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Hypothesizing Fragmented Growth of Mathematical Proficiency for Teaching

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Perspectives

The quality of teachers' mathematical knowledge for teaching (MKT) has become an important area of interest internationally (An, Kulm, & Wu, 2004; Cai, 2005; Ma, 1999; Schmitdt, Houang, & Cogan, 2011), with several US reports arguing that teachers with strong content-related knowledge in every classroom is the key to improving lagging US student performance and maintaining a globally competitive STEM workforce (Conference Board of the Mathematical Sciences, 2001; RAND Mathematics Study Panel, 2002). Teachers' knowledge does not develop in isolation, but knowledge develops along side beliefs about mathematics and teaching (Kilpatrick, Swafford, Findell, 2001). We focus on three belief constructs that comprise motivation for teaching: anxiety, self-concept of ability, and value. Following Kilpatrick and colleagues, we call the intertwined knowledge and beliefs mathematical proficiency for teaching (MPT). One important open question is how changes in one aspect of MPT such as MKT are related to changes in others, such as motivation for teaching.

We hypothesize that novice teachers' MPT is likely to be compartmentalized and topic-specific, because of novice-expert research in cognitive science that has demonstrated striking differences in how novices and experts organize and use their knowledge. Novices tend to perceive surface features of problems in a domain rather than attending to the deep structure perceived by experts (Chase & Simon, 1976; Chi et al., 1981). This hypothesis highlights the

difficulties novice teachers may face developing MPT from learning opportunities in teacher preparation. If novice teachers' MPT is specific and bounded by the mathematical context in which it was learned, then the demands on teacher education are much greater than is now understood. The critical question is whether novice teachers are able to develop motivation and MKT that transcends the particular examples they encounter in teacher preparation.

In this paper we report a study of change in elementary preservice teachers' MPT for multidigit addition and subtraction as a result of a mathematics methods course they took. We addressed two research questions:

- *RQ1:* How do prospective elementary teachers' reported opportunity to learn MKT, their assessed MKT, and their related motivation beliefs differ after taking a methods class?
- *RQ2:* Are changes in knowledge and motivation for specific MKT items consistent with opportunity to learn individual MKT items or with opportunity to learn MKT more generally?

Methods and Data

In this study we developed a new assessment instrument that focused on the topic of multidigit addition and subtraction and is sensitive to item-specific change in MKT and motivation. We paired each multiple choice MKT item with three rating questions, one for each of the motivation beliefs: anxiety, self-concept of ability, and value. For example, the anxiety rating item was, "I often feel nervous when I answer questions like this one." In a preliminary study, we used a sample of 119 preservice teachers to assess the psychometric properties of the new instrument that included a unidimensional MKT scale and motivation scales. We used item response theory to assess the 28-item MKT instrument, and the instrument displayed no evidence

of item or person misfit and had high reliability. The three motivation scales were also highly reliable ($\alpha_{anxiety} = .98$, $\alpha_{self\text{-}concept} = .94$, and $\alpha_{value} = .96$). Finally, for each item we asked about opportunity to learn using this rating question: "During my coursework, I've had the opportunity to learn this concept."

In the present study, a group of 40 preservice elementary teachers who had a range of mathematical ability (as assessed informally by their instructor) were invited to take the pretest. Of these 36 did, and 33 also completed the posttest at the end of the semester. We report data from the 33 preservice teachers who completed both the pretest and the posttest.

The data for each participant included overall pre- and posttest scores for MKT, anxiety, self-concept, value, and opportunity to learn. In addition, 14 MKT items were included on both the pre- and posttest. Each of these 14 items provided an item-specific MKT score (whether the answered was correct or not) as well as item-specific ratings of the preservice teachers' anxiety, self-concept, value, and opportunity to learn. We used paired t-tests to analyze the statistical significance of observed changes in overall scores for MKT, motivation beliefs, and opportunity to learn. We use descriptive analysis to examine change at the item level.

