>It will work for all numbers because the order does not change the result in addition.</td></tr>
<tr><td>Property + Property in words</td><td>Student both states the Commutative Property of Addition and states the property in her/his own words.</td><td>It will work for all numbers because of the Commutative Property. The numbers are just switched around.</td></tr>
<tr><td>Same numbers</td><td>Student states it will work for all numbers because the same two numbers are used (without referring to commutativity or the operation of addition).</td><td>It will work because the same two numbers are used.</td></tr>
</table>

Figure 1. Coding scheme for Items 3b and 3c indicating correctness.

Discussion

Overall, intervention students: (1) used relational thinking and recognized equation structure more frequently; (2) stated that a generalization would hold across a broad domain of numbers, justified their arguments using fundamental properties of number and operations, and generalized using variables more frequently; (3) represented unknown quantities using variables and coordinated their representations of related unknown quantities more frequently; and (4) had higher rates of success writing a function rule in variables and words. The results suggest that students at all standardized mathematics achievement levels can benefit from a grade 3-7 mathematics curriculum containing a purposeful integration of algebraic thinking skills.

References

Baroody, A. J., Cibulskis, M., M., Lai, M.-I., & Li, X. (2004). Comments on the use of learning trajectories in curriculum development and research. *Mathematical Thinking and Learning*, *6*(2), 227-260.

Blanton, M., Stephens, A., Knuth, E., Gardiner Murphy, A., Isler, I., & Kim, J-S. (2015). The development of children's algebraic thinking: The impact of a comprehensive early algebra intervention in third grade. *Journal for Research in Mathematics Education, 46*, 39-88.

Kaput, J. J., D. W. Carraher, D. W., & Blanton, M. L. (2008). *Algebra in the early grades*. New York: Lawrence Erlbaum.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics: A quest for coherence*. Reston, VA: Author.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Washington, DC: Author. Retrieved from http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Shin, N., Stevens, S. Y., Short, H., & Krajcik, J. (2009, June). Learning progressions to support coherence curricula in instructional material, instruction, and assessment design. In *Learning Progressions in Science (LeaPS) Conference, Iowa City, IA.*

Van deWalle, J. A., Karp, K. S., & Bay-Williams, J. M. (2010). *Elementary and middle school mathematics: Teaching developmentally, Seventh edition*. Boston: Allyn & Bacon

Table 3.
Mean (SD) performance on "big ideas" by treatment and general mathematics assessment achievement.

	EEEI				FT			
Intervention	3 Pre	3 Post	4 Post	5 Post	3 Pre	3 Post	4 Post	5 Post
Tier 1	.39	.82	.85	.80	.38	.83	.83	.73
	(.15)	(.15)	(.10)	(.26)	(.20)	(.15)	(.18)	(.23)
Tier 2	.30	.76	.78	.74	.28	.77	.67	.69
	(.14)	(.13)	(.14)	(.20)	(.22)	(.17)	(.24)	(.23)
Tier 3	.18	.61	.64	.60	.19	.63	.46	.49
	(.14)	(.20)	(.24)	(.20)	(.15)	(.16)	(.18)	(.20)
Comparison								
Tier 1	.37	.40	.64	.52	.40	.48	.41	.38
	(.20)	(.20)	(.09)	(.23)	(.12)	(.17)	(.08)	(.24)
Tier 2	.24	.35	.49	.48	.31	.41	.38	.43
	(.13)	(.17)	(.16)	(.13)	(.19)	(.14)	(.10)	(.15)
Tier 3	.17	.22	.38	.34	.26	.38	.28	.37
	(.11)	(.18)	(.20)	(.18)	(.18)	(.19)	(.14)	(.20)
	EEEI, FT, and V				GA			
Intervention	3 Pre	3 Post	4 Post	5 Post	3 Pre	3 Post	4 Post	5 Post
Tier 1	.10	.69	.81	.92	.23	.64	.67	.78
	(.12)	(.37)	(.27)	(.26)	(.19)	(.22)	(.22)	(.29)
Tier 2	.02	.37	.78	.78	.25	.61	.63	.68
	(.14)	(.28)	(.33)	(.27)	(.16)	(.20)	(.25)	(.22)
Tier 3	.05	.23	.43	.54	.15	.45	.47	.65
	(.12)	(.27)	(.35)	(.33)	(.19)	(.22)	(.25)	(.22)
Comparison								
Tier 1	.04	.17	.54	.54	.22	.31	.47	.50
	(.16)	(.18)	(.40)	(.35)	(.21)	(.26)	(.25)	(.30)
Tier 2	.08	.10	.26	.54	.11	.26	.41	.49
	(.08)	(.22)	(.35)	(.39)	(.20)	(.25)	(.24)	(.19)
Tier 3	.02	.04	.26	.35	.18	.25	.38	.37
	(.09)	(.11)	(.38)	(.39)	(.18)	(.20)	(.24)	(.21)

Table 3 continued.

	PR			
Intervention	3 Pre	3 Post	4 Post	5 Post
Tier 1	.14	.36	.43	.64
	(.36)	(.50)	(.51)	(.50)
Tier 2	0	.16	.26	.32
	(0)	(.37)	(.45)	(.48)
Tier 3	.03	.21	.15	.27
	(.12)	(.41)	(.36)	(.45)
Comparison				
Tier 1	0	.14	.43	.57
	(0)	(.38)	(.40)	(.53)
Tier 2	.10	.13	.23	.30
	(.31)	(.35)	(.35)	(.47)
Tier 3	.07	.13	.13	.13
	(.26)	(.35)	(.35)	(.35)