Results

In response to the first research question, we found a statistically significant increase in prospective elementary teachers' opportunity to learn MKT for multidigit addition and subtraction ($t = 4.41$, $df = 32$, $p < 0.01$) and in their MKT ($t = 2.70$, $df = 32$, $p = 0.01$; see also Figure 1, top panel), but we found no statistically significant difference in anxiety ($t = 0.43$, $df = 32$, $p = 0.67$), self-concept ($t = 1.90$, $df = 32$, $p = 0.07$), or value ($t = 0.08$, $df = 32$, p-value = 0.94). Given the high reliability and other strong psychometric characteristics of the scales, these

results suggest that the methods class provided the prospective teachers with opportunities to learn MKT and that these opportunities lead to change in knowledge but not in motivation.

The findings for the first research question rely on the assumption that novice teachers' MKT is a connected, coherent domain. Our second research question investigated this assumption. To examine the consistency of item level changes in knowledge and motivation with increased opportunity to learn individual items, we used line plots of change in responses on the 14 items that appeared on both the pretest and posttest (Figure 1, middle and bottom panel). The first line plot compared the change in opportunity to learn with the change in knowledge, and the second compared the change in opportunity to learn with change in anxiety, self-concept, and value. Change was computed as the mean individual pre- to posttest difference for each item in terms of item specific standard deviation.

The line plot for opportunity and knowledge (Figure 1, middle panel) shows that there was only an increase in knowledge on items for which there was also a large increase in opportunity to learn (> 0.2 SD). None of the items with small increases in opportunity to learn (< 0.2 SD) had large increases in knowledge and all of the items with large increases in knowledge (> 0.2 SD) also had large increases in opportunity to learn (> 0.2 SD). This pattern suggested that item specific opportunity to learn is necessary but not sufficient for item specific change in MKT.

The line plot for opportunity and motivation (Figure 1, bottom panel) showed that the change in item specific anxiety and self-concept consistently followed item specific change in opportunity to learn. That is, anxiety and self-concept on an item improved to the extent that opportunity to learn that same item increased from the beginning to the end of the methods class.

With the exception of two items, the same pattern held for the value outcome. We concluded that increased motivation for item specific MKT was related to item specific opportunity to learn.

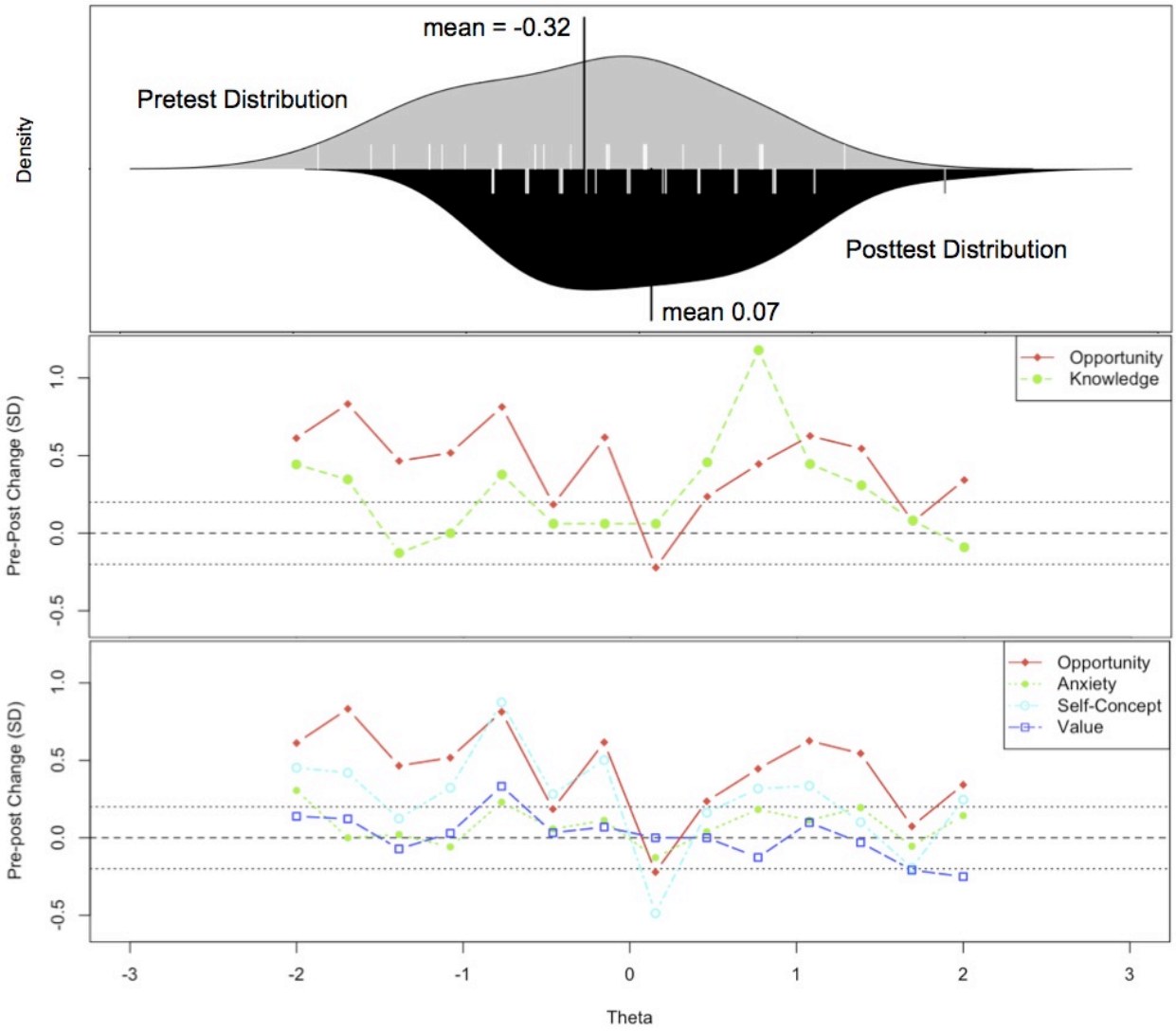

Figure 1. The top panel compares the pretest and posttest MKT distribution, the vertical line segments indicate individual responses (n=33). The middle panel compares the change in opportunity to learn with the change in knowledge on 14 items common to the pre-and posttest. The bottom panel compares change in opportunity with change in anxiety, self-concept, and value on the same 14 items. NOTE: The same theta scale orders individuals by ability (top panel) and items by difficulty (middle & bottom panels); the dotted lines indicate 0.2 SD bands around the zero-change line.

Significance

A central assumption of teacher education is that preservice teachers will be able to generalize what they learn from the specific topics and questions they discuss in a methods class to a larger domain of knowledge. This assumption is also built into item response theory, because individuals are hypothesized to have some amount of a general ability trait that is needed to answer the questions on a test. Those with more ability are able to answer more difficult items. In this study we hypothesized that novice teachers' MPT is fragmented, and we examined change in MKT and motivation for teaching multidigit addition and subtraction to discover whether this assumption appeared to be warranted.

Had we only looked at the overall scores (i.e., research question 1) we would have concluded that opportunities to learn in the methods class influenced MKT but not motivation. This conclusion is an example of the kind of conclusion that is based on the assumption of a coherent body of knowledge or beliefs. Looking more closely at the co-occurrence of changes in opportunity to learn and changes both in MKT and in motivation beliefs suggested another explanation in line with the hypothesis of fragmented novice MPT. Our results suggest that preservice teachers may only learn MKT and develop motivation for the specific examples they encounter during teacher education. More research is needed on how generalization that leads to mathematical proficiency for teaching in a general way might take place in teacher education.

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